

## Physics 351 – Vibrations and Waves

### Problem Set 2

**Due date:** Friday, Oct. 12, 5pm.

**Reading:** French – Chapter 2 and Chapter 3 through page 54

(1, 3 pts.) **French, Problem 2-3.**

(2, 4 pts. total) **A vertical mass & spring setup.** We've considered in class a horizontal setup of a mass on a spring, in which we could neglect the role of gravity. Now, consider a vertical setup. A mass  $m$  hangs from a massless spring of stiffness  $k$ , in a gravitational field whose acceleration is  $g$ . Gravity leads to extension of the spring – at static equilibrium (i.e. if the mass isn't moving), the gravitational force exactly balances the spring's restoring force, and the spring is extended by some length. Now consider a moving mass: Show that vertical oscillations of the mass about the equilibrium point have angular frequency  $\omega = \sqrt{k/m}$ , just like the horizontal case. In other words, determine the equation of motion for the displacement relative to the static extended length, noting that both gravity and the spring's restoring force act on the mass, and show that  $\omega = \sqrt{k/m}$ .

(3, 7 pts.) **A rolling ball.** A ball of mass  $m$  moves in a one-dimensional landscape of hills and valleys with height  $h$  as a function of lateral position  $x$  being given by the function  $h(x) = \frac{a}{\sqrt{x}} + bx$ , where  $a$  and  $b$  are constants. The system is in a uniform gravitational field with acceleration  $g$ , as usual. Consider only  $x > 0$ .

(a, 2 pts.) Given the behavior of  $h(x)$  as  $x \rightarrow \infty$  and  $x \rightarrow 0$ , and the existence of any local extrema, sketch **by hand**, the shape of  $h(x)$ . Exact values of  $h(x)$  aren't important; the general shape is.

(b, 1 pt.) The ball rolls back and forth about a stable equilibrium point. How many such equilibrium points are there?

(c, 4 pts.) For any equilibrium points you found in (b), determine the period  $T$  of small oscillations of the ball about equilibrium. Recall the lectures of Week 1, in which we discussed why "any" oscillation is harmonic. Make sure the dimensions of your answer are correct, first determining the dimensions of constants  $a$  and  $b$ .

(4, 4 pts.) **A spider.** Consider a spider hanging from a silk line of diameter  $d$  approx. 0.1 mm. Like a pendulum, the spider sways back and forth. It also bounces up and down, since the silk is elastic, stretching and compressing like a spring. We find that the period of the back-and-forth swaying is 200 times that of the up-down oscillations. From this information, estimate the Young's modulus,  $Y$ , of spider's silk. First express your answer symbolically, relating  $Y$  to the relevant parameters. Also, estimate the actual numerical value. Make some reasonable assumption about the mass of the spider – you may find it useful to recall that the density of water is 1 gram / cm<sup>3</sup>.

(5, 4 pts.) **French, Problem 3-3a.** Think carefully about why the block leaves the platform. (Think about first-year mechanics.) Try experimenting: hold something in your hand, raise and lower your hand, and ask yourself why it flies off or doesn't fly off.

~~(6, 6 pts.: 1,1,1,3 for each part) **French, Problem 3-17.** Note for part (a) that you'll need to think carefully about how to determine the potential energy relative to the static ( $y=0$ ) state. Since the fluid volume is conserved, if the fluid level rises to  $y$  in the left arm, what is it in the right arm? You may wish to first review the symmetric U-tube discussed in class and on p. 53-54. "Review" means read *and* work through all the steps yourself with pencil and paper.~~

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[Survey, optional] Roughly how many hours did you spend on this assignment?