Prof. Raghuveer Parthasarathy
University of Oregon
Physics 352; Winter 2008

Problem Set 4: SOLUTIONS

(1) Flipping coins.

(a) Possible outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT – there are **eight.**

(b)

k	g(k)
0	1
1	3
2	3
3	1

(c)

k	$\frac{N!}{(N-k)!k!}$
0	$\frac{3!}{(3-0)!0!}=\frac{3!}{3!}=1$
1	$\frac{3!}{(3-1)!1!}=\frac{3!}{2!}=3$
2	$\frac{3!}{(3-2)!2!}=\frac{3!}{2!}=3$
3	$\frac{3!}{(3-3)!3!}=\frac{3!}{3!}=1$

The table is the same as that of (b).

(2) Logarithms.

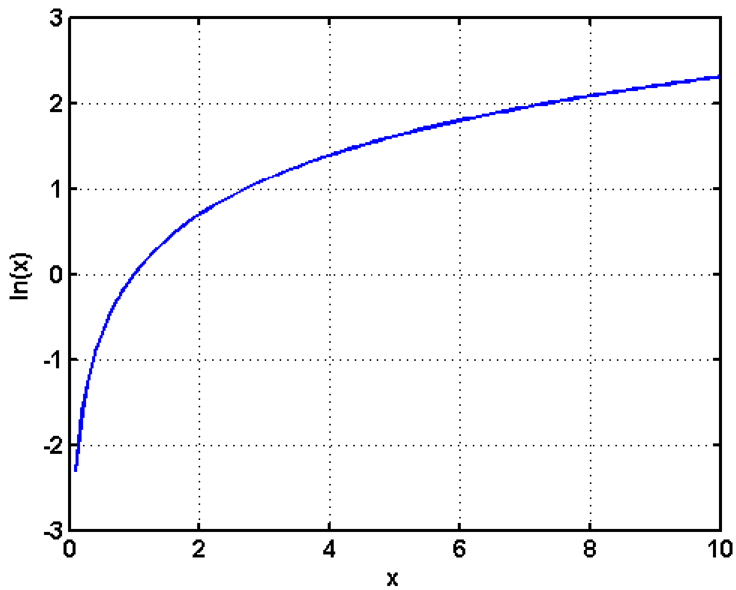
(a) see plot, below. Note that ln(1) = 0.

(b) Start from $e^{\ln x} = x$, [*Equation 1*]. Differentiate. Using the "chain rule" on the left side: $\frac{d}{dx}e^{\ln x} = e^{\ln x}\frac{d}{dx}(\ln x) = x\frac{d}{dx}(\ln x)$. The right side is simply: $\frac{d}{dx}(x)=1$. Therefore: $x\frac{d}{dx}(\ln x)=1$, so $\frac{d}{dx}(\ln x)=\frac{1}{x}$.

(c) From eq. 1, $e^{\ln(xy)} = xy$. $e^{\ln x} = x$ and $e^{\ln y} = y$, therefore $e^{\ln(xy)} = e^{\ln x}e^{\ln y}$. Taking the log of each side, $\ln(xy) = \ln(x) + \ln(y)$.

(d) $\ln(x^n) = \ln\left(xxxx...x\right)$ (n factors of x in the product) $= \ln(x) + \ln(x) + ... \ln(x)$ (n terms in the sum) via part (c), so $\ln(x^n) = n\ln(x)$.

(e) Taylor expansion: $\ln(1+x) = \ln(1+x)\big|_{x=0} + \frac{d}{dx}\ln(1+x)\Big|_{x=0} x + \frac{d^2}{dx^2}\ln(1+x)\Big|_{x=0} x^2 + \ldots$ Using part (b), this is: $\ln(1+x) = 0 + \frac{1}{1+x}\Big|_{x=0} x + \ldots = x + \ldots$ To lowest order, $\ln(1+x) \approx x$.



(3.) Partial Derivatives. $V = \pi r^2 h$. $\frac{\partial V}{\partial r} = 2\pi rh$. $\frac{\partial V}{\partial h} = \pi r^2$.

$$\frac{\partial^2 V}{\partial r \partial h} = \frac{\partial V}{\partial r}\left(\pi r^2\right) = 2\pi r$$
$$\frac{\partial^2 V}{\partial h \partial r} = \frac{\partial V}{\partial h}\left(2\pi rh\right) = 2\pi r$$

The two are the same.