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Problem Set 2: SOLUTIONS

(1, 10 pts.**) 2-slit interference.**

(a) $\vec{E_1} = \vec{E_0} \exp[j(ks-\omega t)]$

$\vec{E_2} = \vec{E_0} \exp[j(k(s+\delta)-\omega t)]$

$\delta = D\sin\theta$

$\therefore \Delta\phi = k\delta = \frac{2\pi}{\lambda} D\sin\theta$

from the problem 6 of PS 1, we have.

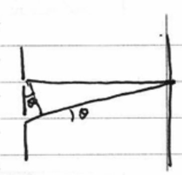
$I = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$

$\therefore I(\theta) = 4I_0 \cos^2\left(\frac{\pi}{\lambda} D\sin\theta\right)$

for the screen is far from the barrier, we have

$\sin\theta \doteq \theta \doteq \tan\theta = \frac{\Delta y}{L}$

$\therefore I(\theta) = 4I_0 \cos^2\left(\pi \frac{D\Delta y}{L\lambda}\right)$



(b) for $D = 10\,\mu m$, $L = 10\,m$, $\lambda = 530\,nm$

for the distance on the screen (Δy) that seperates the interference fringe maxima, we have

$\pi \frac{D\Delta y}{L\lambda} = \pi$

$\therefore \Delta y = \frac{L\lambda}{D} = \frac{10 \times 530 \times 10^{-9}}{10 \times 10^{-6}}$

$= 0.53\,m.$

For parts (c) and (d): Suppose that in front of one of the slits we insert a device that shifts the phase of the light passing through it by ϕ_0 radians.

(c) At $\theta = 0$, the phase shift due solely to geometry is zero. Therefore the total phase shift is π, and so we have destructive interference, and a dark spot.

(d, 5 pts.**)** The total phase difference between the two paths is now $\Delta\phi = \frac{2\pi}{\lambda} D\sin\theta + \phi_0$ -- the first term is from geometry just as in part (a), and the second is our "additional" shift. Again via $I = 4I_0 \cos^2\left(\Delta\varphi/2\right)$ we find: $I = 4I_0 \cos^2\left(\frac{\pi}{\lambda} D\sin\theta + \frac{\phi_0}{2}\right)$. This is the same intensity function as in part (a), but "shifted." Consider the maximum that "used to be" at $\theta = 0$, for which the argument of the cosine was zero. (We could consider any point on the curve; this one is particularly simple.) Now, $\theta = 0$ is no longer a maximum, since the argument of the cosine is not zero. More explicitly: the "new" position of that maximum is at the angle for which $\frac{\pi}{\lambda} D\sin\theta + \frac{\phi_0}{2} = 0$, i.e. $\frac{\pi}{\lambda} D\sin\theta = -\frac{\phi_0}{2}$. Again using small angles and trigonometry, we can write $\sin\theta = \frac{y}{L}$, where y is the position on the screen of this maximum. Therefore: $\frac{\pi}{\lambda} D\frac{y}{L} = -\frac{\phi_0}{2}$, or $\frac{-2\pi Dy}{\lambda L} = \phi_0$ -- this is how the phase shift and the position of the "central" maximum are related. We want $y = \frac{\Delta y}{6}$, where $\Delta y = L\lambda / D$ was the spacing found in part (b). Therefore, we need $\phi_0 = \frac{-2\pi Dy}{\lambda L} = -\frac{2\pi D}{\lambda L}\frac{L\lambda}{6D} = -\frac{\pi}{3}$ radians.

(2, 4 pts.**) No interference.**

The key issue here is **coherence** – see the notes, Section 1.2.4. (I also referred to this in email.) For there to be an interference pattern, we need a well-defined phase difference between the interfering paths – see, for example, Problem 1. There are two reasons that Mr. K.'s light bulb setup will not create interference fringes – <u>each reason alone is sufficient</u> to wash out any interference, but I'll describe them both:

(i) From the notes: "The light from a light bulb is emitted by many independent sources throughout the filament... Each emitted wave has a random phase difference relative to any other." So there can't possibly be any well-defined phase relation between one light bulb and another, since all the individual waves emitted by these incoherent sources have some random phase offset. If one wave is "in phase" and adds constructively, we can find another emitted by some other part of the filament that is "out of phase," and adds destructively, for example. We average over all possible phase relationships.

(ii) The emission is incoherent! (More precisely, it has a short coherence length.) Therefore, even if there is temporarily some well-defined phase relation between a wave emitted by one bulb and a wave emitted by another, it only lasts a distance $L_c \approx 10\mu$m, or a time $t_c \approx L_c / c \approx 10^{-13}$ seconds. After this it is "randomized" and we have some new phase relation. So again, we average over all possible phase relationships.

Some people mistakenly thought that the fact that the screen distance is an integer multiple of L_c is somehow relevant. It isn't. (Why would it be?) Please think about this.

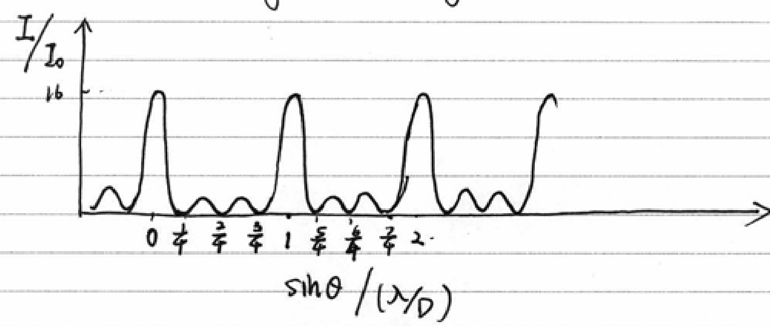
3. 4-slit interference.

$$I = I_0 \frac{\sin^2(N\pi D\sin\theta/\lambda)}{\sin^2(\pi D\sin\theta/\lambda)}$$

$N=4$

$$\therefore I = I_0 \frac{\sin^2(4\pi D\sin\theta/\lambda)}{\sin^2(\pi D\sin\theta/\lambda)}$$

so we can get the figure



(4) Single-slit interference.

For N slits each separated by D:

$$I(\theta) = I_1 \frac{\sin^2(\pi ND\sin\theta/\lambda)}{\sin^2(\pi D\sin\theta/\lambda)},.$$ We are interested in $N\to\infty$, $D\to 0$, and the product $ND\to a$. Note that I_1 goes to zero as $N\to\infty$, which makes it an inconvenient variable for considering the overall amplitude. Therefore as suggested, we think about the intensity we measure at $\theta=0$, which is $I(\theta=0) = N^2 I_1$; we'll call this I_0. Therefore

$$I(\theta) = \frac{I_0}{N^2}\frac{\sin^2(\pi ND\sin\theta/\lambda)}{\sin^2(\pi D\sin\theta/\lambda)}$$ The $ND\to a$ is simplest to consider:

$$I(\theta) \to \frac{I_0}{N^2}\frac{\sin^2(\pi a\sin\theta/\lambda)}{\sin^2(\pi D\sin\theta/\lambda)}.$$

For the rest, we need to be careful. There are (at least) two approaches we could take.

(i) We note $\frac{I_0}{N^2}\to 0$ and $\sin^2(\pi D\sin\theta/\lambda)\to 0$, so we might apply L'Hopital's rule. Our numerator depends on N and our denominator on D, which at first sight appears problematic until we realize that we can just write $N = a/D$. We can apply L'Hopital's rule. It's laborious, but it works.

(ii) We'll be considering $D \to 0$, and so $(\pi D \sin\theta/\lambda) \to 0$, so the dominant term in the Taylor expansion of $\sin(\pi D \sin\theta/\lambda)$ will be the first one, allowing us to write $\sin(\pi D \sin\theta/\lambda) \to \pi D \sin\theta/\lambda$. Therefore

$$I(\theta) \to \frac{I_0}{N^2} \frac{\sin^2(\pi a \sin\theta/\lambda)}{(\pi D \sin\theta/\lambda)^2} = \frac{I_0}{(ND)^2} \frac{\sin^2(\pi a \sin\theta/\lambda)}{(\pi \sin\theta/\lambda)^2}.$$

Defining $\beta = \pi a \sin\theta/\lambda$, we see that

$$I(\theta) = I_0 \frac{\sin^2(\beta)}{\beta^2}.$$

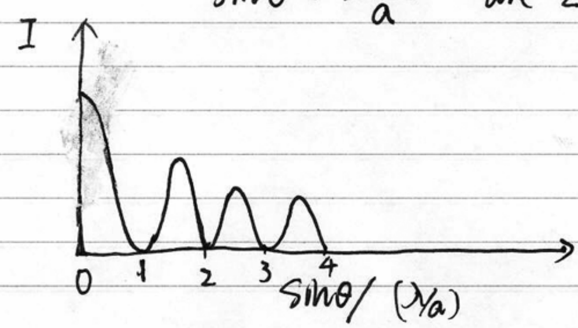
(b) To evaluate $\lim_{x\to 0} \frac{\sin x}{x}$, we can apply L'Hopital's rule, or use Taylor expansion. Let's do the former.

$$\lim_{x\to 0} \frac{\sin x}{x} = \frac{\frac{d}{dx}\sin x}{\frac{d}{dx}x} = \frac{\cos x}{1} = 1$$

c) when $\sin\beta = 0$, $\beta \neq 0$ $I(\theta) = 0$

i.e. $\frac{\pi a \sin\theta}{\lambda} = n\pi$ $n = 1, 2, \cdots$

$\therefore$ $\sin\theta = \frac{n\lambda}{a}$ are zeros of $I(\theta)$



as $\sin\theta$ increases, the maximum value of $I(\theta)$ ~~wat~~ decreases, as shown in the figure left.

(d) when θ is small

$$\sin\theta \approx \tan\theta \approx \theta \doteq \frac{y}{L}$$

$$\therefore I(y) = I_0 \left(\frac{\sin\beta}{\beta}\right)^2$$

$$\beta = \frac{\pi a \sin\theta}{\lambda} \doteq \frac{\pi a y}{L\lambda}$$

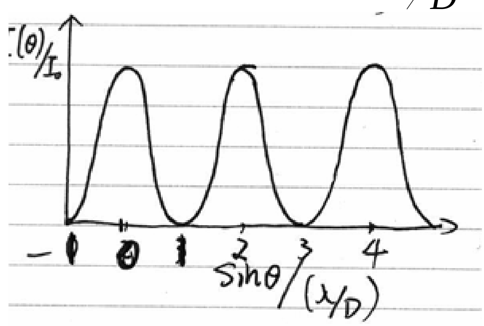
~~the~~ $$\therefore \frac{\pi a \Delta y}{L\lambda} = \pi$$

$$\Delta y = \frac{L\lambda}{a}$$

$$= \frac{10 \times 530 \times 10^{-9}}{0.5 \times 10^{-6}}$$

$$= 10.6 \text{ m.}$$

(5) *Two infinitely narrow slits.* Note that the zeros are at $\sin\theta = \lambda/D, 2\lambda/D, 3\lambda/D, \ldots$



Two finite-width slits. The above graph is "multiplied" by the single slit diffraction pattern, which describes how the light from each slit behaves as a function of angle. Note that the width of the single-slit pattern is λ / a, and since a must be less than D, the single slit "width" is wider in angle than the fringe spacing of the two-slit pattern. Therefore: