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University of Oregon; Winter 2008

Physics 352 – Optics

Problem Set 1

Due date: Wednesday, Jan. 16, **5pm**. (Turn in to the assignment to the box outside my door.)

Reading: Notes (handout), Chapter 1.
Optional Reading: Skim Hecht, Chapters 1-2, most of which should be familiar; Hecht Sections 3.6, 4.1-.4; Steck p. 45-58.

Some trigonometric identities:

$$\sin^2(u) = \frac{1-\cos(2u)}{2}; \quad \cos^2(u) = \frac{1+\cos(2u)}{2}; \quad \sin(2x) = 2\sin(x)\cos(x)$$

(1, 5 pts.**) Spherical Waves.** A point-source emits **spherical waves** – see the Notes, Section 1.1.4. The wave function is $\psi(\vec{r},t) = \frac{A}{r}\exp\left[j\left(kr-\omega t\right)\right]$. (This is written in *spherical coordinates*, in which the position vector $\vec{r}$ is described by three numbers: r is the distance from the point to the origin, θ is the "polar angle" to some particular axis (like a degree of latitude on a globe), and ϕ is the "azimuthal angle" around this axis (like a degree of longitude on a globe). Note that the spherical wave doesn't depend on θ and ϕ -- it's "spherically symmetric."

(a, 2 pts.**)** Show that ψ is a solution to the wave equation, $\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$, with an appropriate relation of the speed v to ω and k. The Laplacian is expressed in spherical coordinates as:

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial r^2} + \frac{2}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

(b, 3 pts.**)** Consider a point-source of light and a spherical shell of radius R that surrounds it. The emitted electric field is $\vec{E}(\vec{r},t) = \frac{\vec{A}}{r}\exp\left[j\left(kr-\omega t\right)\right]$, where $j = \sqrt{-1}$. Show that the total **power** incident on the shell is independent of R. Does this make sense?

(2, 2 pts.**) Wave sizes and speeds.**

(a, 1 pt.**)** Your pupils can contract down to about 1.5 mm. How large is this compared to the free-space wavelength of violet light (frequency ≈ 700 THz)? Radio waves from the campus radio station (88.1 MHz)? (1 THz = 10^{12} Hz; 1 MHz = 10^6 Hz.).

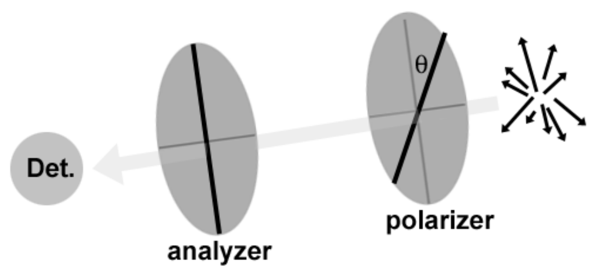
(b, 1 pt.**)** The liquid-filled "chambers" of your eye are the aqueous humor, behind the cornea, and the vitreous humor, behind the lens. Both have indices of refraction of about 1.34. If your eye is about 25 mm deep, how long does it take light to travel across it?

(3, 3 pts**). A light wave.** A linearly polarized plane light wave traveling in a piece of glass has an electric field intensity given by:

$$E_z = E_0 \cos\left[\pi 10^{15}\left(t - \frac{x}{0.65c}\right)\right],$$

if t is measured in seconds and x in meters. (E_z refers to the z-component of the electric field.) Find **(a**, 1 pt.**)** The frequency of the light, f, in Hz.; **(b**, 1 pt.**)** Its wavelength, λ, in meters; **(c**, 1 pt.**)** The index of refraction of the glass.

(4, 3 pts.**) Malus' Law.** A *linear polarizer* transmits light whose electric field is parallel to its *transmission axis.* (Typically, the other component of the electric field is absorbed.) Convince yourself that after passing through a linear polarizer, light is linearly polarized. Consider unpolarized light (e.g. sunlight) that passes through two linear polarizers, after which a detector records the incident light intensity, I. The second polarizer (called the *analyzer*) has a transmission axis oriented at angle θ relative to the first polarizer. (See figure; the black arrows indicate the random polarization directions of the incident light; the black lines indicate the transmission axes of the polarizers.) Show that $I(\theta) = I_0 \cos^2\theta$, where I_0 is the maximal intensity possible in this arrangement (Étienne Malus, 1809).



(5, 6 pts.**) The Blue Sky.** Why is the sky blue? The sun emits light at wavelengths throughout the visible spectrum (and beyond). Gas molecules in the air absorb this radiation and re-emit it in all directions. The molecular sizes (~ Å) are much smaller than the wavelengths of visible light (hundreds of nm), placing them in the regime of "Ralyeigh scattering," named after Lord Ralyeigh, who analyzed this in the 1870s. (*There are lots of interesting phenomena associated with scattering, e.g. the whiteness of clouds and milk, that we won't have time to discuss in this course. Returning to the blue sky...*) Suppose the electric field amplitude of the light incident on a gas molecule is E_{0i}. The electric field amplitude of the scattered light is E_{0s}. We can make some reasonable statements about E_{0s}:

(**i**) E_{0s} should only depend on E_{0i}, the wavelength of the light (λ), the distance from the molecule (r), and V, the volume of the scatterer.

(**ii**) E_{0s} is proportional to E_{0i} – because the greater the incident field amplitude exciting the scatterer, the greater the scattered field amplitude should be.

(**iii**) E_{0s} is proportional to $1/r$ – because the molecule radiates a spherical wave from a "point," and

(**iv**) E_{0s} is proportional to V – because a larger object should absorb more of the incident radiation.;

(a, 3 pts.**)** Show using dimensional arguments that the ***intensity*** of the scattered light depends on λ as λ^{-4}. (If you were not in Phys. 351 last quarter, you might like to read my notes on

Dimensional Analysis, which can be found in the "Notes" section of *http://physics.uoregon.edu/~raghu/Physics351Fall2007.html*.)

(b, 3 pts.**)** How does this explain the blue color of the sky? Draw a diagram illustrating the path of scattered red and blue light from the sun.

(6, 6 pts.**)** **Interference.** Consider the superposition of two plane waves of identical amplitude, frequency, and polarization:

$$\vec{E}_1(x,t) = \vec{E}_0 e^{j(kx-\omega t-\delta_1)}, \quad \vec{E}_2(x,t) = \vec{E}_0 e^{j(kx-\omega t-\delta_2)}, \quad \vec{E} = \vec{E}_1 + \vec{E}_2.$$

(a, 3 pts.**)** Show that the intensity of the resulting wave, *I*, is given by

$$I = 4I_0 \cos^2\left(\Delta\phi / 2\right),$$

where $\Delta\phi=\delta_2-\delta_1$ is the difference in phase between the two waves and I_0 is the intensity of each of the component waves. Recall (see your notes) that *I* is proportional to $\left|\vec{E}\right|^2 = \vec{E}\cdot\vec{E}^*$, where * indicates the complex conjugate. The identities at the top of this assignment may be of use.

(b, 3 pts.**)** A beam of red light (f = 450 THz) is emitted and split into two identical beams, each of intensity I_0. One of these travels 10 μm (10×10^{-6}m) in air (n=1) and the other travels an equal distance in glass (n=1.46), after which the two are recombined. What is the intensity of the combined beam?

(7, 5 pts.**)** **Anti-reflection coating.** Consider light in air (index n_0 = 1) normally incident on a piece of glass (n_g = 1.5) coated with an anti-reflective film of thickness *d* and index n_1 ($n_0 < n_1 < n_g$) (see Figure). The reflected wave is equal to the superposition of the wave reflected at the air / coating interface and the wave reflected at the coating / glass interface. Suppose the wave reflected at the air / coating interface has amplitude *r* relative to the incident wave, and the wave transmitted from the coating to the air (after being reflected at the glass) has amplitude *t* relative to the incident wave. (We'll determine later in the course the values of *r* and *t*.) We'll also prove later in the course that a wave reflecting from a medium with faster propagation speed off a medium in which the speed is slower flips sign (i.e. has an extra phase change of π).

What is the smallest non-zero film thickness, *d*, such that the total reflected intensity is minimal? Write your answer in terms of the free-space wavelength of the incident light, λ_0. (This is how anti-reflective coatings, used for example on camera lenses, work.)

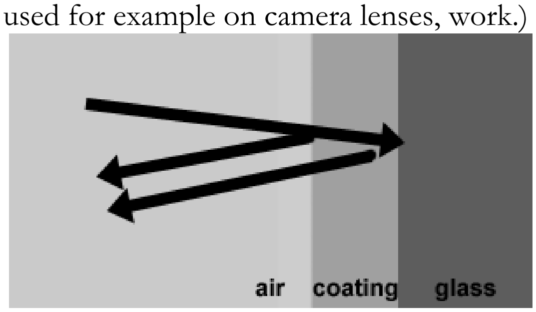


Figure (Problem 7). Interference at an anti-reflective coating. For clarity, the incident and reflected rays have been drawn at non-normal incidence. For the problem, consider normal incidence and reflections (i.e. perpendicular to the surfaces).