Let M be a matroid. The Kazhdan-Lusztig polynomial $P_M(t) \in \mathbb{Z}[t]$ was introduced in [EPW16], and the closely related Z-polynomial $Z_M(t) \in \mathbb{Z}[t]$ was introduced in [PXY18]. Kazhdan-Lusztig polynomials of matroids are neither special cases nor generalizations of classical Kazhdan-Lusztig polynomials. Rather, both classes of polynomials are special cases of Kazhdan-Lusztig-Stanley polynomials; see [Pro18] for more details. The following conjecture appears in [GPY17b, 3.2] and [PXY18, 5.1].

Conjecture 1. *The polynomials $P_M(t)$ and $Z_M(t)$ are real rooted.*

We also have various conjectures that say that the roots of the Kazhdan-Lusztig polynomials or Z-polynomials of various different matroids should interlace. For the conjectural statement about Kazhdan-Lusztig polynomials, see [GPY17b, 3.4 and 3.5]. The statement about Z-polynomials is cleaner. It does not appear in any paper, but it generalizes the statement in [PXY18, 5.2].

Conjecture 2. *If M' is obtained by contracting M along a flat of rank 1, then polynomials $Z_M(t)$ and $Z_{M'}(t)$ have interlacing roots.*

If there is a finite group W acting on M, then these polynomials have equivariant analogues $P_M^W(t)$ [GPY17a] and $Z_M^W(t)$ [PXY18], whose coefficients isomorphism classes of representations of W. We can then formulate equivariant versions of Conjectures 1 and 2. The equivariant analogue of being real rooted is the statement that the minors of the Toeplitz matrix are honest (rather than virtual) representations. The equivariant analogue of (strictly) interlacing is that the principal minors of the Bézoutiant are honest representations.

Conjecture 3. *The polynomials $P_M^W(t)$ and $Z_M^W(t)$ are equivariantly real rooted.*

Conjecture 4. *If M' is obtained by contracting M along a flat of rank 1, then polynomials $Z_M^W(t)$ and $Z_{M'}^W(t)$ equivariantly interlace.*

References

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