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Physics 353 – Statistical Mechanics

Study Guide: Solutions to Suggested Problems

1. Barometric pressure.

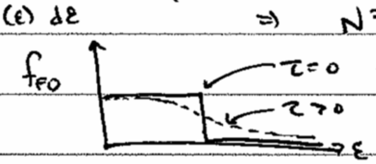
I'll call the height h, to avoid too many "z"s.
Consider "layers" of atmosphere, each with some concentration $n(h)$. Diffusive equilibrium means that the chemical potential of all layers is the same; $\mu(h)$ = constant. The chemical potential has an "internal" and "external" part, where the external chemical potential is any relevant potential energy: $\mu = \mu_{int} + \mu_{ext}$. Here, $\mu_{int} = \tau \ln\left(\frac{n(h)}{n_Q Z_{int}}\right)$ (given) and $\mu_{ext} = mgh$ (gravitational potential). Calling the concentration at height zero n_0, wee see that $\mu(h) = \mu(0)$ implies:

$\tau \ln\left(\frac{n(h)}{n_Q Z_{int}}\right) + mgh = \tau \ln\left(\frac{n_0}{n_Q Z_{int}}\right) + 0$. Therefore $\ln\left(n(h)\right) = -\frac{mgh}{\tau} + \ln\left(n_0\right)$, and $\boxed{n(h) = n_0 \exp\left(-mgh/\tau\right)}$.

2. Fermion chemical potential.

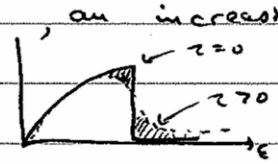
In general, $N = \int p(\varepsilon)\, d\varepsilon$ (probability density) $\Rightarrow N = \int D(\varepsilon) f_{FD}(\varepsilon)\, d\varepsilon$.

$f_{FD}(\varepsilon) = \dfrac{1}{\exp\left(\frac{\varepsilon-\mu}{\tau}\right)+1}$



In 3D, $D(\varepsilon) \propto \varepsilon^{1/2}$, an increasing function.

So $f_{FD}(\varepsilon) D(\varepsilon)$

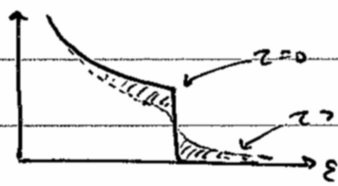


The area under this curve $= N$, which must be constant with τ. $D(\varepsilon)$ increases with ε, so the right shaded area will be bigger than the left shaded area unless μ (the ε at which $f = 1/2$) moves to the left. So to keep N = const, we need μ to decrease as τ increases.

In 2D, $D(\varepsilon)$ is constant as a function of ε, so μ doesn't have to "move" to keep N fixed.

In 1D, $D(\varepsilon)$ decreases with ε

$f_{FD}(\varepsilon)\, D(\varepsilon)$



So the left shaded area is bigger than the right shaded area unless μ moves to the right.

So μ increases as τ increases.