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University of Oregon; Fall 2007

Physics 351 – Vibrations and Waves

Problem Set 1

Due date: Wednesday, Oct. 3, 10 a.m.

Reading: French, Chapters 1-2, Dimensional Analysis notes

Comments. This assignment spans three topics:

(*i*) Simple harmonic motion
(*ii*) Math, especially complex numbers – hopefully a review of things you've learned before
(*iii*) Dimensional analysis

If the answer to a problem isn't immediately apparent, relax – this is, in fact, how most problems beyond elementary first-year exercises are supposed to be. (And it will be especially true later in the course.) Spend time *thinking*.

(1, 4 pts. total**) Simple harmonic motion.**

(a, 1 pt.**)** Verify that $x(t) = A\sin(\omega t) + B\cos(\omega t)$, with $\omega = \sqrt{k/m}$, is a solution to the differential equation of motion for a mass *m* attached to a spring of spring constant *k*, as discussed in class.

(b, 1 pt.**)** Verify that $x(t) = C\sin(\omega t - \varphi)$ is also a solution.

(c, 2 pts.**)** I claimed that a second order differential equation – i.e. one that involves a second derivative of $x(t)$ – has a general solution that has two arbitrary parameters, for example *A* and *B* in part (a), or *C* and φ in part (b). Mr. K, who never took a course on differential equations, claims that he's found a new, more general solution with three parameters, denoted *D*, *E*, and *F*: $x(t) = D\sin(\omega t) + E\cos(\omega t - F)$. Is his $x(t)$ a solution to our differential equation? If not, what's wrong with it? If so, why shouldn't we use it as a more general solution?

(2, 3 pts. total) **Taylor series.**

(a, 1 pt.**)** Derive the Taylor series expansion, up to the x^5 term, for $\sin(x)$ around x=0.

(b, 2 pts.**)** Derive the Taylor series expansion, up to the third term, for $y(x) = \frac{1}{\sqrt{x}} + x$ around the point x_0 at which $y(x)$ is minimal. (First determine x_0.) (Consider only x>0.)

(3, 2 pts. total**) French, Problem 1-3.** Note that you're asked to prove *two* things for any complex number *z* multiplied by $e^{j\theta}$ (where $j = \sqrt{-1}$). First, that the multiplication rotates *z* by angle θ (with respect to the real axis), and second that the multiplication leaves the length of *z* unchanged. Start by writing *z* as $z = a + jb$. Always remember: exp(*j*x) = cos(x) + *j* sin(x). You may find the following identity useful:

$$\tan(u+v)=\frac{\tan u+\tan v}{1-\tan u\tan v}$$

(4, 1 pt.**) A trigonometric identity.** Knowing how to derive trigonometric identities is a useful skill – complex notation for sines and cosines removes much of the headache. As above, note that

$\exp(jx) = \cos(x) + j\sin(x)$; (where $j = \sqrt{-1}$)

$\cos(x) = (1/2)\,[\exp(jx) + \exp(-jx)]$ and $\sin(x) = (1/2j)\,[\exp(jx) - \exp(-jx)]$.

Show that $\sin(u \pm v) = \sin(u)\cos(v) \pm \cos(u)\sin(v)$;

(5, 3 pts.**) French, Problem 1-4a.**

(6, 4 pts.**) French, Problem 1-9.** A good first step is to convert j^j from a power of j to a power of *e*. Recall the algebraic properties of logarithms: if $a^x = b^y$, what is y as a function of a, b, and x? (*Hint*: take logarithms of both sides of the equation.)

(7, 8 pts. total**) French, Problem 1-11.** (a, 4pts; b, 4pts)

(8, 7 pts. total**) Dimensional Analysis – membrane waves.** In class we saw movies of height fluctuations of two-dimensional lipid membranes. We can use dimensional analysis to derive an approximate expression for the period of a fluctuation as a function of its wavelength. Let's assume the timescale is determined by three physical parameters: the wavelength (λ), the rigidity of the membrane, and the viscosity (η) of the fluid (water) in which the membrane exists.

(a, 2 pts**)** The energy required to deform a membrane is a function of the membrane's rigidity (κ) and the radii of curvature (r) of the membrane along two "principle" axes (r_1 and r_2, each with units of length). One integrates the product of rigidity and the square of the sum of the inverses of the radii of curvature throughout the membrane area to calculate the energy (E):

$$E=\int \kappa\left(\frac{1}{r_1}+\frac{1}{r_2}\right)^2 dA$$

Show that κ has the same dimensions as Energy.

(b, 1 pt.**)** Viscosity (η) describes a fluid's resistance to shear. The force needed to drag a sphere slowly through a fluid is proportional to the product of its radius, its velocity, and the fluid's viscosity. Show that η has dimensions of Mass / [Time × Length]

(c, 2 pts**)** Show that the timescale (τ) for membrane deformations must depend on λ, η, and κ by a relation of the form: τ = (dimensionless constant) × $\eta\lambda^3 / \kappa$.

This is, in fact, correct for a free membrane. The numerical prefactor happens to be $(1/2\pi^3)$.

(d, 2 pts**)** For a membrane near a surface, the mean distance to the surface (z) affects the dynamics. How does adding z to our list of parameters affect our ability to create a unique expression for τ ?

[Survey, optional] Roughly how many hours did you spend on this assignment?