1 Kittel and Kroemer Chp. 4 #18. Isentropic expansion of a photon gas.

(a) \[ \frac{\tau_i V_i^{1/3}}{\tau_f V_f^{1/3}} = \frac{V_i}{V_f} \propto V_f^3 \]
\[ \frac{\tau_f}{\tau_i} = 2.7 \quad \frac{\tau_f}{\tau_f} = 3000 \]  
\[ \therefore \frac{\tau_i}{\tau_i} = \frac{\tau_i}{\tau_f} = \frac{3000}{2.7} = 1111 \]

(b) \( dq = 0 \)
\[ dw = -dv \quad w = -sdv \]
\[ w = - (U_{\text{final}} - U_{\text{initial}}) \]
\[ = \frac{\tau_i^2}{c^3 k^3} \left( V_i^4 \tau_i^{4/5} - V_f^4 \tau_f^{4/5} \right) \]
\[ \tau_i V_i^{1/3} = \tau_f V_f^{1/3} \]
\[ \tau_i^3 V_i = \tau_f^3 V_f \]
\[ = \frac{\tau_i^2}{15c^3 k^3} V_i \tau_i^{3/2} \left( \tau_i - \tau_f \right) \]
2 Phonon heat capacity.

\[ U = V \frac{3k}{2\pi^2 C_s^3} \int_0^{\omega_{\text{max}}} \frac{\omega^3 d\omega}{e^{\frac{\omega}{k_B T}} - 1} \]

Low \( \omega \), \( \omega_{\text{max}} \gg \omega \).

\[ U \approx V \frac{3\kappa}{2\pi^2 C_s^3} \left( \frac{\tau}{\kappa} \right)^4 \int_0^{\infty} \frac{\kappa^3 d\kappa}{e^{\kappa} - 1} \]

\[ C_v^3 = \frac{\omega_{\text{max}}^3 V}{6 \pi^2 N} \left( \text{class} \right) \int_0^{\infty} \frac{\kappa^3 d\kappa}{e^\kappa - 1} \]

\[ \rightarrow U \approx V \frac{3 \tau^4}{2 \pi^2 \hbar^2} \frac{6 \pi^2 N}{\hbar^2} \frac{\pi^4}{15} \]

Therefore \( c_v = \frac{\partial U}{\partial T} |_V = \frac{12}{5} \pi^4 N \left( \frac{\tau}{\Theta_D} \right)^3 \), simplifying the algebra and using the definition of the Debye temperature \( \Theta_D = \hbar \omega_{\text{max}} \).

2b

\( \mu_{\text{class}} \equiv \frac{\mu}{N} \)

\[ \exp \left( \frac{\hbar \omega}{k_B T} \right) = 1 + \frac{\hbar \omega}{k_B T} + \frac{\hbar^2 \omega^2}{k_B T^2} \]

\[ \frac{\omega^3}{e^{\omega/k_B T} - 1} \approx \frac{\omega^3}{k_B T} = \frac{\omega^2}{k_B} \]

From class \( c_{\text{max}} = (6 \pi^2 C_s^3 N)/V \)

\[ \mu = \frac{3}{2} \pi^2 \frac{C_s^5 N^2}{V} = 3 N \pi^2 \]

\[ C_v = \frac{3 \pi^2}{3 N} \frac{\partial U}{\partial T} |_V = 3 N \pi^2 \]

\[ \left. \frac{\partial U}{\partial \nu} \right|_\nu = 0 \]
3 Photon Energy.

a. Isothermal steps. 
\[ \delta Q = \delta u + \delta w = \delta u + P \delta v. \]
\[ \delta Q = \frac{\partial u}{\partial v} \delta v + P \delta v \quad \text{(general)} \]
\[ \delta Q = \frac{\partial u}{\partial v} \delta v + P \delta v \quad \text{(isothermal step)} \]

b. Work done over the entire cycle. 
Heat = heat input \( \text{in part (a)}. \) \[ \Rightarrow \eta = \frac{w}{q} \]
\[ \Rightarrow \eta = \frac{\partial P}{\partial v} \frac{\partial v}{\partial P} + P \]
Carnot : \[ \eta = 1 - \frac{T_f}{T_h} \quad \text{(ideal)} = \frac{A^{-2}}{2 + 2} \]
\[ \Rightarrow \frac{\partial W}{\partial q} = \frac{A^{-2}}{2 + 2} \Rightarrow \frac{\partial P}{\partial v} (2P + 2) (2P + 2) \]

Let's now use the fact that \( P = a Z \). \[ \Rightarrow \frac{\partial P}{\partial Z} = 4a Z^2. \]
\[ \Rightarrow 4a Z^2 (2P + 2) = \frac{\partial u}{\partial v} + a Z \]
\[ \Rightarrow 4a Z^2 + 4a Z \frac{\partial Z}{\partial u} = 2 + \frac{\partial u}{\partial v} \]
\[ \Rightarrow 3a Z^2 + 4a Z \frac{\partial Z}{\partial u} = 2 + \frac{\partial u}{\partial v} \]

\( \text{no differentials \& derivatives, a different \& \& a derivative.} \)

In the limit of small differentials, the second term \( \to 0 \).
\[ \Rightarrow 3a Z^2 = \frac{\partial u}{\partial v} \quad \Rightarrow u = 3a v Z^2 \]
(Causing \( u \to 0 \) as \( v \to 0 \)).
Surface temperature of the Earth – Part I. Kittel and Kroemer Chp. 4 #5.

\[ P_0 = \sigma B T_0^4 \frac{4\pi R_0^2}{4\pi D_{SE}^2} \]

Total solar power received by earth = \( P' = P_0 \frac{\pi R_0^2}{4\pi D_{SE}^2} \) sphere

Total power radiated by the earth = \( J_u \times \text{earth surface area} \)

\[ P_E = \sigma B T_E^4 \frac{4\pi R_E^2}{4\pi D_{SE}^2} = \sigma B T_0^4 \frac{4\pi R_0^2}{4\pi D_{SE}^2} \]

Equil: \( P_E = P' \)

\[ \frac{T_0^4 R_0^2}{D_{SE}^2} = T_E^4 \]

or \[ T_E = \left( \frac{T_0^4 R_0^2}{4D_{SE}^2} \right)^{1/4} \]

Numerically: \( T_E = 280 \text{ K} = -18 \text{°C} \)

a bit cold...
5 Surface temperature of the Earth – Part 2. The greenhouse effect.

Earth is in radiative equilibrium with atmosphere:

\[ P_E = P_0' + \frac{1}{2} P_E \]

Atmosphere is in radiative equilibrium with Earth:

\[ P_{atm} = P_E \]

\[ P_E = P_0' + \frac{1}{2} P_E \rightarrow P_E = 2 P_0' \]

\[ \sigma_B T_e^4 \frac{4 \pi R_e^2}{4 \pi D_{se}^2} = 2 \sigma_B T_0^4 \frac{4 \pi R_0^2}{4 \pi D_{se}^2} \]  

(See #4)

\[ T_e^4 = 2 \left( \frac{T_0^4 R_0^2}{4 D_{se}^2} \right) \rightarrow T_e = T_0 2^{1/4} \left( \frac{R_e}{2 D_{se}} \right)^{1/2} \]

i.e. the 7 from #4 times 2^{1/4}.

Numbers: \[ T_e = 333K = 60^\circ C \] A bit warm!
Plate M must be in radiative equilibrium with plates L and U. The radiative flux density within a unit area of it (considering both sides) must equal the radiative flux density of its emission (considering both sides).

\[ J_{\text{UM}} + J_{\text{LM}} = J_{\text{MN}} + J_{\text{MU}} \]

\[ \sigma T_u^4 + \sigma T_L^4 = \sigma T_m^4 + \sigma T_M^4 = 2 \sigma T_m^4 \Rightarrow T_m = \left[ \frac{1}{2} \left( T_u^4 + T_L^4 \right) \right]^{\frac{1}{4}} \]

Energy flux density without the middle plate: \( J_o = \sigma (T_u^4 - T_e^4) \)

Considering the lower half of the drawing, the energy flux density in the vacuum between plates M and L: \( J_{\text{Low}} = \sigma (T_m^4 - T_e^4) = \sigma \left( \frac{1}{2} (T_u^4 + T_L^4) - T_e^4 \right) = \sigma \left( \frac{1}{2} \right) (T_u^4 - T_L^4) \)

\[ J_{\text{Low}} = \frac{1}{2} J_o \]

Similarly, considering the upper half, \( J_{\text{Up}} = \sigma (T_u^4 - T_m^4) = \sigma \left( T_u^4 - \frac{1}{2} T_u^4 - \frac{1}{2} T_L^4 \right) = \sigma \left( \frac{1}{2} \right) (T_u^4 - T_L^4) = \frac{1}{2} J_o \)
b. It is true that $U = \frac{3}{2} N \tau$, and $\mu = \frac{\partial U}{\partial N}$, However we can't simply evaluate $\mu$ by differentiating $U$ since this partial derivative must be taken at constant entropy and volume. It is not at all evident from $U = \frac{3}{2} N \tau$ how $U$ would change as a function of $N$ if entropy and volume were held fixed. Entropy for example varies in a complicated way as a function of $N$, $\tau$, etc. – varying $N$ but keeping entropy fixed, we would certainly have to change $\tau$, which would affect our derivative.