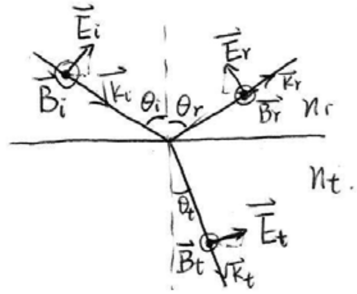
Prof. Raghuveer Parthasarathy
University of Oregon
Physics 352; Winter 2008

Problem Set 5: SOLUTIONS

1. A Fresnel Relation.

The tangential components of the electric field $\vec{E}$ and $\vec{B}/\mu$ are continuous across the boundary.



$$\therefore \begin{cases} B_{oi} + B_{or} = B_{ot} & (\text{non magnetic materials: } \mu \approx \mu_0) \\ E_{oi}\cos\theta_i - E_{or}\cos\theta_r = E_{ot}\cos\theta_t \end{cases}$$

and $\quad B_0 = E_0/v \qquad v_i = v_r \qquad \theta_i = \theta_r$

$$v_i = \frac{c}{n_i}$$

$$\therefore \begin{cases} E_{oi}\cos\theta_i - E_{or}\cos\theta_i = E_{ot}\cos\theta_t \\ \dfrac{E_{oi}n_i}{c} + \dfrac{E_{or}n_i}{c} = \dfrac{E_{ot}n_t}{c} \end{cases} \Rightarrow E_{ot} = \frac{E_{or}n_i + E_{oi}n_i}{c}\, c$$

$$\therefore E_{oi}\cos\theta_i - E_{or}\cos\theta_i = (E_{oi}n_i + E_{or}n_i)/n_t \cdot \cos\theta_t.$$

$$E_{oi}\left(\cos\theta_i - \frac{n_i}{n_t}\cos\theta_t\right) = E_{or}\left(\frac{n_i}{n_t}\cos\theta_t + \cos\theta_i\right)$$

$$\therefore r_{\parallel} = \left(\frac{E_{or}}{E_{oi}}\right)_{\parallel} = \frac{\cos\theta_i - \frac{n_i}{n_t}\cos\theta_t}{\cos\theta_i + \frac{n_i}{n_t}\cos\theta_t}$$

$$= \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t}$$

2. Air / glass interface.

$$r_{\parallel} = \left(\frac{E_{or}}{E_{oi}}\right)_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

$$r_{\perp} = \left(\frac{E_{or}}{E_{oi}}\right)_{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

for $n_i = 1 \quad n_t = 1.5$

$$r_{\parallel} = \frac{1.5\cos\theta_i - \cos\theta_t}{1.5\cos\theta_i + \cos\theta_t}$$

$$n_i \sin\theta_i = n_t \sin\theta_t = n_t\sqrt{1-\cos^2\theta_t}$$

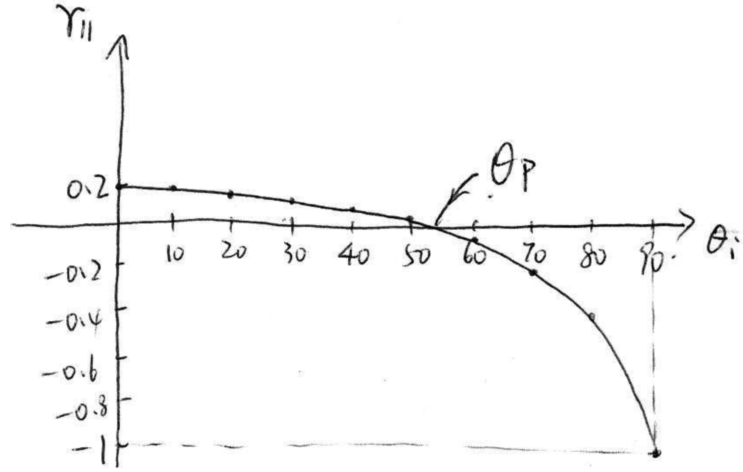
$$\therefore \cos\theta_t = \sqrt{1-\left(\frac{n_i}{n_t}\right)^2 \sin^2\theta_i} = \sqrt{1-\frac{4}{9}\sin^2\theta_i} = \sqrt{\frac{5}{9}+\frac{4}{9}\cos^2\theta_i}$$

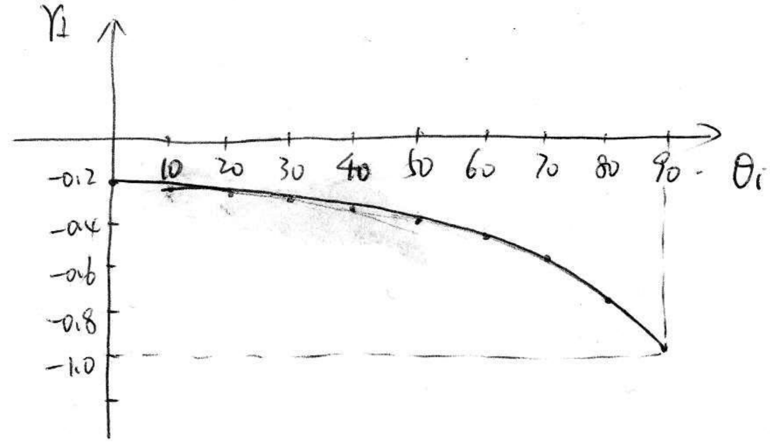
$$\therefore r_{\parallel} = \frac{1.5\cos\theta_i - \sqrt{\frac{5}{9}+\frac{4}{9}\cos^2\theta_i}}{1.5\cos\theta_i + \sqrt{\frac{5}{9}+\frac{4}{9}\cos^2\theta_i}}$$

$\theta =$	0	10	20	30	40	50	60	70	80	90
$r_{\parallel}$	0.2	0.19	0.18	0.16	0.12	0.06	-0.04	-0.2	-0.49	-1

$$r_{\perp} = \frac{\cos\theta_i - 0.5\sqrt{5+4\cos^2\theta_i}}{\cos\theta_i + \frac{1}{2}\sqrt{5+4\cos^2\theta_i}}$$

θ	0	10	20	30	40	50	60	70	80
$r_{\perp}$	-0.2	-0.204	-0.215	-0.237	-0.27	-0.33	-0.42	-0.55	-0.74





(b) the Brewster Angle is the θ_i when $r_{\parallel} = 0$

from the plot, we can see $\theta_p \doteq 56°$

it agrees with $\theta_p = \arctan\left(\frac{n_t}{n_i}\right) = \arctan 1.5$

$\doteq 56.3°$.