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University of Oregon; Winter 2008

Physics 352: The Binomial Distribution

For those of you who haven't seen the **binomial distribution** before (or for those who want a refresher): **A BRIEF DERIVATION OF THE BINOMIAL DISTRIBUTION**.

First, the notation $N!$ ("*N* factorial") means $N(N-1)(N-2)...1$. So $4! = 4\times3\times2\times1 = 24$ By definition, $0! = 1$.

Consider flipping *N* coins, which can either land heads (H) or tails (T). We are interested in the number of configurations which have *k* heads. For example, if *N*=4, there are 4 configurations with *k*=3, namely HHHT, HHTH, HTHH, and THHH.

If we don't care about heads and tails, and just count the number of ways of choosing *N* coins from a box, we find that the number of configurations is $N!$. This is because there are *N* possibilities for picking the first coin, *N*-1 for the second, *N*-2 for the third, etc.

Now let's count all the ways of arranging *k* heads and *N*-*k* tails. When "picking" the first H coin, we can choose any of our *k* heads; when picking the second, we can choose any of the remaining *k*-1, the third, *k*-2, etc. Picking the $k+1^{th}$ coin, i.e. the first T coin, we can choose any of the *N*-*k* tails; picking the $k+2^{nd}$ coin, any of the remaining *N*-*k*-1, etc. Thus the total number of permutations in which *k* coins are heads and *N*-*k* are tails is $k!(N-k)!$. We don't care "which head is which," so all these permutations are equivalent for our goal of counting the number of configurations with a given number of heads.

Therefore $\Omega(N,k)$, the number of configurations which have *k* heads, is given by

$$\Omega(N,k) = \frac{N!}{k!(N-k)!}$$

This is the **binomial distribution**, and is often written

$\Omega(N,k) = \binom{N}{k}$, pronounced "*N* choose *k*."

You can verify that the above distribution gives the correct values for configurations of flipped coins. Try doing the following exercise:

Flipping coins. Suppose you flip *N*=4 fair coins (i.e. coins with an equal probability of "heads (H)" and "tails (T)"). Make a list of all possible outcomes (you should find 2^4=16). Make a table of the number, Ω (*k*), of outcomes with *k* heads, where *k* = {0, 1, 2, 3, 4}, and show that this agrees with that given by the binomial distribution.

In addition, you might like to read Chapter 1 (*Principles of Probability*) of the Dill and Bromberg book on reserve in the Science Library (see the syllabus). You will learn, for example, how to calculate the probability of drawing a royal flush in poker.