Flip through a calculus textbook, or online references, to do the following problems:

**(1)** Let $p$ be a real number strictly between 0 and 1, and let $a_n = cp^n$ for all integes $n \geq 0$. If $\sum_{n=0}^{\infty} a_n = 1$, what is the value of $c$?

**(2)** Using the fact that

$$f(x) = (1+x)^n = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} x^k,$$

(the *Binomial Theorem*) find the value of

$$\sum_{k=0}^{n} k \frac{n!}{k!(n-k)!} x^{k-1}.$$

*Hint:* differentiate $f$.

**(3)** Sketch the curve $y = \log(1-x)$ and its tangent at $x = 0$, where log denotes the natural logarithm (sometimes written ln). Explain intuitively or quote a theorem that explains why there exists a number $C > 0$ such that

$$|\log(1-x) - x| \leq Cx^2$$

for all $|x| \leq \frac{1}{2}$.