The Wonderful Geometry of Matroids

Lecture 28: The Kazhdan–Lusztig polynomial of a matroid

We defined the Kazhdan–Lusztig polynomial of a matroid, which coincides with the intersection cohomology Poincaré polynomial of the reciprocal plane in the realizable case. We gave a full proof that it is well-defined for arbitrary matroids.

1. Show that the constant term of the Kazhdan–Lusztig polynomial of a matroid is always 1.

2. Show that the coefficient of t in the Kazhdan–Lusztig polynomial is equal to the number of flats of corank one minus the number of flats of rank one. (We did this for realizable matroids in the previous lecture.)

3. Can you derive an analogous formula for the coefficient of t^2?

Hint: Your formula should involve counting flags of flats of length 2 with certain prescribed ranks. More generally, there is a formula for the coefficient of t^k that involves counting flags of flats of length k. This formula will have $2 \cdot 3^{k-1}$ terms, half of which are positive and half of which are negative.