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University of Oregon; Winter 2008

Physics 352 – Optics

Problem Set 3

Due date: Wednesday, Jan. 30, **5pm**. (Turn in to the assignment to the box outside my door.)
NOTE: Due to the upcoming midterm, **no late homework will be accepted.**

Reading: Notes Sections 2.5, 3 (all), and 4.1-4.5.

(**1**, 2 pts.**)** ***N*-slit interference.** In class we derived an expression for the *N* slit diffraction pattern:

$$I(\theta) = I_1 \frac{\sin^2(\pi N D \sin\theta / \lambda)}{\sin^2(\pi D \sin\theta / \lambda)},$$

where D is the spacing between slits, λ is the wavelength, θ is the angle with respect to the barrier normal, and I_1 is the intensity of a single beam at a single slit. Show that the maximal intensity is $I_{\max} = N^2 I_1$. (Note, as we discussed in class, that the maxima occur at the angles for which the denominator of the above expression is zero, e.g. $\theta = 0$.) *Hint:* One way to do this is to Taylor expand $I(\theta)$ for small $\sin\theta$.

(2, 4 pts.**) Diffraction gratings and extrasolar planets.** As discussed in class and the notes, a diffraction grating (an *N*-slit barrier) can be used to measure the spectrum of light, i.e. the intensity as a function of wavelength. The precision of the spectral measurement depends on *N*. One dramatic recent use of this technique has been the discovery of planets outside the solar system.

The motion of stars alters the wavelength of the light they emit via the Doppler effect. (If you've never heard of the Doppler effect, look it up in a first-year textbook.) A large, orbiting planet will alter the star's speed due to gravitational interactions, and hence will alter the spectrum. For speeds much smaller than c, the speed of light, the **shift in wavelength** of the emitted light is given by $\Delta\lambda = \frac{\Delta v}{c}\lambda$, where Δv is the perturbation of the speed and λ is the wavelength of the light in the absence of any perturbation.

Suppose you have a diffraction grating with $N = 10^5$ slits attached to your telescope. Suppose some weird sort of star only emits light at one wavelength. (If you want, pretend it's orange, $\lambda \approx 600$ nm.) **Determine a rough value for the precision** with which you and your grating can determine Δv. This problem requires little math, but some clear thinking. Think about the positions of the zeros of the diffraction pattern, and read Sections 2.2-2.3 of the notes. Consider small

(*Note:* The above *N* is a realistic value for the diffraction gratings that scientists use. However, the precision in Δv that one can obtain is around 10 m/s, much better than what you've derived above. The reason: real stars emit a continuous spectrum, and we can determine the shift in wavelength of the entire "shape" more precisely than we can resolve the position of a single wavelength. This isn't an obvious statement, but you don't have to worry about it.)

(3, 1 pt.**) Single-slit diffraction figure.** You showed in the last problem set that for single-slit interference, $I(\theta) = I_0\left(\sin(\beta)/\beta\right)^2$, where $\beta = \pi a \sin\theta / \lambda$. We derived this by building on the *N*-slit case, and considered a plane wave diffracted by an aperture with the light then reaching a far-off slit. I mentioned in class that all of these derivations work just as well for "receiving" light from a far-off source. (See also section 2.3 of the notes.) **Draw a figure,** oriented like Fig. 2.3 of the notes, that illustrates three different paths for parallel rays traveling left **from** a distant source on the right, reaching the aperture at three different points. Indicate the different path lengths on your figure. (Notice that the show the same path length differences as rays traveling right, and so $I(\theta)$ is the same function shown above.)

(4, 4 pts.**) The Sun.**

(a, 1 pt.**)** Measure the angular diameter of the sun – i.e. the angle it spans in the sky – to one significant figure. No, this doesn't have anything to do with what you've learned in the course so far, but it's a simple measurement that should only take you a few minutes, armed with a ruler. Think. (Don't stare at the sun; use a few layers of sunglasses, or wait for thin cloud cover. You could instead measure the angular diameter of the moon – the two are nearly identical, as we know from the fact that total solar eclipses are dramatic. The moon is full Jan. 22, and so is up at night this week. Describe your procedure.

(b, 1 pt.**)** Can your eyes "resolve" the moon? In other words, is the angular resolution of your eyes less than the angular size of the moon? *Comment #1:* For parts (*b*)-(*d*), provide rough numerical estimates – don't use a calculator. *Comment #2:* Note the implications of Problem 3, as discussed in class and in the notes, Section 2.5. *Comment #3.* Hopefully you realize that the answer to this question is "yes" – otherwise, the moon would look fuzzy!

(c, 1 pt.**)** Supposing you "saw" with radio waves, e.g. $\lambda = 1$ meter. Could you resolve the moon?

(d, 1 pt.**)** One way of improving the angular resolution of a telescope is to build a big telescope. Another way is to build two (or more) telescopes and combine their detected signals so that the "effective" aperture size is equal to the separation between the telescopes. This is the principle behind interferometry, especially popular in radio astronomy. Suppose you're building a two-dish radio interferometer, operating at a frequency of *f* = 1400 MHz. Roughly what dish separation will you need to be capable of resolving the sun (i.e. providing an angular resolution smaller than the angular diameter of the sun)?

(5, 3 pts.**) Fourier Transforms and Apertures.** Last quarter we learned about Fourier Series, writing an arbitrary function *y*(*x*) defined over a finite interval as an infinite sum of waves with discrete wavenumbers, *k*. The continuum analog of this, for *y*(*x*) defined over an infinite interval, is the Fourier Transform; all wavenumbers are allowed, and the infinite sum becomes an integral. The Fourier Transform *Y*(*k*) of the function *y*(*x*) is:

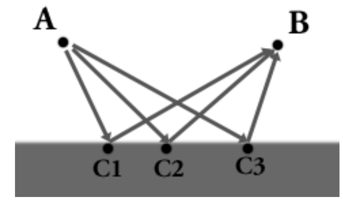
$$Y(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y(x) e^{-jkx} dx.$$

Show that the **square** of the Fourier Transform of a "rectangular aperture" function:

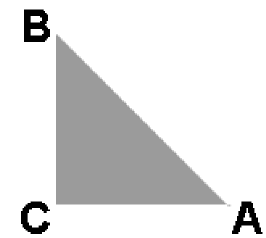
$$y(x) = \begin{cases} 1/a, \; if \; |x| \le a/2 \\ 0, \; if \; |x| > a/2 \end{cases}$$

has the same mathematical form as the diffraction pattern of a single slit, especially if we assign this "*k*" to correspond physically to $2\pi\sin\theta/\lambda$. The principle illustrated in this problem is in fact generally true: the intensity pattern of an aperture is given by the aperture's Fourier Transform.

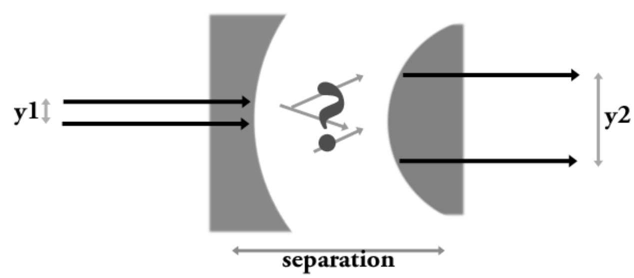
(6, 3 pts.**) Law of Reflection.** In class, we used Fermat's principle to derive Snell's Law of Refraction. Here, use Fermat's principle to **prove the Law of Reflection:** that the incident and reflected rays at a surface make the same angle with respect to the normal. The figure shows three of infinitely many possible paths that connect A and B by bouncing off the surface; they strike the surface at various distances along it, denoted by the points C_1-C_3.



(7, 4 pts.**) A prism.** A prism, ABC, of index of refraction n is configured such that angle BCA = 90° and angle CBA is 45°. Mr. K. will pay you $100 if you can send a beam through face AC (at any incident angle you want) and have light be transmitted through face BC. In other words, if your beam is totally internally reflected at BC, you get nothing. Your beam **is not** allowed to hit face AB during its travels. Can Mr. K. find an n such that he certainly won't have to pay you? If so, what's the smallest n he can choose?



(8, 9 pts.**) Galilean Telescope.** Consider the pair of thin lenses shown – an arrangement, by the way, is known as a Galilean Telescope. The first, a planar-concave lens, has a radius of curvature $|R|$ =200 cm on the curved side. The second, a planar-convex lens, has a radius of curvature $|R|$ =450 cm on the curved side. Both lenses are made of glass (n =1.5).



(a, 1 pt.**)** What are R_1 and R_2 for each lens, using the "proper" sign conventions?

(For b-d) Suppose a plane wave is incident from the left, as indicated by the rays drawn, and we want a plane wave to exit from the right, as drawn.

(b, 3 pts.**)** What separation must the lenses have? Express your answer (i) symbolically in terms of the focal lengths and separation of the lenses, and (ii) numerically given the above parameters. You may wish to consider each lens independently first. See also part (c).

(c, 2 pts.**)** Draw a diagram of the lenses that includes the rays in between the lenses, and indicate any real or virtual image points.

(d, 3 pts.**)** Suppose the incident plane wave has width y_1 (for example, it comes from a laser whose diameter is y_1). What is the width y_2 of the exiting beam? Express your answer (i) symbolically in terms of the focal lengths of the lenses, and (ii) numerically given the above parameters. A clear ray diagram will help you figure this out. This arrangement, by the way, is commonly used as a "beam expander."