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University of Oregon; Spring 2008

Physics 353: Problem Set 3

Due date: Wednesday, April 23, **5pm**. (Turn in to the assignment to the box outside my door.)
Reading: Kittel & Kroemer Chapter 4; start Chapter 5.

1, 5 pts. **Kittel and Kroemer Chp. 4 #18. Isentropic expansion of a photon gas.** Comments: (*1*) The relation "$\tau V^{1/3} = \text{constant}$" for an isentropic expansion follows trivially from PS2 #4d. (*2*) In part (a), the idea is this: We observe the cosmic background radiation, which has a temperature of 2.7 K. This is a remnant of the early universe, when the temperature was about 3000 K, at which point the average photon could ionize hydrogen atoms (see PS 8 #2 from Phys. 352). The universe has expanded and cooled since then. This expansion has been isentropic "by definition" – there's no place outside the universe that heat could have flowed in from!

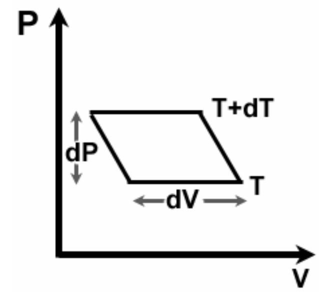
2, 3 pts. **Phonon heat capacity.** As discussed in class, the thermal vibrations of a solid can be analyzed very similarly to blackbody radiation. We showed that $U = V\frac{3\hbar}{2\pi^2 c_s^3}\int_0^{\omega_{max}} \frac{\omega^3 d\omega}{\exp(\hbar\omega/\tau)-1}$, where the symbols were defined in class. Let's consider this expression in various temperature regimes. We'll define the Debye temperature: $\Theta_D = \hbar\omega_{max}$.

(a, 1 pt.**)** At low τ, we can treat $\omega_{max} \approx \infty$ for the integration. Show that in this regime the specific heat $c_V = \frac{12}{5}\pi^4 N\left(\frac{\tau}{\Theta_D}\right)^3$. (We've sketched most of this in class and in the text.)

(b, 2 pts.**)** Show that at high temperature, $\tau \gg \Theta_D$, the specific heat $c_V = 3N$. (Consider the lowest order expansion of $\exp(\hbar\omega/\tau)$.)

3, 6 pts. **Photon Energy.** The "equation of state" for a photon gas is $P = a\tau^4$, where a is some constant $\left(\frac{\pi^2}{45\hbar^3 c^3}\right)$, as you showed on the previous problem set. This is the photon analog of the ideal gas equation of state, $PV = N\tau$, as it relates various observable physical parameters. From the equation of state alone we can deduce that the energy $U = 3aV\tau^4$ in various ways. Let's consider a Carnot cycle applied to the photon gas.

Consider the P-V diagram shown, of an infinitesimal Carnot cycle. Note that the isotherms are also stages of constant pressure. (Why?) The work done is $đW = dP\, dV$. (Why?)



(a, 2 pts.**)** Calculate the heat absorbed during one of the isothermal stages. Express your answer in terms of $P, dV,$ and $\frac{\partial U}{\partial V}$. (Note: You can always write any $U(x,y)$ as $\frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy$.)

(b, 2 pts.**)** Using the Carnot efficiency, relate your work and heat to T and dT .

(c, 2 pts.**)** Assuming $U = 0$ at $\tau = 0$, solve your expression from (b) to determine $U(V,\tau)$.

ABOUT PROBLEMS 4 AND 5: We'll examine the surface temperature of the Earth. The mean surface temperature of the Earth is about 288 K, or 15 °C – this corresponds to a peak wavelength of the blackbody spectrum in the infrared range. The sun's peak is in the visible, as noted in class.

4, 5 pts. **Surface temperature of the Earth – *Part 1.* Kittel and Kroemer Chp. 4 #5.** Please first express the temperature of the Earth symbolically in terms of $T_{\odot}$ (the temperature of the sun), $R_{\odot}$ (the radius of the sun), D_{SE} (the Earth-sun distance), and any other necessary symbols, before plugging in numbers.

5, 6 pts. **Surface temperature of the Earth – *Part 2.* The greenhouse effect.** The temperature of the Earth is warmer than the value you (hopefully) found in the previous problem. Why? The Earth's atmosphere contains gases like CO_2 and H_2O that absorb infrared radiation. The peak of the sun's spectrum ($T_{\odot}$ = 5800 K) lies in the visible range, so most of the sun's radiation passes through the atmosphere. The peak of the Earth's spectrum lies in the infrared range, and so some of it is absorbed by the atmosphere. This is the "greenhouse effect." Let's calculate the temperature the Earth would have using an extreme treatment of the atmospheric "blanket."

Consider the atmosphere to be a layer that is transparent to **all** the sun's radiation, and perfectly absorbent to **all** the Earth's radiation. The atmosphere radiates its energy outward to space and inward back to Earth, equally, and is in radiative equilibrium. The Earth is also in radiative equilibrium. The separation between the atmosphere and the Earth's surface is negligible – you can think of them as planes with sunlight incident. Calculate the temperature of the Earth. (Hopefully, you should find a value that is larger than the true temperature of the Earth[1].)

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[1] The fraction of infrared radiation absorbed by the atmosphere is a crucial determinant of the Earth's climate. A large body of evidence points to increasing amounts of greenhouse gases in the atmosphere generated largely by fossil fuel burning and having the consequence of increasing temperatures. By "large body of evidence" I mean so much that even Shell Oil, well known for its intense disregard for "social" concerns, writes that "We acknowledge that this problem is related to the burning of fossil fuels and believe urgent action is needed to stabilize atmospheric concentrations of greenhouse gas..." (http://www.shell.com/static/au-en/downloads/corporate/annual_review_2003.pdf). If you'd like

6, 6 pts. **Kittel and Kroemer Chp. 4 #8.**

7, 5 pts. **Chemical Potential of an Ideal Gas.** We will calculate the chemical potential, μ, for an ideal gas. The gas may have internal degrees of freedom.

Starting point: We derived several useful relations regarding ideal gases in Phys. 352. We showed that for a single monatomic particle in a box of volume V, the partition function $Z_{1,PB} = n_Q V$, where n_Q is the "quantum concentration" (PS8 no. 3). We showed that if there are internal degrees of freedom, $Z_1 = Z_{1,PB} Z_{\text{int}}$, where Z_{int} is the partition function of the "internal states" (Final Exam, no. 5). For N particles, we might expect $Z = (Z_1)^N$, since the particles are non-interacting and their energies simply add, giving simple products of exponentials in the sum over states that makes up the partition function. This is almost right, except that we are overcounting the number of states available to N particles – just as we did when deriving the ideal gas law. (See also K&K p 74-76). The true partition function is $\boxed{Z = \frac{(Z_1)^N}{N!}}$.

(a, 4 pts.**)** From the partition function, calculate the Helmholtz Free Energy and then the **chemical potential**, μ, as a function of $n, n_Q, \tau,$ and Z_{int}, relating μ and F as discussed in class. Use the Stirling approximation for factorials. Note that $n \equiv N/V$.

(b, 1 pt.**)** Mr. K. thinks: For a monatomic ideal gas, I know that $U = \frac{3}{2} N\tau$. I also know that $\mu = \left.\frac{\partial U}{\partial N}\right|_{V,\sigma}$. Therefore $\mu = \frac{3}{2}\tau$. Why is this wrong? (No calculations are necessary.)

8, 0 pts. **Final Project Reading.** Please read the final project description carefully. Read the list of suggested topics

9, 0 pts. **One short literature search.** Last week (PS2 #6c), you searched INSPEC for the TOPIC "superconductivity" in the YEARS PUBLISHED "1999-2008," and hopefully found an enormous number of papers (> 20,000). It's useful to be able to further limit the papers one finds by some criteria. **Figure out** how to limit this to papers that are "Review" papers – i.e. papers that review or comment on papers describing original research. In INSPEC, this is done by refining the "treatment type." Look through the "help" index and find what the "treatment types" are. This exercise is optional, since it should be covered in the upcoming Science Library tutorial. But if you'd like to try it now, or if you can't attend the tutorials, go ahead.

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to read more there are several sources, most notably the IPCC – see the Final Project list, and also http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_FAQs.pdf .