

## Physics 353 – Statistical Mechanics

### Study Guide for the Midterm Exam

**NOTE:** This list is intended only as an approximate guide to the topics with which you should be familiar, in preparation for the upcoming exam. There will certainly be topics listed here that are not present on the exam. And of course, this list won't spell out exactly everything that will show up in the midterm – it is meant only as an approximate guide.

**Format of the exam:** The exam (Monday, May 5) will be closed book / closed notes, and 50 minutes long. Some information will be supplied to you (see below).

**General advice:** Study your homework assignments and notes. Be sure to study the *solutions* to homework problems you didn't understand. Understand the *derivations* by which we determined various expressions.

REMEMBER IMPORTANT RESULTS FROM LAST QUARTER, **FOR EXAMPLE:**

- The Boltzmann Distribution  $p_i = \frac{1}{Z} \exp\left(-E_i/\tau\right)$ , and how to use it.
- The partition function  $Z = \sum_i \exp\left(-E_i/\tau\right)$ , and how to use it.
- How to calculate the average value of any property.
- The thermodynamic identity  $\tau d\sigma = dU + PdV$ , or  $dQ = dU + dW$ , and what it means.
- The ideal gas law.

### TOPICS TO STUDY

- **Heat Engines and Refrigerators.** Be able to derive and explain, for example, the ideal (Carnot) efficiency of a heat engine or refrigerator.
- **PV -diagrams.** Be able to analyze heat and work along various stages.
- **Blackbody radiation.** Understand the derivation and general features of the blackbody spectrum. Note that I would not expect you to derive the spectrum on an exam – it would take too much time. It would be reasonable, however, to give you the expression we derived for the “density of modes”  $d\Omega_0(\omega)$  (the number of modes in the range  $[\omega, \omega + d\omega]$ ) and ask, given that, for a derivation of the blackbody spectrum. Recall what “pieces” went into the spectrum: the energy per photon  $\times$  the average number of photons per mode  $\times$  the number of modes per  $d\omega$ .
- **Phonons:** Understand the similarities and differences to the “photon gas.”
- **Radiative equilibrium.** Understand what the energy flux density,  $J_u$ , means and how it depends on temperature. Understand radiative equilibrium.
- **Chemical potential and diffusive equilibrium.**

- **The Grand Partition Function,  $\mathbb{Z}$**  (I can't type the curly Z) and how to calculate probabilities from it.
- **Gibbs Free Energy and Chemical Equilibrium.** Remember and understand the equilibrium condition  $\sum_j \mu_j \nu_j = 0$ . Be able to relate equilibrium concentrations of "reactants" and "products" for reactions.

**RELATIONS YOU WILL BE GIVEN:**

NOTATION:  $P$  = pressure;  $\tau$  = temperature (absolute, "fundamental" temperature);  $T$  = conventional temperature;  $\beta$  = inverse temperature =  $1/\tau$ ;  $E$  = energy;  $U$  = mean energy;  $\sigma$  = entropy;  $V$  = volume;  $F$  = Helmholtz Free Energy;  $Z$  = partition function;  $N$  = number of particles,  $n = N/V$ , chemical potential  $\mu$ ,  $\mathbb{Z}$  = grand partition function.

- Stirling's approximation for factorials:  
 $\ln(N!) \approx N \ln(N) - N$ .
- Geometric series:  $\sum_{m=0}^{\infty} r^m = \frac{1}{1-r}$
- $F = -\tau \ln(\mathbb{Z})$
- Energy of a Monatomic Ideal Gas:  $U = \frac{3}{2} N \tau$
- Entropy of a Monatomic Ideal Gas:  
$$\sigma = N \left( \frac{5}{2} + \ln \left( \frac{n_Q V}{N} \right) \right)$$
- The quantum concentration for a particle of mass  $m$ :  $n_Q = \left( \frac{m \tau}{2 \pi \hbar^2} \right)^{3/2}$
- Chemical potential of an Ideal Gas:  
$$\mu = \tau \ln \left( \frac{n}{n_Q \mathbb{Z}_{\text{int}}} \right)$$
- The activity  $\lambda \equiv e^{\mu/\tau}$
- A relation concerning the chemical potential:  
$$\mu = \left. \frac{\partial F}{\partial N} \right|_{\tau, V}$$
- $\langle N \rangle = \frac{1}{\mathbb{Z}} \lambda \left( \frac{\partial}{\partial \lambda} \mathbb{Z} \right)$ . Be sure to understand how this was derived.