
Credit: smbc-comics.com

# Symmetries in Physics (classrooms)

UO Quarknet 2021
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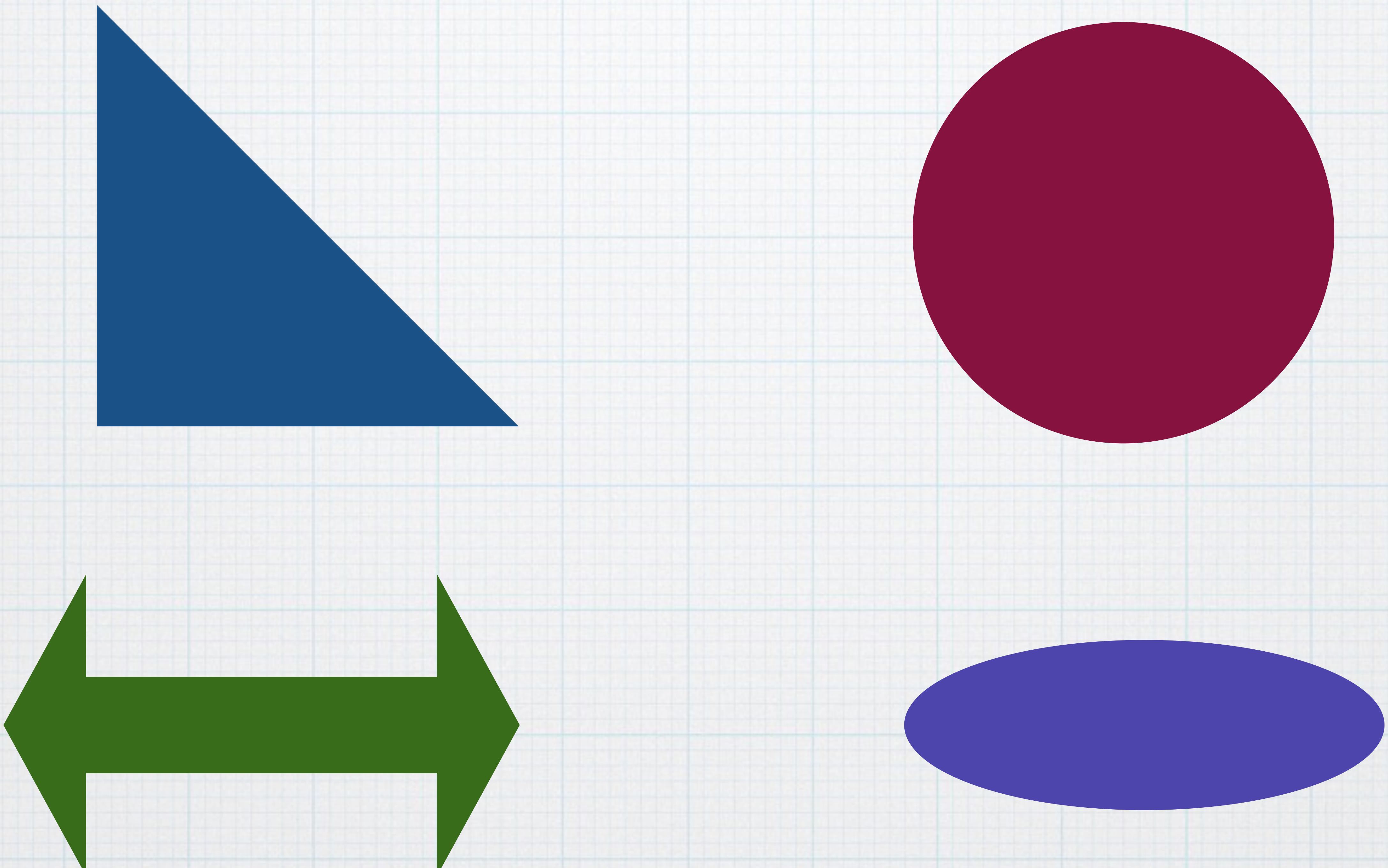
# Outline

* Use of symmetries in physics
    * Revisit results from a different (more general) viewpoint
    * Check results/calculations
    * Symmetries and conservation laws
    * Organize theories/experiments

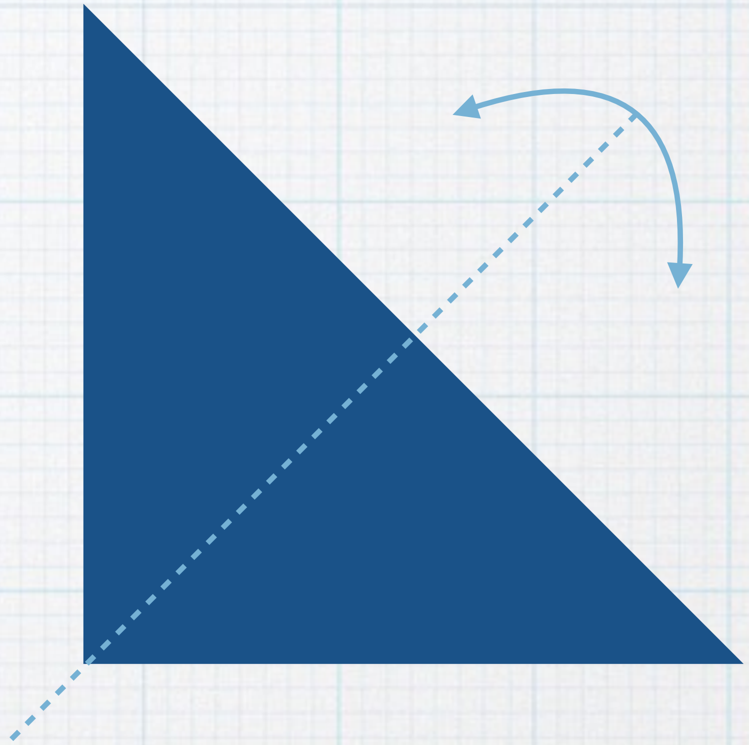
# Objects

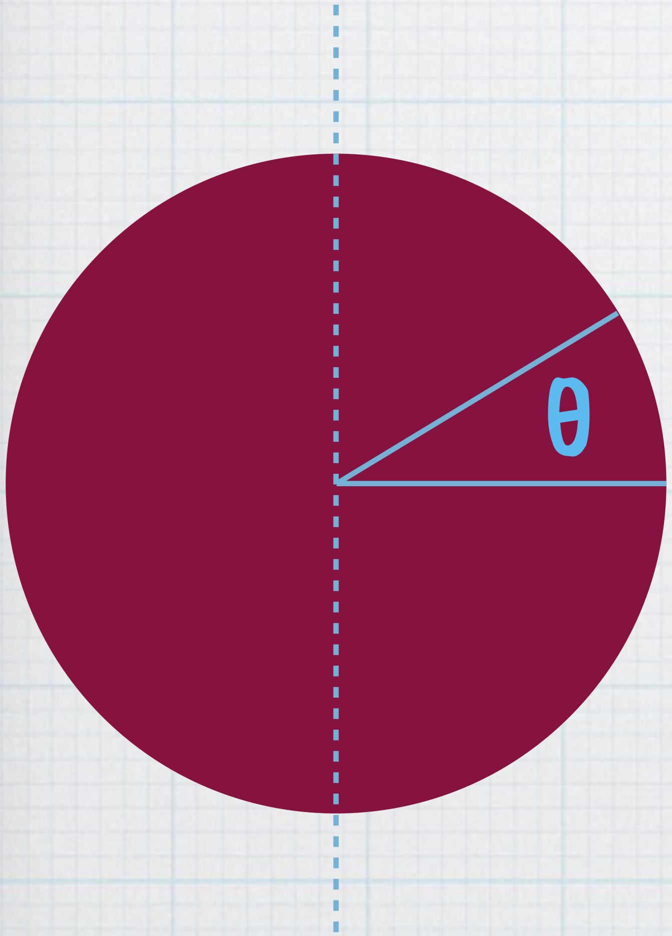


Which of these objects is symmetric? Which one is most symmetric?

# Symmetries

Symmetries of objects can be characterized by how many transformations of it leave it unchanged (invariant)

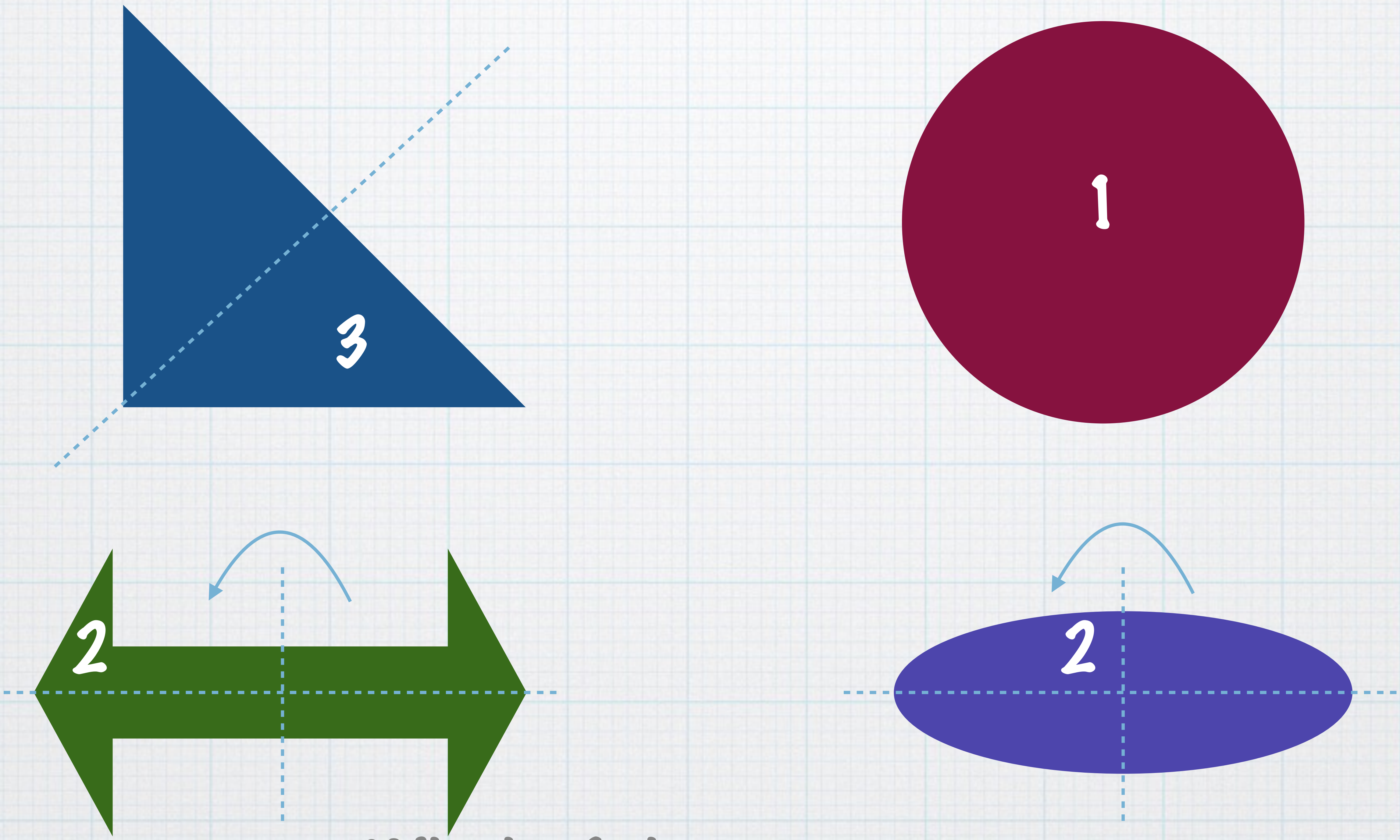





Circle can be reflected across any diameter
It can also be rotated by any angle around center

To count these transformations we can use one reflection (discrete) and a rotation angle (continuous)

# Objects




Which of these objects is symmetric? Which one is most symmetric?

# Symmetries in Physics

Symmetries in physics are transformations that do not change the laws of physics

For example, all evidence suggests that the strength of the force of gravity does not change with time of day or position in space

Technically, we would say that gravity is invariant under translations of time or space (or in laymen's terms, it doesn't depend on when or where you do an experiment)

# Example of Spatial Translations

Motion doesn't matter on "absolute" value of coordinates, just "relative" distances, which is unchanged if you translate all spatial coordinates by same distance

$$F = (Gm_1\ m_2)/r^2$$

# List of Symmetries of Nature

## Discrete

Parity ($x \rightarrow -x$, $y \rightarrow -y$, $z \rightarrow -z$)

Time Reversal ($t \rightarrow -t$)

Charge conjugation (particle goes to antiparticle)
(Caveat: Weak interactions are only invariant under all three at the same time (i.e. CPT))

## Continuous

Time translations ($t \rightarrow t + \Delta t$)

Spatial translation ($x \rightarrow x + \Delta x$, $y \rightarrow y + \Delta y$, $z \rightarrow z + \Delta z$)
Rotations (Angle and Axis of Rotation)

# Consequence of Continuous Symmetry

All continuous symmetries result in a conserved quantity

Time translations ↔ Energy conservation

Spatial Translation ↔ Momentum conservation

Rotations ↔ Angular Momentum conservation

Noether's Theorem by mathematician Emmy Noether made this connection using least action principle form of classical mechanics (see Feynman lectures Vol. II Ch. 19, "Arrival" Movie or short story "Story of Your Life" by T. Chiang)

# Conservation example

Two masses that interact via a translationally invariant potential, $V = f(x_1 - x_2)$

Because of this special form
$F_1 = -dV/dx_1 = + dV/dx_2 = -F_2$
the forces obey Newton's third law which implies momentum conservation

# Energy Non-conservation (arXiv:physics/0001061)

Imagine strength of gravity was weaker on Tuesdays....

Then you could create more energy by lifting water on Tuesdays and letting it out on other days

Bonneville Dam by Eric Guinther

# Understanding Our World

The electron and proton are stable particles, why is that?

Electron is the lightest (in mass) charged particle, decay has to conserve charge and energy
$e \rightarrow X$, but X has to have same charge as electron and due to energy from rest mass, Energy (X) >= $m_e c^2$

However, proton could decay into
proton $\rightarrow$ positron + neutral pion, but
limits on the rate of this is > 1 per $10^{34}$ years
(aside on Tim's effective field theory talk)

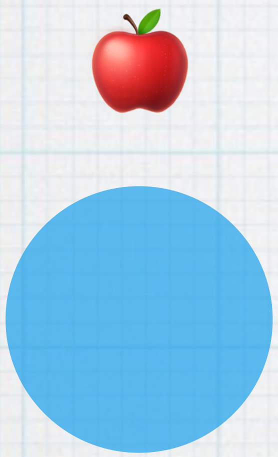
# Takeaway 1:
# Continuous Symmetries Imply Conserved Quantities (Thanks to E. Noether)

# Transforming Solutions

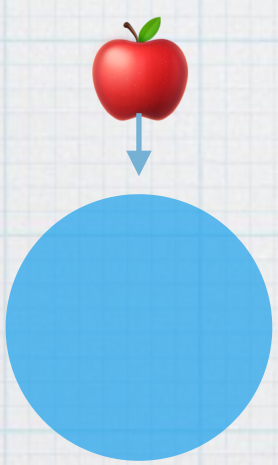
Since the laws are unchanged, transformations of solutions to the dynamical equations when transformed give valid (potentially new) solutions

In classical mechanics, if initial conditions are the same, there is a unique solution, potentially allowing one to determine solution

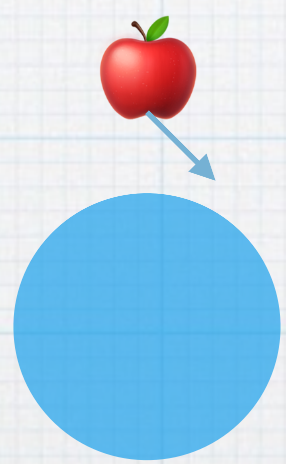
# Classical Example



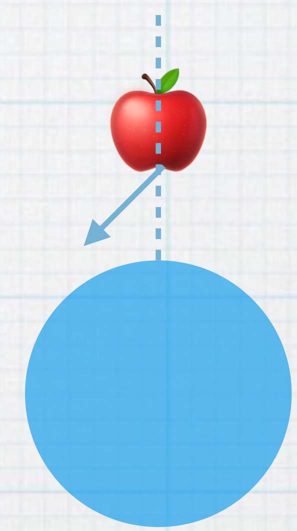
Q: Apple is dropped above Earth, with no initial velocity, from symmetry which way does it start moving?



A: Of course downwards!

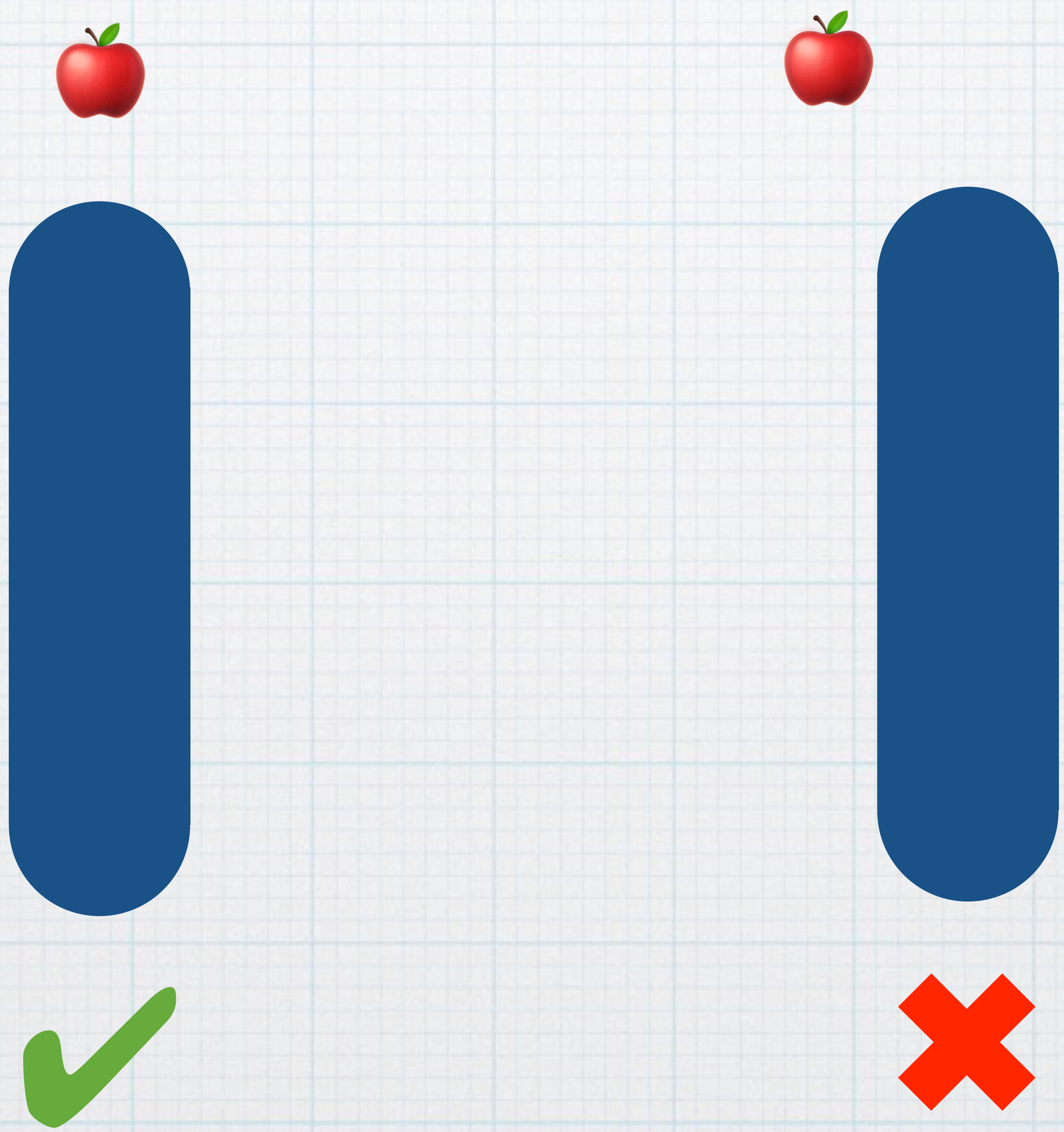


A': But what is wrong with this?



A'': By rotating around vertical all non-downward directions are solutions!

# Works with anything with axial symmetry



In 2nd you can determine that path has no z (in/out page) component by reflection z -> -z

# Quantum Transformations lead to new solutions!

In quantum mechanics, can only specify total angular momentum and along some axis

e.g. Orbital angular momentum $|L|^2 = \ell(\ell+1)\,\hbar^2$, $L_z = m\,\hbar$

where $\ell = 0, 1, 2, 3\ldots$ and $m = -\ell, -\ell+1, \ldots, \ell$

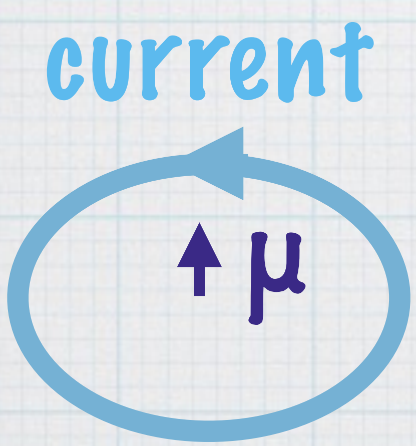
Starting with electron orbital in a single $\ell$, $m$ orbital, rotations change $m$, showing they must have the same energy and these degeneracies can be seen by intensities in emission, absorption lines

# Takeaway 2: Symmetries on solutions generate other solutions

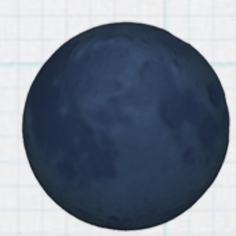
# Physics of Magnetic Dipole moments

Magnetic Dipole moments of fundamental particles (see talks by Jim, Graham) are allowed by these symmetries

Here, let's consider time reversal (TR) and rotations
Under time reversal: positions are unchanged, but velocities are reversed



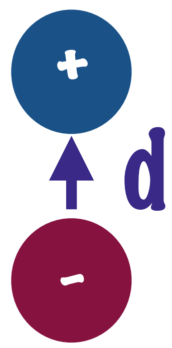

Magnetic Dipole Moment
μ is a vector whose direction changes with TR



Only vector a particle at rest has is spin S, but S -> -S under TR, so $\mu = \lambda S$ with $\lambda$ constant works

# Physics of Electric Dipole moments

Electric Dipole moments of fundamental particles are not allowed by these symmetries!




Electric Dipole Moment
d is a vector whose direction does not change with TR

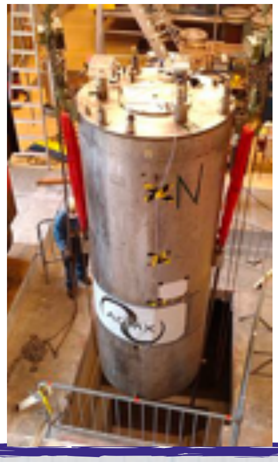
d = λ S
with λ constant doesn't work since it changes with TR!

Discovery of EDM would indicate TR symmetry breaking!
Best experimental limits are on neutron and electron, probing charge separations of $10^{-26}$ cm, $10^{-29}$ cm!

# Cosmological Connections

Strong Interactions allows a term that would give much too large of a neutron EDM

Peccei-Quinn solution leads to axion particle which can be dark matter



ADMX expt. at UW

To generate a Matter-Antimatter Asymmetry requires Time Reveral violation beyond what has been observed

Many places to look for TR violation, EDMs, neutrino oscillations, B-meson factories

# Takeaway 3:
# Symmetries organize directions in theoretical and experimental research

# Takeaways

* Symmetries allow us to reinterpret and principles
* Symmetries can be used to check/determine answers
* Motivate directions in theories and experiment (e.g. search for proton decay, electric dipole moments, cosmological connections)

El Hedim Square, Morocco


Symmetry Magazine

# Thanks for your time and have a great summer!