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University of Oregon; Spring 2008

Physics 353: Problem Set 2

Due date: Wednesday, April 16, **5pm**. (Turn in to the assignment to the box outside my door.)
Reading: Kittel & Kroemer Chapter 4.

1 (2 pts.**) Graphs.** To get a "feeling" for what the blackbody spectrum, u_ω, looks like, plot $u_\omega(\omega)$ over the range $\omega = 0$ to 10^{15} radians/sec. for two temperatures – T = 300 and 600 K – on the same graph. You can use a computer to plot, or plot by hand.

2 (4 pts.**) The blackbody peak.** Consider the blackbody spectrum, u_ω.

(a, 1 pt.**)** Prove that the peak of the spectrum is at $\omega = 2.82\dfrac{\tau}{\hbar}$.

(b, 3 pts.**)** Noting that $\omega = 2\pi f$ and that the wavelength, λ, and speed, c, of light are related by $\lambda f = c$, express u_ω as a function of wavelength, rather than frequency. Where is the peak? Is it at $\lambda = \dfrac{2\pi\hbar c}{2.82\tau}$? Explain.

3 (5 pts.**) Numbers of photons**

(a, 2 pts.**)** Show that the number of photons in a box of volume V at temperature τ is given by $N = \dfrac{V}{\pi^2}\left(\dfrac{\tau}{\hbar c}\right)^3 \displaystyle\int_0^\infty \left(\dfrac{x^2 dx}{\exp(x)-1}\right)$, where $x \equiv \hbar\omega/\tau$. Note that you may use results derived in class, which lead us almost directly to this expression. The integral is of course just a number – look it up, or estimate it either numerically or graphically.

(b, 1 pt.**)** Consider a typical oven ($V \approx 0.1$ m^3) at 400 °F (200 °C). Calculate the number of photons inside.

(c, 2 pts.**)** The oven is also filled with air; assume it's at atmospheric pressure ($P \approx 10^5$ Pa). Are there more photons or gas molecules in the oven? Assuming V and P are held fixed, what would the temperature have to be for the number of photons and the number of gas molecules to be the same? Express your answer in Kelvin.

4 (12 pts.**)** Free energy, entropy, and pressure of a photon "gas." We'll calculate all of these useful characteristics of blackbody radiation. (Note that if you have trouble with the earlier parts, you can still do the later parts.)

(a, 2 pts.**)** Show that the partition function of a photon gas is given by $Z = \prod_m \left[1-\exp(-\hbar\omega_m/\tau)\right]^{-1}$, where m is some label that counts all the modes. The notation "Π" indicates a product; for example $\prod_{i=1}^{3}(x+i)=(x+1)(x+2)(x+3)$. This is the same as Kittel & Kroemer **4.7a,** but without the use of the symbol "n" to identify modes. You may find it helpful to write out terms as you start to think of what the partition function should be; then think of how terms can be rearranged.

(b, 2 pts.**)** Using the answer from (a), show that the Helmholtz free energy $F=\tau\sum_m \ln\left[1-\exp\left(-\hbar\omega_m/\tau\right)\right]$. This is the same exercise as Kittel & Kroemer **4.7b**.

(c, 3 pts.**)** Transform the sum into an integral; integrate by parts to find $F=-\dfrac{\pi^2 V\tau^4}{45\hbar^3 c^3}$. This is the same exercise as Kittel & Kroemer **4.7c**.

(d, 2 pts.**)** Using our insights about F from last term and the result from (c), derive an expression for the **entropy** of the photon gas.

(e, 2 pts.**)** Using our insights about F from last term and the result from (c), derive an expression for the **pressure** of the photon gas. (Note, by the way, that we didn't have to "tell" our calculation that light carries momentum (true) – we automatically "discover" from statistical mechanics that light exerts pressure on the walls of its container!)

(f, 1 pt.**)** We claimed last term that the pressure of a "relativistic gas" should be given by $P=\dfrac{U}{3V}$. Does your result from (e) agree with this?

5 (6 pts.**) A one-dimensional blackbody.**

In various contexts it is useful to consider the radiation of a one-dimensional blackbody. (This arises for example, when considering electrical noise in wires, and in new sorts of structures people build that confine light in various geometries.) Derive the temperature dependence of the total energy for a 1D container of length L, i.e. the exponent p in the relation $U\propto\tau^p$. Derive as much of the 1D blackbody spectrum as you need to in order to determine this.

6 (3 pts.**) Three short literature searches.**

(a, 1 pt.**)** From the UO library web site (http://libweb.uoregon.edu/) do a KEYWORD search for "superconductivity." How many hits do you find? Next do the same, but search by TITLE.

(b, 1 pt.**)** Go to the web site for the journal "*Reviews of Modern Physics*." (Use Google, e.g., to find it.) Find the Table of Contents for January - March 2008. Pick an article, and make sure you can download and see the PDF. (You don't need to read the article!) Indicate that you've done this.

(c, 1 pt.**)** Go to the UO Library's collection of on-line databases (http://libweb.uoregon.edu/general/resources/articles.html). Find "INSPEC." There are many ways to do this, e.g. clicking on "browse databases by category" and then choosing "Physics" as a category. When you get to INSPEC, figure out how to search for the TOPIC "superconductivity" in the YEARS PUBLISHED "1999-2008." How many results do you find?