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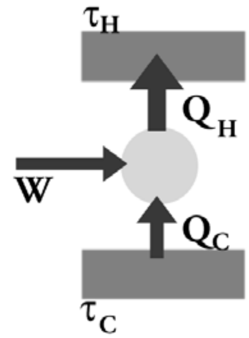
Physics 353: Problem Set 1 – corrected Apr. 2, 2008

Due date: Wednesday, April 9, **5pm**. (Turn in to the assignment to the box outside my door.)
Reading: Kittel & Kroemer Chapter 8 through p. 244 (Irreversible work).

1 (4 pts.**) Refrigerators**. In class, we considered the operation of a heat engine – a device that uses heat to perform work. A refrigerator is essentially a heat engine operated in reverse. We have a cold reservoir at temperature τ_C and a hot reservoir at τ_H. The cold reservoir is the inside of your kitchen fridge, for example, and the hot reservoir is the room. We perform work W in order to extract heat Q_C from the cold reservoir[1]. Some waste heat Q_H is dumped into the kitchen. See the diagram. We define the "coefficient of performance" (γ) of the refrigerator as $\gamma \equiv Q_C / W$.



The higher the COP, the less work it takes to extract a given amount of heat.

(a, 3 pts.**)** Determine the fundamental limits on the performance of a refrigerator – **show that** $\gamma \le \dfrac{\tau_C}{\tau_H - \tau_C}$. Proceed as we did for the heat engine, applying the thermodynamic identity to a cyclic process and noting that the entropy "leaving" the machinery must be at least equal to the entropy "entering" it. Note, by the way, that γ is not bounded by 1, and is largest if the reservoir temperatures are similar.

(b, 1 pt.**)** For a kitchen refrigerator, estimate the maximum possible γ. (Use reasonable values for the reservoir temperatures, and don't forget that we always need to deal with the absolute temperature.) In practice, of course, γ is considerably less than this.

(2, 7 pts.**) An ideal diatomic gas.** This problem may look familiar... For a diatomic molecule, (e.g. "dumbbells" like nitrogen, N_2), there are **two** internal degrees of freedom, corresponding to vibration along the dumbbell axis and rotation perpendicular to the axis. This means there are **five** total degrees of freedom, since the diatomic molecule has the same ability to move (translate) in three dimensions as does a monatomic molecule.

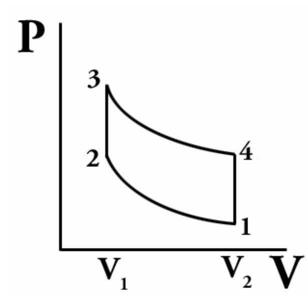
(a, 1 pt.**)** Using the Equipartition Theorem: What is the energy, U, of the gas of ***N*** **diatomic molecules** as a function of the number of particles, N, and the temperature, τ ?

(b, 1 pt.**)** Show that in general for an ideal gas with d degrees of freedom, $dU = c_V\, d\tau$, and determine c_V in terms of d and N.

(c, 5 pts.**)** Defining γ as c_P / c_V, the ratio of the specific heat capacities at constant pressure and constant volume, show that γ for a diatomic ideal gas is $7/5$. The work done by a change in volume dV is PdV, just as for a monatomic gas (as it must be, since this is just the mechanical work associated with a weight on a piston, for example).

[1] Our analysis applies to any refrigeration scheme. We won't explore the methods by which actual refrigerators work. Basically, your kitchen refrigerator pumps and compresses liquids and gases to transfer heat.

3 (11 pts.**) The Otto cycle**. In class we introduced the Otto cycle, used in internal combustion engines. Here we will calculate the efficiency of an ideal Otto cycle, acting on an ideal gas. The $P-V$ diagram is shown; the engine cycles between points 1→2→3→4→1...



- Gasoline vapor and air are compressed, isentropically (1→ 2).
- A spark causes rapid combustion at roughly constant volume (2→ 3). Heat Q_H is absorbed from the burning fuel.
- The gas expands ("power stroke") isentropically (3→ 4).
- Heat is expelled to reservoirs / exhaust (4→ 1), at constant V.

(We're glossing over many details of engine design, but they don't concern us.) Consider the working substance to be an ideal gas, so $PV = N\tau$. The gas may have internal degrees of freedom (e.g. rotational modes) characterized by some particular γ, where $\gamma \equiv c_p / c_v$.

(a, 2 pts.**)** Call the temperatures at points 1, 2, 3, and 4 $\tau_1, \tau_2, \tau_3,$ and τ_4, respectively. What temperature is hottest? What is coldest? You may find it helpful to consider each "leg" of the cycle separately and consider which of its endpoints is hotter or colder.

(b, 2 pts.**)** What is the heat input, Q_H, for the engine? Express your answer in terms of c_V and two of the temperatures ($\tau_1, \tau_2, \tau_3,$ and τ_4). *Hints:* Only one leg of the cycle is relevant; use your answer from (2b).)

(c, 1 pt.**)** What is the heat output, Q_L, for the engine? Express your answer in terms of c_V and two of the temperatures ($\tau_1, \tau_2, \tau_3,$ and τ_4).

(d, 2 pts.**)** Calculate the efficiency, η, of the engine using your answers to (c) and (d). Leave your expression in terms of $\tau_1, \tau_2, \tau_3,$ and τ_4. *Hint:* You can do this without calculating the work done. In fact, it's easier if you don't calculate W. How can you remove W from the definition of η? (Of course, there's nothing *wrong* with calculating W.)

~~**(e,** 2 pts.**)** Consider a Carnot cycle operating with reservoirs at the hottest and coldest temperatures of this Otto Cycle (see part (a)). Which is more efficient, the Carnot or Otto cycle? Explain.~~

(f, 3 pts.**)** We learned last term how temperature and volume are related for adiabatic expansions. Using this, show that the efficiency of the Otto cycle is $\eta = 1 - r^{1-\gamma}$, where the "compression ratio" $r = V_2 / V_1$, the ratio of the volumes of the constant-volume stages.

(g, 1 pt.**)** Look up a typical value of r for a car's engine and determine the ideal Otto cycle efficiency this gives. Note Problem 2; air is mostly diatomic gas. In practice, by the way, η for a car is about 20-25%. (Cite your source for r; a quick search on the internet is fine.).

4 (8 pts.**) A heat engine and a refrigerator.** The efficiency of a heat engine is greater if the temperature difference between the reservoirs is greater, Mr. K. learns. "Eureka!" he says. "Instead of running my heat engine between reservoirs at temperature τ_H and room temperature, τ_R, I'll use a refrigerator to cool the cold reservoir to temperature τ_F. Brilliantly, I'll just use some of the work output by the heat engine to power the fridge! I'll improve the efficiency of my heat engine system!" What is wrong with this scheme? Calculate its efficiency and compare to that of the heat engine alone. (Consider ideal, Carnot efficiencies.)

This problem is more involved than its short length might imply. Some suggestions:

- Make a clear diagram of the heat and work flows. The heat engine and the refrigerator share the "cold reservoir" at τ_F.
- You may wish to call the output work W_o and the work used by the fridge W_x. Apply conservation of energy to the heat engine and the fridge, separately. One way to proceed is to combine these expressions to eliminate W_x.
- Note that the efficiency we're trying to calculate is $\eta = W_o / Q_1$, where Q_1 is the heat flow from the hot reservoir to the heat engine. Derive an expression for η in terms of temperatures and heats.
- Compare this efficiency to the Carnot efficiency for the system without the fridge – i.e. operating between τ_R and τ_H. Is it larger or smaller? Note the following: you'll have to think of the "cold reservoir" as a finite system, rather than an "ideal" reservoir that can input or output arbitrary amounts of heat. Another way of saying this is that during a cyclic process, the entropy "created" by heat flow from the cold reservoir must be at least equal to the entropy "destroyed" by heat flow into the cold reservoir.