The Wonderful Geometry of Matroids

Lecture 3: Direct sum and truncation

In this lecture we defined the matroid operations of direct sum, which we interpreted for vector configurations and graphs, and truncation, which we interpreted for vector configurations.

1. For any $n \geq d \geq 0$, let $U_{n,d} := \operatorname{tr}^{n-d} \operatorname{Boo}_n$ be the matroid obtained from the Boolean matroid of rank n by truncating $n-d$ times. This is called a **uniform matroid**.

(i) Describe the independent sets, bases, circuits, flats, and rank function of $U_{n,d}$.
(ii) Show that $U_{n,n-1}$ is isomorphic to $M(C_n)$.

2. Let K_5 be the complete graph on 5 vertices. Show that $\operatorname{tr} M(K_5)$ is not isomorphic to $M(G)$ for any graph G. In particular, there is no purely graphical interpretation of truncation.

Hint: How many vertices would G have to have (assuming that G is connected)? How many edges? Would G have any loops or parallel edges?