Prof. Raghuveer Parthasarathy
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Physics 353 – Statistical Mechanics

Study Guide for the Final Exam

NOTE: This list is intended only as an approximate guide to the topics with which you should be familiar, in preparation for the upcoming exam. There will certainly be topics listed here that are not present on the exam. And of course, this list won't spell out exactly everything that will show up in the midterm – it is meant only as an approximate guide.

Format of the exam: The exam (Tuesday, June 10) will be closed book / closed notes, and 2 hours long. Some information will be supplied to you (see below).

General advice: Study your homework assignments and notes. Be sure to study the *solutions* to homework problems you didn't understand. Understand the *derivations* by which we determined various expressions.

REMEMBER IMPORTANT RESULTS FROM LAST QUARTER, **FOR EXAMPLE:**

- The Boltzmann Distribution $p_i = \frac{1}{Z}\exp\left(-E_i/\tau\right)$, and how to use it.
- The partition function $Z = \sum_i \exp\left(-E_i/\tau\right)$, and how to use it.
- How to calculate the average value of any property: $\langle A \rangle = \sum_i A_i p_i = \int A(x)\, p(x)\, dx$.
- The thermodynamic identity $\tau\, d\sigma = dU + PdV$, or $đQ = dU + đW$, and what it means.
- The ideal gas law.
- The Equipartition theorem.

TOPICS TO STUDY

- **Heat Engines and Refrigerators.** Be able to derive and explain, for example, the ideal (Carnot) efficiency of a heat engine or refrigerator.
- PV **-diagrams.** Be able to analyze heat and work along various stages.
- **Blackbody radiation.** Understand the derivation and general features of the blackbody spectrum. Note that I would not expect you to derive the spectrum on an exam – it would take too much time. It would be reasonable, however, to give you the expression we derived for the "density of modes" $d\Omega_0(\omega)$ (the number of modes in the range $[\omega, \omega + d\omega]$) and ask, given that, for a derivation of the blackbody spectrum. Recall what "pieces" went into the spectrum: the energy per photon × the average number of photons per mode × the number of modes per $d\omega$.
- **Phonons:** Understand the similarities and differences to the "photon gas."
- **Radiative equilibrium.** Understand what the energy flux density , J_u, means and how it depends on temperature. Understand radiative equilibrium.

- **Chemical potential and diffusive equilibrium.** Understand these.
- **The Grand Partition Function** and how to calculate probabilities from it.
- **Gibbs Free Energy and Chemical Equilibrium.** Remember the definition of the Gibbs Free Energy, and that $G = N\mu$. Remember and understand the equilibrium condition $\sum_j \mu_j \nu_j = 0$. Be able to relate equilibrium concentrations of "reactants" and "products" for reactions.
- **Quantum Statistics.** Understand how to calculate the density of states, given the energy level structure of a system. Understand the differences between fermions and bosons.
- **Fermi Gases.** Be able to derive the Fermi-Dirac distribution, f_{FD} from the grand partition function. Understand the meaning of the Fermi energy and the chemical potential. From the density of states, be able to evaluate the Fermi Energy, ε_F, and the degeneracy pressure. Know how to calculate properties of Fermi gases, e.g. the mean energy, U. (Note that I would not ask for an involved calculation using the Sommerfeld Expansion on the exam, but you should know what "pieces" go into this.)
- **Bose Gases.** Be able to derive the Bose-Einstein distribution, f_{BE} from the grand partition function. Understand the steps involved in our discussion of Bose-Einstein condensation.
- **Ising magnets.** Understand what the "Ising model" means – I'll provide the expression for its energy (see the following page).
- **Mean Field Theory.** Understand what mean field theory means. Be able to apply it to the Ising model to derive its phase-transition behavior of the mean-field Ising model – e.g. an equation describing the mean spin value, and the existence of a phase transition.
- **Miscibility.** Understand entropy of mixing, how it is calculated, and how it affects miscibility phase transitions.
- Be able to consider phase transitions from the perspective of Free Energy.
- Understand the derivation of the Clausius-Clapeyron relation (and how it uses relations derived from simple expressions for "dG") and understand what the Clausius-Clapeyron relation tells us.

TWO "SUGGESTED PROBLEMS."

Both of these are short, but are useful for reviewing important concepts.

1. **Barometric pressure.** In Physics 352 we showed via the Boltzmann relation that the concentration of molecules in a gravitational field of acceleration g (e.g. in the lower atmosphere at constant temperature) should be an exponential function of height, z. Really, we should have considered chemical potentials, treating thin "layers" of the atmosphere as being in diffusive equilibrium with each other. Do this. You should again find that n is an exponential function of height, z.
2. **Fermion chemical potential.** For a "particle-in-a-box" we've shown that the density of states $D(\varepsilon) \propto \varepsilon^{-1/2}, \varepsilon^0$, and $\varepsilon^{1/2}$ in 1, 2, and 3 dimensions, respectively. Consider low temperature. Explain <u>graphically</u>, <u>without evaluating any integrals</u>, that the chemical potential $\mu(\tau)$ must be an increasing, constant, and decreasing function of temperature in 1, 2, and 3 dimensions, respectively. Recall how expressing N as an integral over a probability distribution relates N and μ. (This is the same as one of the text problems in Chapter 5 or 6, I think.)

RELATIONS YOU WILL BE GIVEN:

NOTATION: P = pressure; τ = temperature (absolute, "fundamental" temperature); T = conventional temperature; β=inverse temperature = $1/\tau$; E = energy; U = mean energy; σ = entropy; V = volume; F = Helmholtz Free Energy; Z = partition function; N = number of particles, $n = N/V$, chemical potential μ, $\mathbb{Z}$ =grand partition function.

- Geometric series: $\sum_{m=0}^{\infty} r^m = \frac{1}{1-r}$
- $F = -\tau \ln(Z)$
- The quantum concentration for a particle of mass m: $n_Q = \left(\frac{m\tau}{2\pi\hbar^2}\right)^{3/2}$
- Entropy of a Monatomic Ideal Gas: $\sigma = N\left(\frac{5}{2} + \ln\left(\frac{n_Q V}{N}\right)\right)$
- Chemical potential of an Ideal Gas: $\mu = \tau \ln\left(\frac{n}{n_Q Z_{\text{int}}}\right)$
- The activity $\lambda \equiv e^{\mu/\tau}$
- A relation concerning the chemical potential: $\mu = \left.\frac{\partial F}{\partial N}\right|_{\tau,V}$.
- $\langle N \rangle = \frac{1}{\mathbb{Z}} \lambda \left(\frac{\partial}{\partial \lambda} \mathbb{Z}\right)$. Be sure to understand how this was derived.
- Energy of a single spin, i, in the Ising model: $E_i = -J \sum_j s_i s_j$, where the sum runs over the nearest neighbor spins.