Emily Clader

Title: Ehrhart Fans

Abstract: For any complete unimodular fan, the Euler characteristic provides a linear map from the K-ring of vector bundles on the associated toric variety to the integers, which can be combinatorially encoded in terms of lattice-point counts in certain polytopes. On the other hand, recent work of Larson, Li, Payne, and Proudfoot constructs an Euler characteristic on Bergman fans of matroids, despite the fact that these fans are incomplete. Motivated by the structural properties of lattice-point counts, we introduce a new class of fans whose K-rings admit a canonical linear map to the integers, which includes complete unimodular fans and Bergman fans of matroids as special cases, and thus creates a general framework for studying Euler characteristics of matroids and smooth proper toric varieties together. This is joint work with Melody Chan, Caroline Klivans, and Dusty Ross.

Martin Ulirsch

Title: From tropical linear algebra to vector bundles

Abstract: In this talk I will explain how our perspective on tropical linear algebra shapes our understanding of tropical vector bundles. An elementary approach to this story is based on tropical matrices. In this case we find an elementary and geometrically appealing theory of tropical vector bundles, which allows us to give a satisfying treatment of the process of tropicalization in abelian situations, e.g. in the case of the Tate curve or for semihomogenous vector bundles on abelian varieties. Expanding on these developments, as a first step towards the more general non-abelian situation, I will outline a framework to functorially tropicalize linear maps between finite-dimensional vector spaces using the geometry of affine Bruhat–Tits buildings. This will provide us with a pathway to study the tropical geometry of vector bundles on more general base spaces.

Most of the new results in the talk will be based on joint works with A. Gross and D. Zakharov; A. Gross, A. Kuhrs, and D. Zakharov; I. Kaur, A. Gross, and A. Werner; as well as with L. Battistella, K. Kuehn, A. Kuhrs, and A. Vargas.

Kris Shaw

Title: Tropical homology in geometry and combinatorics

Abstract: In this lecture I will survey partial results and open problems surrounding the questions:

- When do tropicalisations know something about the cohomology and Hodge structures of algebraic varieties? Can tropical homology theories be useful in studying objects arising in combinatorics?

- Tropical homology was introduced as a way to recover information about mixed Hodge structures of families of complex varieties under certain conditions. It has also found applications to the topology of real algebraic varieties arising from Viro's patchwoking. In addition, this homology theory can be applied to combinatorial objects far from the world of algebraic geometry and tropicalisations, for example matroids and non-regular subdivisions, leaving many applications and open questions in the realm of combinatorics.

Daniel Erman

Title: Long Live the King (Conjecture)

Abstract: The King Conjecture proposed that every toric variety has a full, strong exceptional collection of line bundles. While the conjecture turned out to be false, it has continued to inspire much research on derived categories and toric varieties. I will explain how King's Conjecture can be remedied, and proven, if one incorporates a birational geometry perspective. My talk will cover joint work with Ballard, Berkesch, Brown, Cranton Heller, Favero, Ganatra, Hanlon, and Huang.

Sam Payne

Title: Combinatorial and motivic structures in unstable cohomology

Abstract: I will survey recent advances in understanding unstable cohomology groups of moduli spaces of curves and abelian varieties. The proofs involve an intricate interplay between algebraic geometry, topology, and arithmetic, governed by the combinatorial and inductive structure of the boundaries of suitable compactifications. The results reveal the advantages of studying the cohomology of these moduli spaces "one motive at a time"; each irreducible motivic structure appears in combinatorial patterns described in terms of graph complexes and Hopf algebras.

Dhruv Ranganathan

Title: Logarithmic GW/DT and tropical geometry

Abstract: The Gromov-Witten/Donaldson-Thomas correspondence, introduced by Maulik-Nekrasov-Okounkov-Pandharipande in 2003, is a series of conjectures relating two enumerative geometric theories associated to projective threefolds. The correspondence reflects the relationship between two ways to study curves: via equations and via parameterizations. I will explain how to extend this conjecture to threefold pairs, or "logarithmic" targets, and also how this allows the original conjectures to be studied via normal crossings degeneration. I will then outline a proof of this new logarithmic conjecture in the toric setting, where tropical methods play a crucial role, and discuss implications for GW/DT correspondence. Based on joint work with Davesh Maulik.

Chris Eur

Title: How matroids behave like smooth projective toric varieties

Abstract: Matroids are combinatorial abstractions of linear subspaces. Hodge theory of matroids showed that matroids satisfy a Chow-theoretic positivity. We show a K-theoretic positivity for matroids by establishing strong cohomology vanishing properties for matroids. Our result in particular resolves the 20-year old f-vector conjecture of Speyer. Forthcoming joint work with Alex Fink and Matt Larson.

Renzo Cavalieri

Title: From Hurwitz numbers to log Chow

Abstract: Double Hurwitz numbers count ramified covers of the projective line with prescribed profiles over 0 and ∞. In this talk I will review work that, on the one hand, develops a combinatorial framework to compute Hurwitz numbers in terms of a sum of weighted graphs, and on the other it connects these enumerative invariants to the tautological intersection theory of the logarithmic DR cycle. This perspective then opens up a path to new enumerative geometric problems. This is based on joint work with H. Markwig, D. Ranganathan and J. Schmitt.

Prakash Belkale

Title: Quantum Schubert Calculus and Rigid Local Systems

Abstract: Quantum Schubert calculus describes the small quantum cohomology rings of Grassmannians. It is related to representation theory (ranks of conformal blocks, equivalently of global sections of line bundles on the moduli of parabolic bundles on the projective line) by results of Gepner and Witten. Since Grassmannians have an evident symmetry (interchanging ranks of subs and quotients), these results give rise to a strange duality phenomenon for ranks of line bundles on moduli of parabolic bundles. I will describe the following recent results (here a rigid local system is an irreducible local system on a multiply punctured Riemann sphere $\mathbb{P}^1 - \{p_1, \ldots, p_s\}$ which is uniquely determined by its local monodromies, classical examples come from the Euler/Gauss hypergeometric functions).

(1) We give a construction which produces irreducible complex rigid unitary local systems via quantum Schubert calculus and strange duality. In particular, certain enumeratively defined codimension one loci in the moduli of parabolic bundles (e.g. a locus of flags for which an unexpected (by dimension counts, and by one) intersection of Schubert varieties in a Grassmannian is non-empty) give rise to all unitary rigid local systems on the projective line. The local monodromies and the corresponding multiplicities can be determined explicitly using Schubert calculus.

(2) Answering a question of Nicholas Katz, we show that there are no irreducible rigid local systems of rank greater than one, with finite global monodromy , whose local monodromies have orders dividing n, when n is a prime number.

(3) All unitary irreducible rigid local systems with finite local monodromies arise as solutions to the Knizhnik-Zamalodchikov equations on conformal blocks for the special linear groups.

Matt Baker

Title: Band schemes and moduli spaces of matroids

Abstract: We introduce a generalization of commutative rings called bands, along with the ry corresponding geometric theory of band schemes. Among other things, band schemes provide a new viewpoint on tropical geometry and Berkovich analytifications, and they enable a partial explanation for phenomena observed by Jacques Tits concerning algebraic groups over the "field of one element". Band schemes also furnish a natural algebro-geometric setting for studying matroid theory; in particular, they allow us to construct a moduli space of matroids, and they provide new tools for studying realization spaces of matroids.

Aaron Pixton

Title: Counting boundary strata and tautological relations

Abstract: The tautological ring of the moduli space of smooth curves of genus g is the subring of its Chow ring generated by the kappa classes. When g = 3d-1, there is a unique "earliest" relation between the kappa classes, in codimension d (first discovered by Ionel in 2003). It turns out that the coefficients of this relation have a natural interpretation as counting dimension 0 boundary strata in moduli spaces of stable curves. Later relations between the kappa classes can also conjecturally be interpreted in terms of other boundary strata counts (making use of work of Yin from 2014). After explaining these connections, I will then discuss tautological relations on the universal Jacobian. Part of this talk presents joint work with Y. Bae.

Chris Manon

Title: Toric tropical vector bundles

Abstract: A toric vector bundle is a vector bundle over a toric variety which is equipped with a lift of the action action of the associated torus. As a source of examples, toric vector bundles and their projectivizations provide a rich class of spaces that still manage to admit a combinatorial characterization. Toric vector bundles were first classified by Kaneyama, and later by Klyachko using the data of decorated subspace arrangements. Klyachko's classification is the foundation of many interesting results on toric vector bundles and has recently led to a connection between toric vector bundles, matroids, and tropical geometry.

After explaining some of this background, I'll introduce the notion of a tropical toric vector bundle over a toric variety. These objects are discrete analogues of vector bundles which still have notions of positivity, a sheaf of sections, an Euler characteristic, and Chern classes. Finally, I'll explain how each matroid determines a tautological tropical vector bundle over the permutahedral toric variety. I'll discuss some properties of these bundles, and I'll show that the characteristic classes of the tautological bundle recover the tautological classes of matroids used by Berget, Eur, Spink, and Tseng to prove log-concavity properties of the Tutte polynomial.