

Physics 353: Problem Set 5

Due date: Wednesday, May 14, 5pm.

Reading: Kittel & Kroemer Chapter 6 (first part) and Chapter 7 (first part).

1, 6 pts. **Particle states**¹. Consider a system of six particles in some sort of container in which the energy levels are non-degenerate and evenly spaced (e.g. the levels of a 1D harmonic oscillator). A schematic diagram of the energy levels is shown in Fig. 1a (right).

Figure 1a.



(a, 1 pt.) Describe the ground state (i.e. the lowest energy configuration) of this system if the particles are **fermions**. Draw a diagram like the one shown, and illustrate occupied levels with a number indicating the number of particles in this state. E.g. for 2 fermions, your drawing would look like:

Figure 1b.



(b, 1 pt.) Draw the ground state of this system if the particles are **bosons**.

(c, 2 pts.) Suppose the system has one unit of energy (i.e. an amount equal to the level spacing) above the ground state. For the fermion case, how many possible configurations of the system are there? For the boson case? Don't forget that particles are **indistinguishable**, so "switching" the "1"s in Fig. 1b **does not** give a new state.

(d, 2 pts.) Repeat part (c) for two units of energy above the ground state.

2, 2 pts. **Derivative of the Fermi-Dirac function.** Kittel & Kroemer #6.1.

3, 3 pts. **Symmetry of filled and vacant orbitals.** Kittel & Kroemer #6.2.

4, 8 pts. **Density of orbitals in one and two dimensions.** Kittel & Kroemer #7.1.

5, 6 pts. **Energy of a relativistic Fermi gas.** Kittel & Kroemer #7.2. The energy levels for relativistic particles-in-a-3D-box are $E_{n_1, n_2, n_3} = \frac{\hbar\pi}{L} (n_1^2 + n_2^2 + n_3^2)^{1/2}$, where the "n's" are integers just as in the non-relativistic case.

¹ #1 is based on problems in D. Schroeder, *Thermal Physics*, in turn based on J. Arnaud et al. *Am. J. Phys.* **67**, 215 (1999).