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University of Oregon; Winter 2008

# Physics 352

## Problem Set 4

**Due date:** Friday, Feb. 8, **5pm**. (Turn in to the assignment to the box outside my door.)

**Reading:** Notes on the Binomial Distribution.

**PLEASE NOTE:** This problem set is due in **two days.** However, it is **very, very short.** The exercises are intended to review concepts that you've seen earlier.

**(1,** 3 pts.**) Flipping coins.** Suppose you flip $N$=3 fair coins (i.e. coins with an equal probability of "heads (H)" and "tails (T)").

**(a,** 1 pt.**)** Make a list of all possible outcomes. How many are there?

**(b,** 1 pt.**)** Make a table of the number, $g(k)$, of outcomes with $k$ heads, where $k = \{0, 1, 2, 3\}$. This is referred to as the multiplicity of the "macrostate" with $k$ heads.

**(c,** 1 pt.**)** Show that the $g(k)$ you counted agrees with that given by the binomial distribution:

$$g(k) = \frac{N!}{(N-k)!k!}$$

(The exclamation marks indicate factorials).

**(2,** 7 pts.**) Logarithms.** The natural logarithm function, $\ln$, is the inverse of the exponential function. It can be defined by the relation

$$e^{\ln x} = x \text{ , } [\textit{Equation 1}]$$

or equivalently

$$\ln\left(e^{x}\right) = x \quad [\textit{Equation 2}]$$

for any positive real number $x$.

**(a,** 1 pt.**)** Graph $\ln(x)$.

**(b,** 1 pt.**)** Starting from Equation 1, prove that $\frac{d}{dx}\ln x = \frac{1}{x}$. (*Hint:* Differentiate each side of Eq. 1 with respect to $x$.)

**(c,** 2 pts.**) Prove** from either Eq. 1 or 2 that $\ln(xy) = \ln(x) + \ln(y)$ for any positive numbers $x$, $y$.

**(d,** 1 pt.**)** Using your result from part (c), prove that $\ln(x^n) = n\ln(x)$ for integer $n$.

**(e,** 2 pts.**)** Show that for small $x$, $\ln(1+x) \approx x$.

**(3,** 1 pt.**) Partial Derivatives.** The partial derivative $\frac{\partial}{\partial x} f(x, y)$ is the derivative with respect to *x*, keeping *y* fixed. For example: $\frac{\partial}{\partial y}\left(x^2 + xy + y^3\right) = x + 3y^2$. The notation $\frac{\partial^2}{\partial x \partial y}$ indicates taking the partial derivative with respect to *y*, then the partial derivative of this with respect to *x*. The volume of a cylinder of radius $r$ and height $h$ is $V = \pi r^2 h$. Show that $\frac{\partial^2 V}{\partial r \partial h} = \frac{\partial^2 V}{\partial h \partial r}$. (In general, for "nice" functions, the order of differentiation does not matter.)