Additional comments on Problem Set 4 #4

4 Biopolymer growth: Kittel & Kroemer 9.4.

Comment #1.

This problem concerns a reaction of the sort we discussed in class. Just as in class, we can relate the chemical potentials of the species. Treating the reactants and products as ideal gases (or more accurately speaking, ideal solutions), we can turn our relation for chemical potentials into a relation concerning concentrations. In class we denoted the concentration of species “A” by \( n_A \). In this problem, adopt the more common and more concise notation \([A]\). There are many different species -- a monomer, with concentration [1], a 2-mer, with concentration [2], a 3-mer, with concentration [3], etc.

Comment #2.

As suggested in my statement of the problem, I recommend first considering the equilibrium of \( \text{1mer} + N\text{mer} \leftrightarrow (N+1)\text{mer} \). Determine a relation between [1], [N], and [N+1]. The answer is given below\(^1\) -- don't look at it and don't move on to Comment #3 until working on this.

Comment #3.

One bit of “notation” we have not defined in class, but that is defined in the text: All the pieces of your equilibrium relation that don’t depend on concentration can be grouped together into an “equilibrium constant” \( K \). If you want, you can see the section on “equilibrium for ideal gases” for a relations about \( K \), but it’s not at all necessary. Just notice that you can write your answer for comment #2 as \([N+1] = \frac{[1][N]}{K_N}\), where \( K_N \) absorbs many concentration-independent factors.

\(^1\) \([N+1] = [1][N] \left( \frac{n_Q^{(N+1)}}{n_Q^{(1)} n_Q^{(N)}} \right) \left( \frac{Z_{\text{int}}^{(N+1)}}{Z_{\text{int}}^{(1)} Z_{\text{int}}^{(N)}} \right) \). Note the definition in the problem of \( n_Q^{(N)} \) as the quantum concentration of an N-mer.