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University of Oregon; Fall 2007

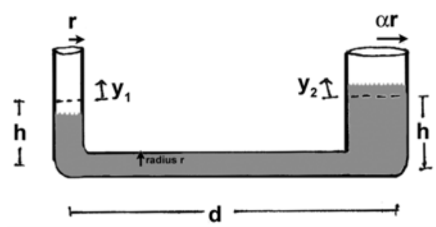
Physics 351 – Vibrations and Waves

Problem Set 3 – *Problem #2 modified Oct. 17, 2007*

Due date: Friday, Oct. 19, 5 pm.
Reading: French, Chapter 3

(1, 8 pts.**) A pendulum.** Consider a pendulum with a bob of mass *m* and a string of length *L*.
(a, 1 pt.**) Derive** the general solution to the equation of motion $\theta(t)$ (i.e. for arbitrary initial conditions). Determine the period as a function of g and L. Start by considering the force acting on the mass, applying Newton's laws, obtaining a differential equation, etc. Yes, I know we did this in class.
(b, 3 pts.**)** Derive the general solution to the equation of motion $\theta(t)$ by considering the **energetics** of the system – i.e. noting that energy is conserved and considering the kinetic and potential energies of the system. Drawing clear diagrams of the pendulum geometry will help. Again, obtain a differential equation, and find (and verify) its general solution.
(c, 2 pts.**)** The pendulum is set up such that at time t=0 it has zero velocity and is at angle θ_0 radians with respect to the vertical. What is $\theta(t)$ for these **particular** initial conditions?
(d, 2 pts.**)** Continuing the setup of part (c), suppose θ_0 = 15 degrees, m = 1kg, and L = 1 meter. How fast is the pendulum bob moving as it passes the lowest point of its arc? (Note that I'm asking for the actual speed, e.g. meters/second, not the angular velocity.)

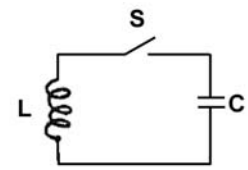
(2, 9 pts.**) A modified U-tube.** In class (and in the text) we considered oscillations of a fluid of density ρ about its equilibrium position in a U-shaped tube. Consider a modified U-tube, in which the left arm has radius r, the bottom tube has radius r, and the right arm has radius αr, where α is some constant. (See figure.) At equilibrium, water reaches to height *h* in each arm. The length of the base of the tube is *d*. (So in terms of our earlier simple U-tube, $L = d + 2h$). The water sloshes back and forth, its height being denoted by $y_1(t)$ in the left arm and $y_2(t)$ in the right arm (both defined as positive upwards from the equilibrium height – see Figure). As usual, ignore any complications due to the shape of the tube at the corners; consider the corners to be small compared to other lengths.



(a, 1 pt.**)** How are y_1 and y_2 related? (In other words, if the water level drops by some height in the left arm, how much does it rise in the right arm?)
(b, 3 pts.**)** What is the potential energy of the system? Express your answer as a function of y_1 and constant parameters only – i.e. removing any explicit dependence on y_2 using your result for part (a).
(c, 2 pts.**)** What is the kinetic energy? (Again, as a function of y_1 and constant parameters only.)
(d, 3 pts.**)** Consider **small oscillations** – i.e. $y_1 << h$ and $y_2 << h$. What is the angular frequency of oscillations, ω, for the system? Show also that your answer reduces to the ω of a simple U-tube (discussed in class) if α = 1. (Note: small oscillations mean that you should consider the smallest nonzero terms in y. So if you have an expression like $\left(1+\frac{y}{h}\right)\dot{y}$, this just becomes $\dot{y}$, neglecting the second term.)

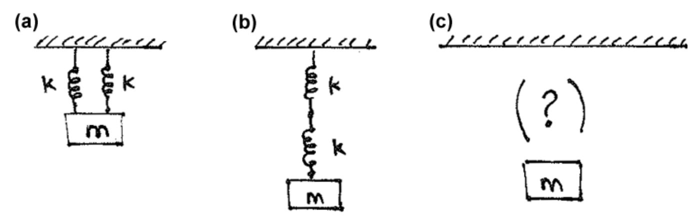
(3, 4 pts**) Average Energy.** Show that for a simple harmonic oscillator, e.g. a mass attached to a spring, described as usual by $x(t) = A\sin(\omega t-\varphi)$, the time-average of the kinetic energy and the time-average of the potential energy are both equal to $\frac{1}{4}kA^2$, where k is the spring constant and A is the amplitude. Note that you can just average expressions for kinetic and potential energy over one period, since the motion is periodic. (You may wish to remind yourself, via an elementary calculus text, how to calculate the average value of a function. Though there's a "quick" way to solve this problem doesn't require such a calculation...)

(4, 6 pts**) An LC circuit.** Oscillations form an important part of electrical circuit design. Let's consider a simple "LC circuit," in which an inductor (of inductance L) and a capacitor (of capacitor C) are arranged in series (see figure). Let's say there is no voltage supply in the circuit, but that the capacitor is initially charged to voltage V_0, and the switch (S) is closed, completing the circuit, at time t=0. Recall[1] that the total voltage drop (ΣV) as one traces any "loop" of any circuit is zero, the voltage across an inductor is $V_L = L\, dI/dt$, the voltage across a capacitor is $V_C = Q/C$, and the current flowing in the circuit $I = dQ/dt$ – the rate of change of the electrical charge.



(a, 2 pts**)** Show that the condition $\Sigma V = 0$ implies that Q is described by a differential equation that has the same form as our familiar "mass on a spring" equation.
(b, 2 pts**)** Show that the current (I) oscillates with an angular frequency $\omega = \sqrt{1/LC}$.
(c, 2 pts**)** Express Q(t) in terms of only L, C, and V_0 – i.e. incorporating the "initial conditions" to determine the amplitude and phase of the oscillations, as well as the frequency.

(5, 5 pts.**) Arrangements of Springs.** We know that a mass *m* hanging from a spring of spring constant *k* oscillates with angular frequency $\omega = \sqrt{(k/m)}$. (E.g., Problem Set 2.)



(a, 2 pts.**)** Determine the frequency of oscillation for a mass *m* hanging from two springs of spring constant *k* "in parallel," as shown in part (a) of the Figure. Think carefully about how force and extension are related – if you want to move the mass by some amount, how much force do you need to apply relative to the single spring case? If you find yourself doing much calculation, you're probably on the wrong track.
(b, 2 pts.**)** Determine the frequency of oscillation for a mass *m* hanging from two springs of spring constant *k* "in series," as in Figure (b). Think carefully: if you apply some force, how much will each spring stretch?
(c, 1 pt.**)** Think of an arrangement of **four** springs of spring constant *k* that will lead to the same oscillation frequency as that of a single spring, i.e. $\omega = \sqrt{(k/m)}$. Draw it.

(6, 3 pts.**) Critical Damping.** In class we stated (or will state) that a critically damped oscillator (i.e. one for which $\gamma = 2\omega_0$) is described by $x(t) = (A+Bt)e^{-\gamma t/2}$. Verify that this $x(t)$ is a solution to the equation of motion $\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = 0$.

[Survey, optional] Roughly how many hours did you spend on this assignment?

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[1] Familiarity with circuits is not a prerequisite for the course, though you've probable seen simple circuit elements like these in earlier classes. If not, don't worry: all the information you need is contained within the problem.