**Prof. Raghuveer Parthasarathy**
University of Oregon; Winter 2008

# Physics 352 – Optics

## Problem Set 2

**Due date:** Wednesday, Jan. 23, **5pm**. (Turn in to the assignment to the box outside my door.)

**Reading:** Diffraction Notes

**(1,** 10 pts.**) 2-slit interference.** Consider light (a plane wave) of wavelength $\lambda$ diffracted by two identical slits in a barrier, separated by distance *D*, as discussed in class. We're interested in the intensity $I$ incident on a far-off screen as a function of $\theta$, the angle with respect to the normal to the barrier.

**(a,** 2 pts.**)** As discussed in class, light from "slit 2" travels an extra distance $\delta$ compared to light from slit 1, where $\delta$ depends on $D$ and $\theta$ – see Figure 2.3 of the Diffraction Notes. Determine the phase shift, $\Delta\phi$, that this distance corresponds to, and then use your result from Problem 6 of Problem Set 1 to simply derive $I(\theta)$. (This should, of course, be the same $I(\theta)$ we derived in class.)

For the rest of this problem consider small $\theta$, so that $\sin(\theta) \approx \tan(\theta) \approx \theta$. Consider the screen to be a distance *L* from the barrier.

**(b,** 2 pts.**)** For D = 10 μm, L = 10 m, and $\lambda$ = 530 nm (green light), determine the distance on the screen ($\Delta$y) that separates the interference fringe maxima.

For parts (c) and (d): Suppose that in front of one of the slits we insert a device that shifts the phase of the light passing through it by $\phi_0$ radians.

**(c,** 1 pt.**)** If $\phi_0 = \pi$, would you expect a bright or dark spot on the screen at $\theta = 0$? (A one-sentence answer is sufficient.)

**(d,** 5 pts.**)** Determine the value of $\phi_0$ that shifts the interference fringes in position on the screen by $\Delta$y /6 relative to their unshifted positions.

**(2,** 4 pts.**) No interference.** Mr. K. places a (very small) flashlight in front of one slit of a two-slit interferometer and another flashlight in front of the other slit. (Use the same notation and numbers as the previous problem: *D* = 10 μm, *L* = 10 m, and $\lambda$ = 530 nm – the flashlights have green filters.) Looking at the screen, Mr. K. does not see any interference fringes. Why?

**(3,** 3 pts.**) 4-slit interference.** For light of wavelength $\lambda$ passing through a 4-slit interferometer with inter-slit separation D, graph the intensity distribution – plot $I$ as a function of $\sin\theta$. Clearly indicate the positions of maxima and minima. (You can use expressions derived in class.)

**(4,** 10 pts.**) Single-slit interference.** So far, we have considered infinitely narrow slits, and so have not had to worry about interference between beams passing through different regions of the *same* slit. However, this certainly happens! In this problem we will derive the diffraction pattern of a single slit. This is not hard to do, given the expression we derived for $N$ slits each separated by $D$:

$$I(\theta) = I_1 \frac{\sin^2(\pi ND\sin\theta/\lambda)}{\sin^2(\pi D\sin\theta/\lambda)},$$

where $I_1$ is the intensity of a single beam (denoted $I_0$ in class). A finite aperture is simply the limit of the above expression as $N \to \infty$, $D \to 0$, and the product $ND \to a$, where $a$ is the width of the aperture (slit). The "single beam" intensity is a bit meaningless in this case, since each of the infinitely many waves interfering in the slit has negligibly small intensity, but we can avoid it by noting that we showed in class that the intensity we measure at $\theta$=0 is $I(\theta=0) = N^2 I_1$; let's call this $I_0$. Note that $I_0$ is easy to measure – just place a detector at $\theta$=0.

**(a,** 5 pts.**)** Show that for single-slit interference,

$$I(\theta) = I_0\left(\frac{\sin(\beta)}{\beta}\right)^2,$$

where $\beta = \pi a \sin\theta/\lambda$. *Hint*: To take the above "limits" simply replace $ND$ by $a$ and $I_1$ by ..., and continue from there. Think about what factors are "small" and what that means, mathematically.

**(b,** 1 pt.**)** The function $\sin(x)/x$ arises often, as is often referred to as "sinc($x$)." Show that as $x \to 0$, sinc($x$)$\to$1.

**(c,** 2 pts.**)** Where are the zeros of $I(\theta)$ for single-slit diffraction? Graph $I(\sin\theta)$, indicating the zeros, and showing roughly how the magnitude of $I$ changes with increasing angle.

**(d,** 2 pts.**)** Consider light passing through a slit of width a = 0.5 μm hitting a screen a distance L = 10 m from the slit, and consider small $\theta$, so that $\sin(\theta) \approx \tan(\theta) \approx \theta$. For $\lambda$ = 530 nm (green light), determine the separation on the screen ($\Delta$y) of the two central intensity minima (i.e. the total width of the central intensity peak).

**(5,** 3 pts.**)** When considering two-slit diffraction from infinitely narrow slits, we have concluded that infinitely many equally strong interference maxima exist as a function of $\theta$. Given what we've concluded in Problem 4 about finite slit size, how would you expect the two-slit interference pattern from real slits to look? You needn't derive a mathematical expression, but sketch the patterns from (i) two infinitely narrow slits and (ii) two finite slits. You may find Problems *1ab* and *4d* useful.