Our goal: Understand and compute $W$ with morphisms $\text{blow up}$ in order to answer the Big Q.

Easiest way to work $\text{w/}$ an algebra = generators + relations. Start with something abstract, like $\text{HTT}$.

Choose symbols for certain elements — i.e. $T_H$. Now can form $[\text{words}]$ $T_H T_H T_H$ (4 linear combos).

Relations are rules for replacing certain words $\text{w/}$ others, i.e. $T_H T_H = (T_H + T_H T_H)^2$.

Key technology & words: What makes it tick? Associativity + (unit) axiom.

But now look at a monoidal category... words aren't the right thing!! $\text{If: } A \to B$

Then $(\text{id}_A, 1_B) : A \times B \to B$ and your brain hurts. 2 kinds of composition...

...need some kind of planar diagram,

planar diagrams are the correct tool for the job! $\text{It's a notational convention, but as modality}$

Analogy: Words for categories (algebra w/ multiple objects)

planar diagram for 2-categories

Linear diagrams for 1-categories

Our convention is the dual of what you're used to,

Old way: $P \to N \leftarrow M$

"dual": $P \leftarrow M \to N$

In picture: A (generic) point is actually an object.

A ( $\text{\mathbf{n}}$ ) interval is a morphism $\text{\mathbf{f}}$.

Composition $[\text{Diagram}]$ $\rightarrow$ identity $[\text{Linear isotropy}]$

Axioms of category $\iff$ diagram $\rightarrow$ linear isotopy

Unambiguously represents a morphism. (positioning is irrelevant)

Like if $f$ represents a brown map $g$ then $g \circ f$ is also brown.
First, some examples of 2-cats:

**CAT**
- **Ob**: categories
- 1-mor: functors
- 2-mor: natural transformations

**BIM**
- **Ob**: rings
- 1-mor: bimodules
- 2-mor: bimodule maps

**Example in BIM**:

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Also, have map

\[ R \xrightarrow{f} R \]

Excuse as \( \mu_R R \)

Then

\[ Envelope \]

\[ \begin{array}{c}
\text{is a bimorphism of } B_5, \\
\text{and is } R \end{array} \]

Meanwhile,

\[ \begin{array}{c}
\text{is some bimorphism of } B_5, \\
\text{and is } R \end{array} \]

But these are equal. No accident!

Axioms of 2-cats \( \leftrightarrow \) Diagram / Restrilinearization unambiguously represents a 2-morphism.
Ex: $ABC$ a prob. ext

Equipped with 4 maps

Try better notation: $\text{id}_{\text{Ind}} = A$, $\text{id}_{\text{Res}} = B$

with this convention, we have a meaning for any connected labeled 1-manifolds!

Axioms of Fibr G~

Formalize: Whenever $E \cong E'$ we have chosen $1 \mapsto E \mapsto 1$, draw as

but if also $E \cong E'$, get $E$ as well.

However, sparse $E \cong E'$ $\rightarrow$ $F \cong F'$ $\leftarrow$ Nothing guarantees that $E \cong E'$

If they are equal, say $E$ is cyclic with the fixed order of adjunction. Draw with

left side as $\frac{1}{2}$, without this, could never impose $E$ in our isotopies.
If all 1-morphisms have fixed biadjoints AND all 2-morphisms are cyclic.

Then the axioms of biadjunction reduce to the property

\[ \Rightarrow \text{Diagram is isomorphic to a 2-morphism.} \]

Given such a thing, you **should** use diagrams, because isomorphisms make your life easier!

Rekt: When "taking biadjoints" is functorial, all 2-morphisms are automatically cyclic.

Common situation in geometry and representation.

Ex. Full Rep of \( \mathbb{C} \times \mathbb{C} \) or biadjunct \( \mathcal{V} \) is a functor.

\( \mathcal{OB} \) is such a monoidal category.