The Wonderful Geometry of Matroids
Lecture 13: Putting it together

In the last two lectures, we left two things undone: the proof that the differential in the long exact sequence is zero, and the proof that the natural map from the Orlik-Solomon algebra to the cohomology ring is an isomorphism. In this lecture we showed how to prove these two statements simultaneously, as part of one big induction.

Sadly, we left out the proof of the most technical step, which is to show that we have a short exact sequence of Orlik-Solomon algebras. Proving this involves constructing the “no-broken-circuit basis” and studying the maps with respect to this basis.

As a corollary of this technical step, we know that the coefficients of the characteristic polynomial of $M$ are (up to sign) dimensions of graded pieces of the Orlik-Solomon algebra of $M$, even when $M$ is not $\mathbb{C}$-realizable.

The only exercise today is to finish the exercises from Lecture 12.