We defined the Orlik-Solomon algebra of a matroid $M = (E, I)$:

$$\text{OS}^*(M) := \Lambda_{\mathbb{C}}[x_e \mid e \in E]/\left\langle \sum_{i=1}^{k} (-1)^{i} x_{e_1} \cdots \hat{x}_{e_i} \cdots x_{e_k} \mid \{e_1, \ldots, e_k\} \notin I \right\rangle.$$ 

We showed that, if $M$ is $\mathbb{C}$-realizable, the Orlik-Solomon algebra of $M$ comes with a natural homomorphism to the cohomology of the complement of the corresponding hyperplane arrangement. We stated (but did not prove) that this homomorphism is an isomorphism.

1. Show that, if $\{e_1, \ldots, e_k\} \notin I$, then $x_{e_1} \cdots x_{e_k} = 0 \in \text{OS}^*(M)$.

2. Show that the ideal of relations in the Orlik-Solomon algebra is in fact generated by elements corresponding to circuits.

**Bonus Exercise**

3. Write down a presentation for the cohomology ring of the configuration space of 4 distinct points in $\mathbb{C}$. Your presentation should have $\binom{4}{2} = 6$ generators and $7 = 4 + 3$ relations corresponding to circuits of size 3 and 4 for the matroid $M(K_4)$. Can you show that the first 4 relations are in fact sufficient?