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Physics 352 – Winter 2008

OPTICS Notes

1 INTRODUCTION: ELECTROMAGNETIC WAVES

1.1 Wave motion

This section deals with issues of notation and geometry that pertain to waves in general – not just light, but also waves in fluids, vibrations of solids, etc. Some of it is review. See your notes from last quarter, and also Hecht Chapter 2. It may be more inspirational to first look at Section 1.3, on the electromagnetic spectrum, before reading on.

1.1.1 WAVE EQUATIONS

As we learned last quarter, a function $\psi(x,t)$ is a solution to the one-dimensional **wave equation**

$$\boxed{\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}}$$

if it has the form $\psi(x,t) = f(x - vt)$. As usual, x is position and t is time; v is the **speed** of the wave. (This was described by d'Alembert around the mid 1700's.)

The more general wave equation, in >1 dimension, is

$$\boxed{\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}},$$

where ∇ is the Laplacian. In 3D, in Cartesian coordinates, $\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$.

The simplest solution to the wave equation is a traveling sinusoidal wave: $\psi(x,t) = A\cos(kx - \omega t)$. The **wavelength** $\boxed{\lambda = 2\pi/k}$ – if we consider the wave at a particular time, e.g. $t = 0$, the crests are separated by distance $\Delta x = \lambda$. The **frequency** $f = \omega/2\pi$ -- if we consider the wave at a particular position, e.g. $x = 0$, it oscillates with a **period** $T = 1/f$. The **amplitude** is A. The wave speed is related to the other variables by $\boxed{v = \omega/k = \lambda f}$, as we can see by thinking about what the wave looks like at various times (see Figure 1.1). This sinusoidal traveling wave has a single frequency – it is **monochromatic**.

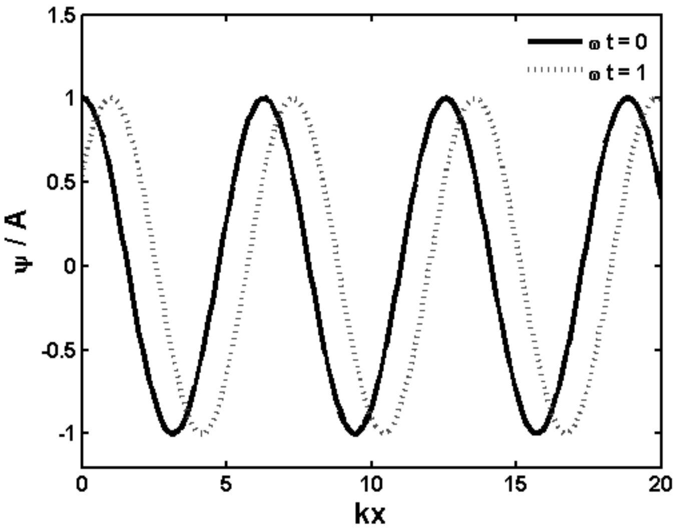

Figure 1.1. The traveling wave $\psi(x,t) = A\cos(kx - \omega t)$ plotted at time $t = 0$ (black solid curve) and $t > 0$ (gray dotted curve). As time increases, the wave appears displaced to the right.

The **phase** of the wave is $\phi = kx - \omega t$. More generally, we can write $\boxed{\psi(x,t) = A\cos(kx - \omega t - \delta)}$, where δ is the initial phase, i.e. what sets the value of ψ at $x = 0, t = 0$. Therefore $\boxed{\phi(x,t) = kx - \omega t - \delta}$. Generally, **differences** in ϕ are important (as we'll see, for example, below when discussing interference); the absolute value of ϕ is not.

1.1.2 SUPERPOSITIONS

The wave equation is linear in ψ, therefore its solutions obey the principle of **superposition**: If ψ_1 and ψ_2 each satisfy the wave equation, then $\psi = \psi_1 + \psi_2$ is also a solution. The relative phase difference between ψ_1 and ψ_2 is important in determining their **interference**:

Figure 1.2 shows an illustration of the superposition of two sine waves (like Hecht Figure 2.14). I've plotted $\psi_1 = 1.0\cos(kx)$, $\psi_2 = 0.9\cos(kx - \delta)$, and $\psi = \psi_1 + \psi_2$ for various values of δ. (I've chosen slightly different amplitudes for these two waves, to make the illustrations clearer.)

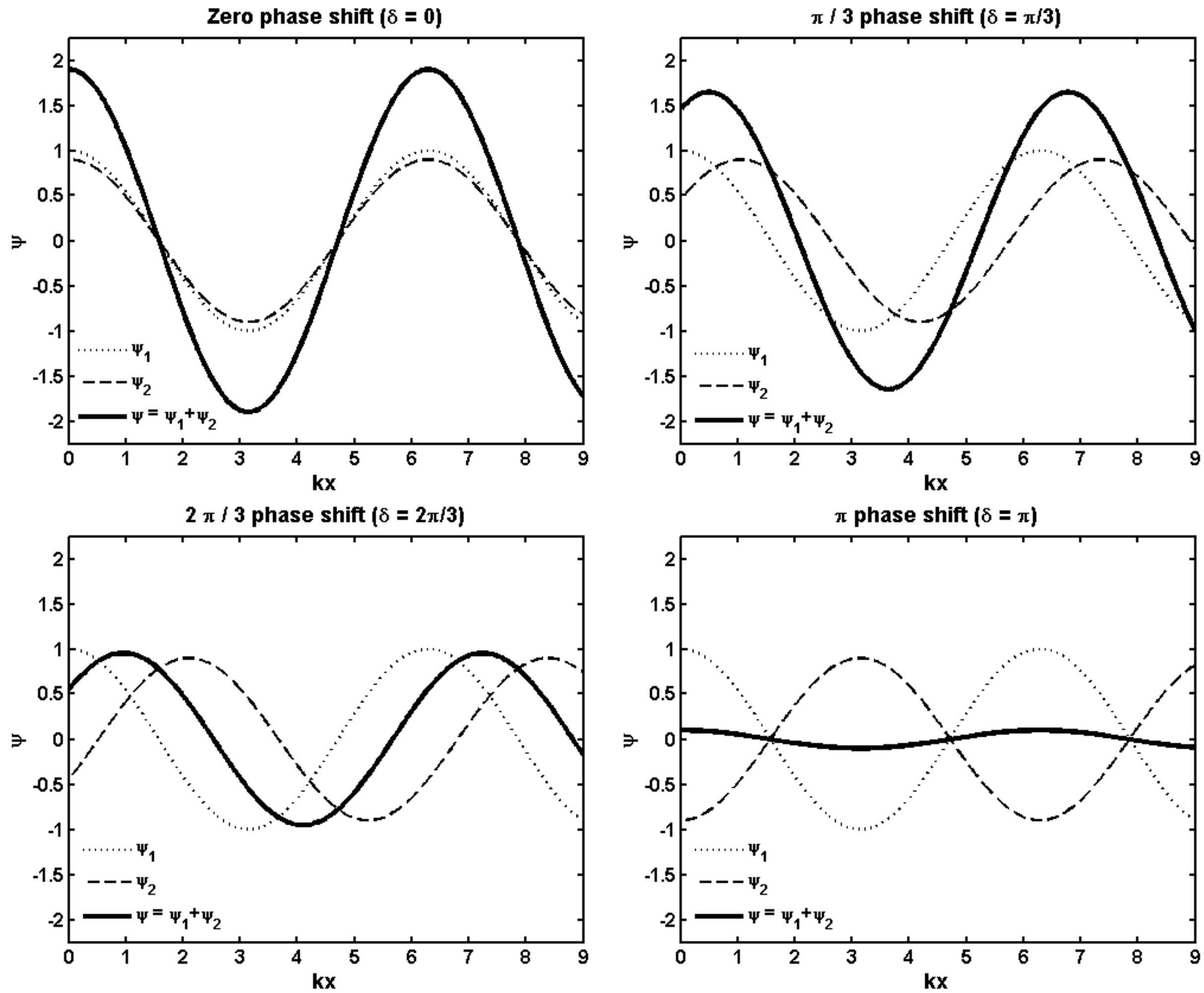

Figure 1.2. The sum (superposition) of the waves $\psi_1 = 1.0\cos(kx)$, and $\psi_2 = 0.9\cos(kx - \delta)$, and $\psi = \psi_1 + \psi_2$ for various values of δ. Upper left: $\delta = 0$; upper right: $\delta = \pi / 3$; lower left: $\delta = 2\pi / 3$; lower right $\delta = \pi$.

Note that a phase difference $\delta = 0$ leads to **constructive interference**, and a phase difference $\delta = \pi$ leads to **destructive interference**.

1.1.3 Complex Notation

As in Physics 351, we'll simplify out lives by using complex exponentials:

$$\boxed{\psi(x,t) = A\cos(kx - \omega t - \delta) = \mathrm{Re}\left[Ae^{j(kx-\omega t-\delta)} \right]}.$$

We'll typically just write

$$\boxed{\psi(x,t) = Ae^{j(kx-\omega t-\delta)} = Ae^{j\phi}},$$

and just take the real part when needed.

Note that the **magnitude** of ψ, denoted $|\psi|$, is given by $|\psi|^2 = \psi\,\psi^*$, where ψ^* is the complex conjugate of ψ (i.e. making all the j's into $-j$'s). If ψ is a vector, $|\vec{\psi}|^2 = \vec{\psi}\cdot\vec{\psi}^*$. [Review last quarter's math if necessary.]

1.1.4 TYPES OF WAVES

For the one-dimensional traveling wave illustrated above, each point corresponds to a particular phase. In 2- or 3-D, there can be more complex structures. It will be useful to consider points of equal phase, which we'll refer to as **wavefronts**.

Plane waves. A simple and very useful construction is the **plane wave**. Let's first illustrate this for a two-dimensional wave (Figure 1.3), in which we can plot the value of ψ along the third dimension:

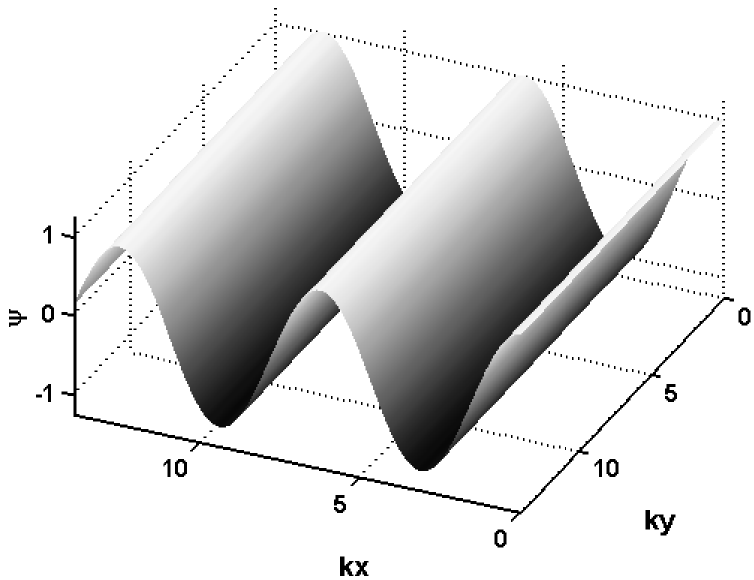

Figure 1.3. A two-dimensional plane wave: $\psi(x,y) = \cos(kx - \omega t)$, plotted at time $t = 0$.

Note that ψ only varies along one spatial dimension (in this case, x). Contours of equal phase (i.e. wavefronts) are **lines** in the *xy* plane. As the wave travels, for the example shown in Figure 1.3, it moves in the x direction – i.e. parallel to a wavevector, $\vec{k}$, that is perpendicular to these lines of constant phase and parallel to $\hat{x}$. We can write $\boxed{\psi(x,y) = A\cos(kx - \omega t)}$, or $\boxed{\psi(\vec{r}) = A\cos(\vec{k}\cdot\vec{r} - \omega t)}$, where $\vec{r}$ is a vector in the *xy* plane – think about how the dot product selects the x-component of $\vec{r}$. (We'll discuss this further below.)

For a three-dimensional plane wave, positions of constant phase (i.e. wavefronts) form a set of parallel **planes** (see Figure 1.4). (We can't plot ψ very easily – try it!) This is a good description of many sorts of light beams. Also, any 3D wave can be expressed as a combination of plane waves (by Fourier analysis).

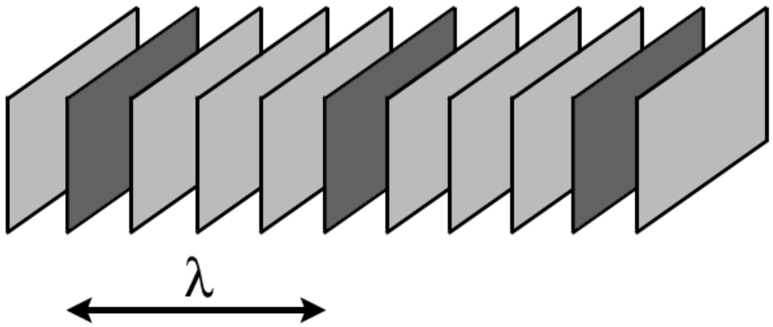

Figure 1.4. Positions of equal phase form a series of parallel planes for a three-dimensional plane wave – each shaded plane has the same value of the phase of the wave function, ψ. The darker planes differ by a phase shift of 2π, and so are separated by distance λ (one wavelength). (Typically, we'll only draw wavefronts that are separated by $\Delta\phi = 2\pi$ -- i.e. only the dark planes.) The **velocity** is perpendicular to the planes, as is the **wavevector**, $\vec{k}$.

The **three-dimensional plane wave** is described by

$$\boxed{\psi(\vec{r}) = A\cos(\vec{k}\cdot\vec{r} - \omega t)},$$

where $\vec{r}$ is any vector in 3D space, as we'll now show. Consider a position vector $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$, where ^ indicates a unit vector, and some particular vector $\vec{r}_0$. Their difference:

$$\vec{r} - \vec{r}_0 = (x - x_0)\hat{x} + (y - y_0)\hat{y} + (z - z_0)\hat{z}.$$

(See Figure 1.5, based on Hecht Fig. 2.19).

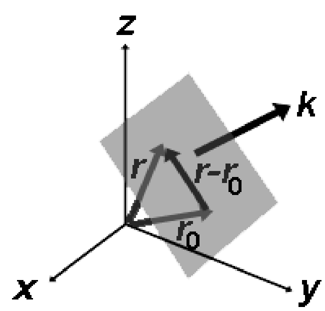

Figure 1.5. A plane in 3D.

Consider the set of points $\{\vec{r}\}$ described by $(\vec{r} - \vec{r}_0)\cdot\vec{k} = 0$.

As $\vec{r}$ varies, this sweeps out a plane perpendicular to $\vec{k}$ (See Figure 5). Expanding this: $(\vec{r} - \vec{r}_0)\cdot\vec{k} = k_x(x - x_0) + k_y(y - y_0) + k_z(z - z_0) = 0$, or $k_x x + k_y y + k_z z = a$, where $a = k_x x_0 + k_y y_0 + k_z z_0$ is a constant. Therefore the equation of a plane perpendicular to $\vec{k}$ is $\vec{k}\cdot\vec{r} = \text{constant} = a$. The set of planes over which $\psi(\vec{r})$ (at $t = 0$) varies sinusoidally is $\psi(\vec{r}) = A\cos(\vec{k}\cdot\vec{r})$, or $\psi(\vec{r}) = A\exp(j\vec{k}\cdot\vec{r})$. This function is periodic if $\vec{k}\cdot\vec{r}$ changes by 2π, i.e. $|\vec{k}|\lambda = 2\pi$, or $k = |\vec{k}| = 2\pi/\lambda$, as expected. The traveling plane wave is described by $\psi(\vec{r}) = A\exp(j(\vec{k}\cdot\vec{r} - \omega t))$. If it's moving along the x-axis, for example, $\psi(\vec{r}) = A\exp(j(kx - \omega t))$.

To reiterate: the wavefronts of a 3D plane wave are planes. Typically, we'll only draw wavefronts that are separated in phase by $\Delta\phi = 2\pi$, which are therefore spatially separated by distance λ. The wavevector $\vec{k}$ points perpendicular to these planes.

Often, we describe the wave by a **ray** that points along $\vec{k}$.

Figure 1.6. Wavefronts for a spherical wave are concentric spheres centered on the source.

Spherical waves. A point-source of light emits **spherical waves** – the wavefronts are concentric spheres that travel away from the point (Figure 1.6). The wave function is

$$\boxed{\psi(\vec{r},t) = \frac{A}{r}\exp\left[j\left(kr - \omega t\right)\right]},$$

where A is a constant and r is the distance from the source. Note that the amplitude decreases with r. (Think about why this might be – we'll return to it later.)

Cylindrical waves. A line-source of light, for example a slit, emits **cylindrical waves** – the wavefronts are concentric cylinders that travel away from the line. The wave function is

$$\boxed{\psi(\vec{r},t) = \frac{A}{\sqrt{r}}\exp\left[j\left(kr - \omega t\right)\right]},$$

where A is a constant and r is the distance from the line. Again, the amplitude decreases with r.

1.2 Light

1.2.1 ELECTROMAGNETIC WAVES

Light is an electromagnetic wave. As you'll see in an electromagnetism course, in the 1800's Michael Faraday conducted beautiful and groundbreaking experiments that revealed that electric and magnetic fields are coupled to one another. These insights led James Maxwell to formulate what are known as Maxwell's equations, which can be combined to yield:

$$\nabla^2 \vec{E} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

Here, $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields, respectively, ϵ_0 is the permittivity of free space (a constant), and μ_0 is the permeability of free space (another constant). We see that $\vec{E}$ and $\vec{B}$ obey **wave equations** – the fields can propagate as traveling waves!

The **speed** of the waves is $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$. Plugging in the known values of ϵ_0 and μ_0 yields $c = 3.0\times 10^8$ m/s, exactly equal to the experimentally measured speed of light! With this insight, it was realized that **light is an electromagnetic wave.**

Light in matter. The above speed is correct for light traveling in a vacuum. In matter of index of refraction n (related to the electrical response of the material), the speed of light is $v = c/n$. (The index of refraction is always ≥ 1.) For air at 20 °C and atmospheric pressure, n=1.0003. For water at 20 °C, n = 1.33. For typical glass, n=1.46. The *frequency* of the wave is unchanged from its value in vacuum – the rate of oscillation of the atoms excited by the electric field is constant. The wavelength of the light is different from its value in vacuum, and obeys the general relation encountered in Section 1.1: $v = \lambda f$. Therefore waves in matter are shorter than in free space: $\lambda = v / f = c / nf = \lambda_0 / n$, where λ_0 is the free space wavelength.

1.2.2 Polarization

We noted above that electric and magnetic fields can propagate as waves. It turns out that $\vec{E}$ and $\vec{B}$ are **perpendicular** to one another and to the propagation direction (Figure 1.7) (something else you'll learn about in an electromagnetism course.) The magnitudes of the field amplitudes are related by: $|\vec{E}| = v|\vec{B}|$, where v is the speed.

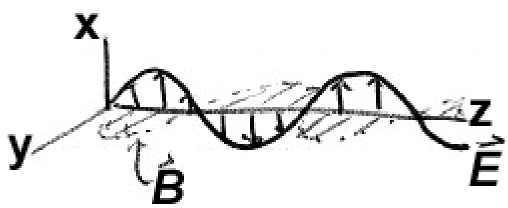

Figure 1.7. Electric and magnetic fields of a plane-polarized EM wave traveling along the z direction.

The direction of $\vec{E}$ specifies the **polarization** of the wave. If this **direction** is constant, as in Figure 1.7, we say the wave is **linearly polarized** (or **plane polarized**). In Figure 1.7, for example, note that $\vec{E}$ is always parallel to the x-axis (in other words, $\vec{E} = E(z,t)\hat{x}$). Waves don't have to be plane polarized, and can do a variety of interesting things. If the direction of $\vec{E}$ rotates as the wave propagates, then we have **circular** or elliptical polarization. (We won't go into the difference between the two – it's a simple thing to look into if you like.)

1.2.3 Energy and Intensity

You'll also learn in an electromagnetism class that electromagnetic waves carry energy and momentum. The power per unit area crossing a surface is $\vec{S} = c^2 \epsilon_0 \vec{E} \times \vec{B}$, known as the Poynting vector. Note that it points along the propagation direction (i.e. parallel to $\vec{k}$), not surprisingly. (See the figure of the electromagnetic wave.) We won't use $\vec{S}$ much in this course. Note however that since $|\vec{E}| = v|\vec{B}|$, $\vec{S}$ is proportional to $|\vec{E}|^2$.

The **intensity** (or **irradiance**), I, of the wave is the average energy carried per unit area per unit time, i.e. the power per unit area. **It is the intensity, not the electric field directly, that we "see" as brightness.** "Average" means that we consider the average power over a period. (Note that the intensity is a *number*, not a vector.) Since $\vec{S}$ is proportional to $|\vec{E}|^2$, the intensity of an electromagnetic wave is proportional to $|\vec{E}|^2$ as well -- **this is important**. We can also think about this, more simply, from last quarter's perspective: the energy of a wave is proportional to the square of its amplitude, and therefore

$$\boxed{I \propto |\vec{E}|^2}.$$

Recall that $|\vec{E}|^2 = \vec{E} \cdot \vec{E}^*$ -- this will prove useful.

What is the constant of proportionality? For the purposes of this course, we don't care, since all we're concerned with are relative intensities. However, for completeness: $I = \frac{1}{2}\epsilon_0 c E_0^2$ in vacuum, where E_0 is the electric field amplitude. In matter, $I = \frac{1}{2}\epsilon_0 v E_0{}^2$, where v is the speed of the wave and ε is the permittivity of the medium.

1.2.4 COHERENCE

The light from a light bulb is emitted by many independent sources throughout the filament (see Figure 1.8). Each emitted wave has a random phase difference relative to any other. The light bulb is an **incoherent** light source. (As is the sun, a fluorescent bulb, or anything besides a laser.) Moreover, any single wave from a light bulb doesn't extend perfectly to "$\pm\infty$." What do we mean by this? The wavefronts for a *perfectly coherent* plane wave are planes separated by λ (see Figure 1.4) – the separation is *always* λ over the entire extent of the wave. The wavefronts of light from a real, imperfect wave are separated by λ if we consider some finite span of size approximately L_C, but if we look at larger lengths the phase relations appear "randomized" – see Figure 1.9. L_C is called the coherence length – it's about 10 μm (around 20 λ) for a light bulb. (There's a more precise way to define the coherence length that won't concern us here – feel free to ask.)

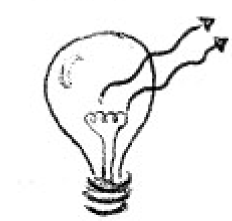

Figure 1.8. A light bulb – an incoherent light source

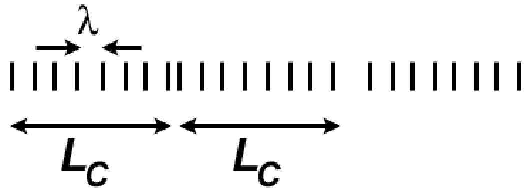

Figure 1.9. The coherence length, L_C, describes the spatial extent over which wavefronts (planes that differ by a phase shift of 2π) are separated by integer multiples of the wavelength. Over distances larger than $\approx L_C$, the phase "resets" and the coherence of the wave with itself – the ability to translate by an integer number of wavelengths and "match up" – is lost.

A **laser** is a **coherent** light source – **all** the waves emitted by all parts of the device have the same phase. Moreover, L_C is typically around 1 meter (> 10^6 λ), and can even be kilometers in length. This is what makes lasers great! (Lasers were invented in the 1960's. At the time, they were perceived as scientifically remarkable, but of questionable practical utility – they were "an answer in search of a question." Now, of course, lasers are ubiquitous and underpin an enormous amount of modern technology!)

1.3 The Electromagnetic Spectrum

Light is an electromagnetic wave, and there are a wide variety of wavelengths that we encounter. "Visible light" comprises a small fraction of this range. The electromagnetic spectrum is a continuum of wavelengths and frequencies. For convenience, we divide it into various artificial categories. The tables below, from Hecht, give values for the frequency (ν, which we've been denoting f) and the free-space wavelength (λ, of course related to f by $\lambda f = c$) for a range of electromagnetic radiation. It also gives the energy of a single photon – the smallest "packet" of energy that the light can carry – which is proportional to its frequency; this is a consequence of quantum mechanics, and won't concern us in this course.

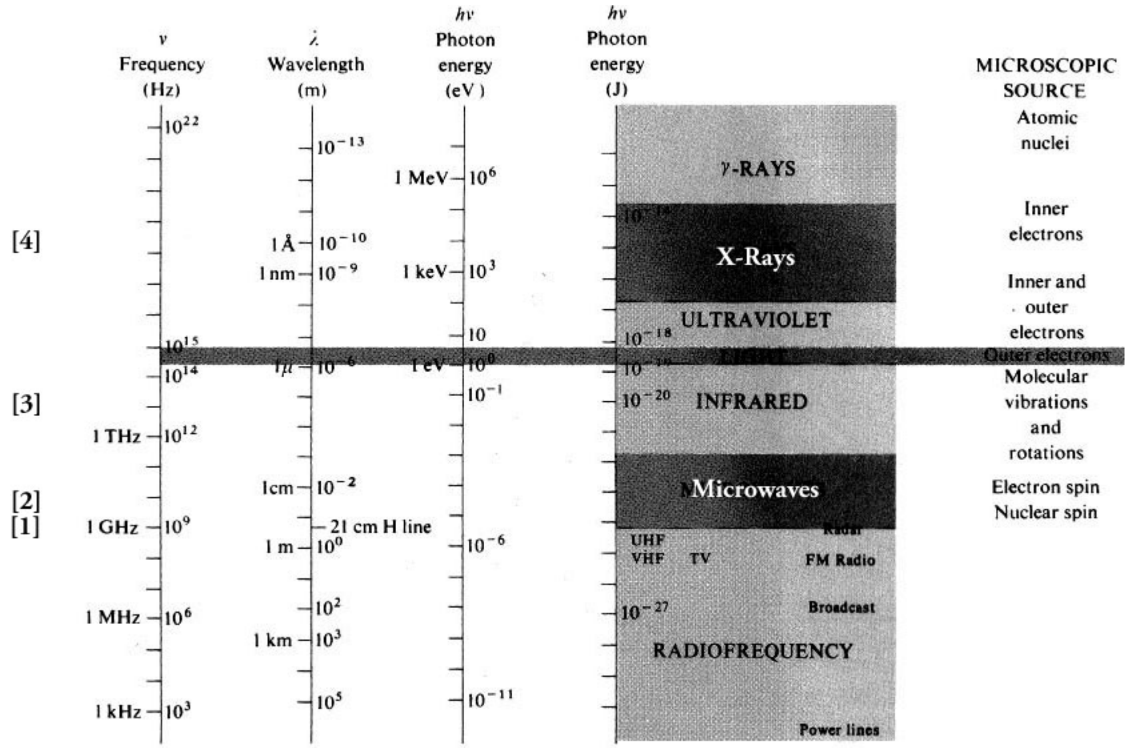

Figure 3.43 The electromagnetic-photon spectrum.

TABLE 3.4 Approximate Frequency and Vacuum Wavelength Ranges for the Various Colors

Color	λ_0 (nm)	ν (THz)*
Red	780–622	384–482
Orange	622–597	482–503
Yellow	597–577	503–520
Green	577–492	520–610
Blue	492–455	610–659
Violet	455–390	659–769

*1 terahertz (THz) = 10^{12} Hz, 1 nanometer (nm) = 10^{-9} m.

from Hecht, "Optics," 4th Ed.

Additional comments:

[*1*] Hydrogen is the most abundant element in space, and its 21-cm atomic emission line is commonly used to image the dynamics of galaxies and other structures.

[*2*] The frequency emitted by microwave ovens is about 2.5 GHz, resonant with the vibration frequency of water.

[*3*] The dominant wavelengths of "room temperature" thermal radiation are in the infrared, as we'll see later this quarter or in Physis 353.

[*4*] Since the wavelength of X-Rays are comparable to interatomic spacings in materials, X-Rays can reveal the molecular structure of crystals.