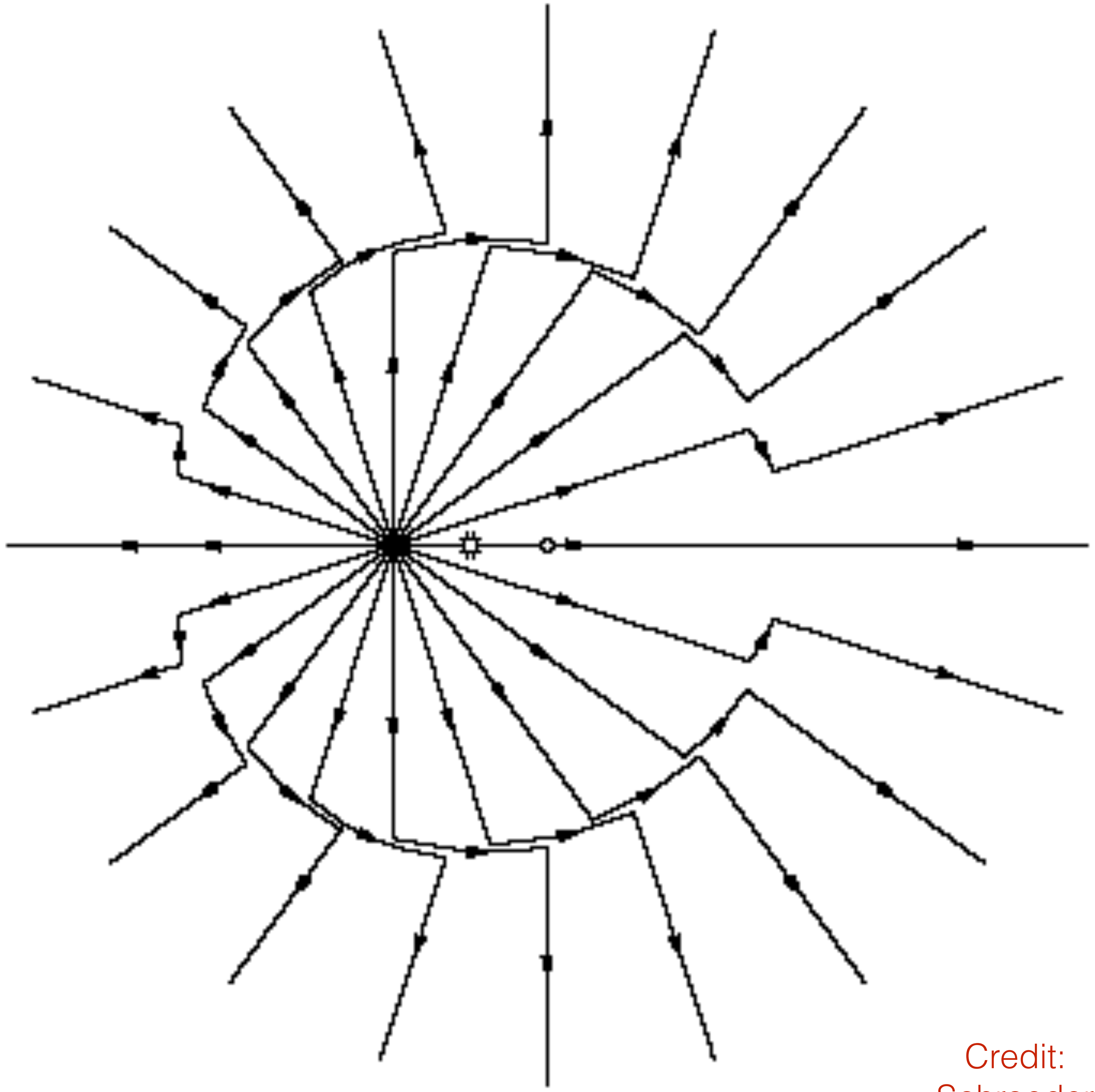
Lessons from Accelerating Electrons in Theory and Practice



Credit: Schroeder

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U. Oregon Quarknet 2026

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My Goals for Today's Discussion

- Motivate why charged particles undergoing acceleration emit electromagnetic radiation
- Derive Larmor formula, power of emitted radiation for a given acceleration, using dimensional analysis
- Applications
  - Classical instability of atoms
  - Relativistic generalization of Larmor
    - Power constraints on circular lepton colliders
    - Synchrotron Radiation

Pictorial Explanation of Electromagnetic (EM) Radiation

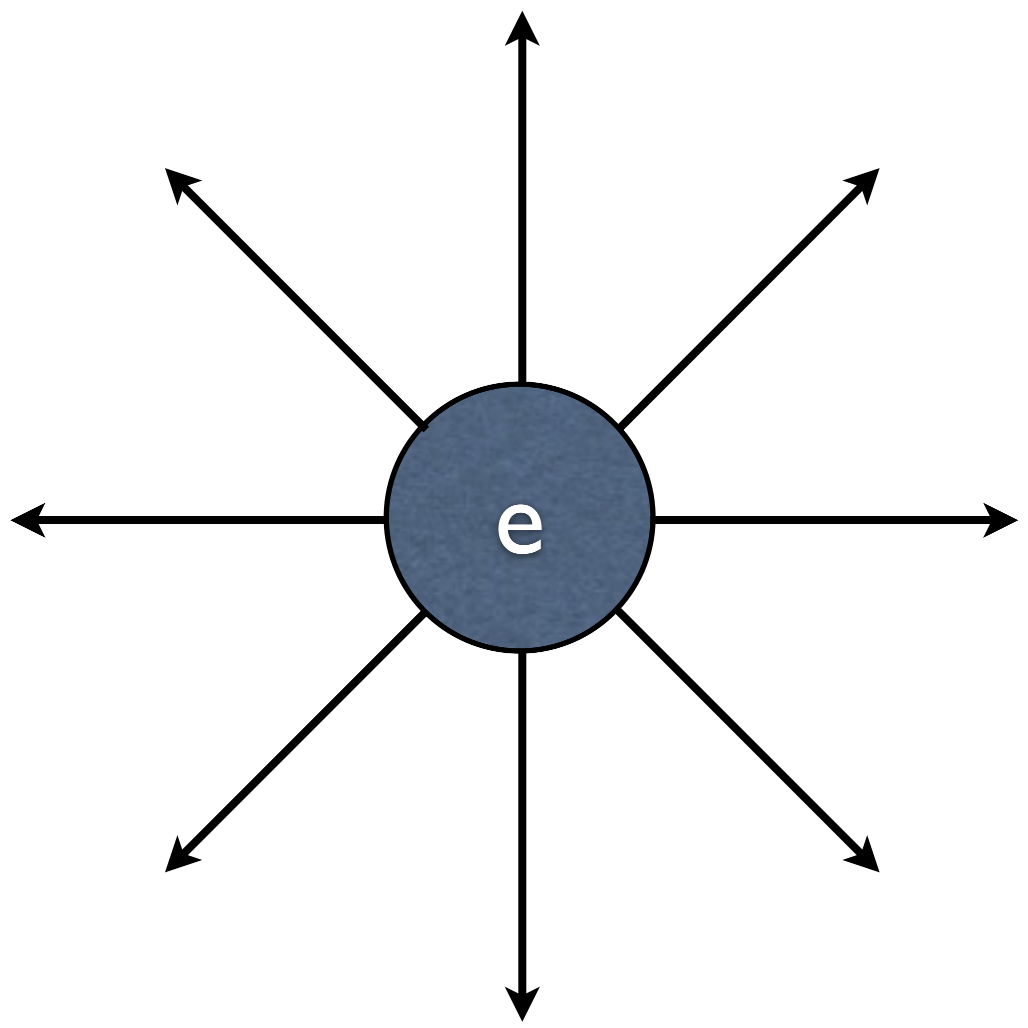
I presented this in 2016 Quarknet
see "Electricity and Magnetism" by Purcell and discussion from Schroeder (link)

Key Facts

No signals travel faster than speed of light (special relativity)

Electric fields around an electric charge point outward

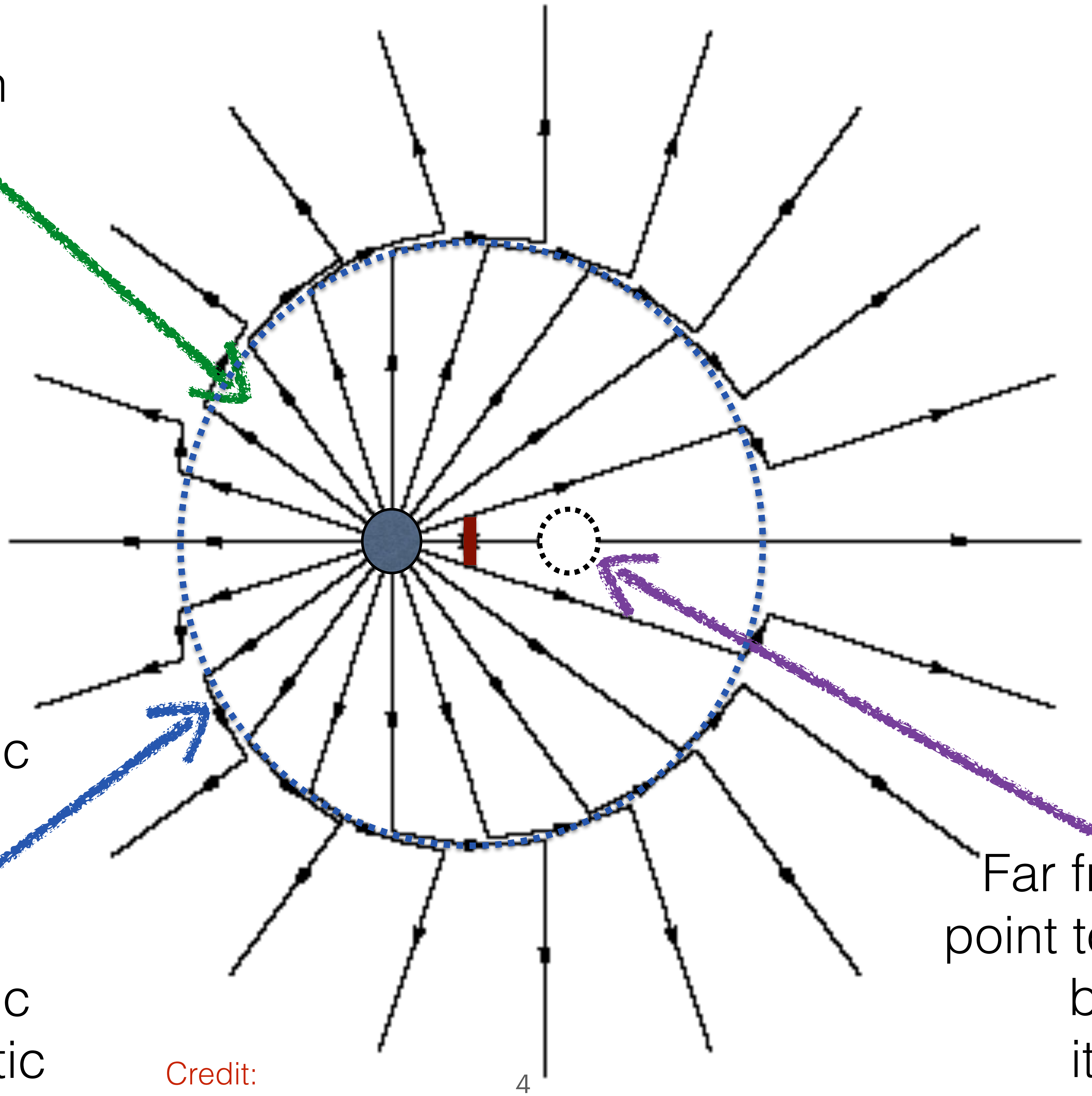
Electric fields cannot end



Consider a thought experiment where an electron moves to the right at constant speed, bounces off a wall, and goes left at same speed

Thought Experiment

Close to electron, electric fields point to where charge is because they are close enough to receive the information that the bounce occurred and “know” where the electron is



Analogy
To create water waves, you can’t create them by smoothly moving hand through water, you need to accelerate through it

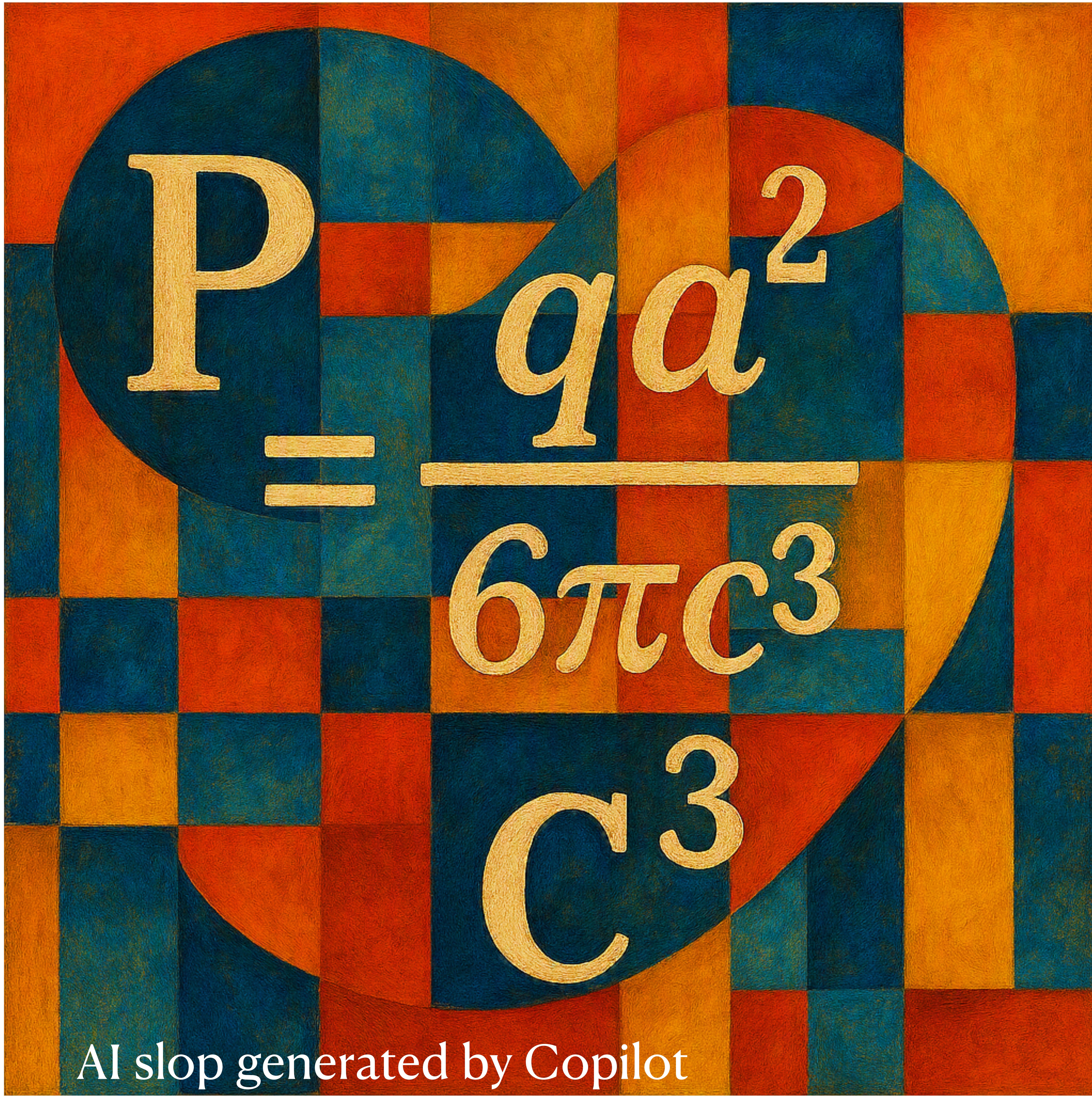
At the transition circle, electric fields must bend and connect.
This circle expands at speed of light and this electric field kink is the electromagnetic radiation

Credit: Schroeder

Far from electron, electric fields point to where electron would have been w/o bounce, since it is too far to receive the information of the bounce

Larmor Formula

How to connect acceleration to power emitted in EM radiation



Larmor Formula Derivation by Dimensional Analysis

As we've argued, a charge in motion
will emit EM radiation only if it is accelerating

Thus, we don't expect power in EM radiation to depend on particle's velocity (or position)

Lets use dimensional analysis to pin down dependence, where [X] = units of X

We expect that the power (P) of EM radiation emitted can depend on charge of particle (q), speed of light (c), and acceleration (a)

Units of charge

To get us in the right direction, lets remember the electrostatic potential energy between two particles of charge q_1, q_2, separated by a distance r

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

So $[U] = \mathrm{J} = \left[\frac{q_1 q_2}{4\pi\epsilon_0 r}\right] = \left[\frac{q_1 q_2}{4\pi\epsilon_0}\right]/\mathrm{m}$ or $\left[\frac{q^2}{4\pi\epsilon_0}\right] = \mathrm{J}\cdot\mathrm{m} = \mathrm{kg}\cdot\mathrm{m}^3/\mathrm{s}^2$

Thus, if Larmor formula only depends on powers of these three variables, we have $P = \left(\frac{q^2}{4\pi\epsilon_0}\right)^x c^y a^z$

or $P = \mathrm{J/s} = \mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}^3 = (\mathrm{kg}\cdot\mathrm{m}^3/\mathrm{s}^2)^x(\mathrm{m/s})^y(\mathrm{m/s}^2)^z$

From kg units, we get x=1
then from m units (y+z = -1)
and s units (-y-2z = -1), so
y = -3 and z = 2

Larmor Formula Prediction by Dim'l Analysis

$$P_{dim'l} = \left(\frac{q^2}{4\pi\epsilon_0}\right)^1 c^{-3}a^2 = \frac{q^2a^2}{4\pi\epsilon_0 c^3}$$

Precise Larmor formula is off only by factor of (2/3)

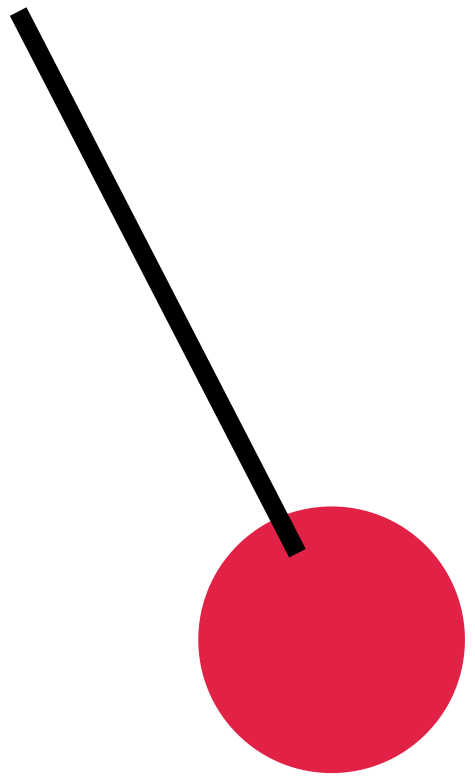
$$P = \frac{q^2a^2}{6\pi\epsilon_0 c^3}$$

Caveats about dim'l analysis

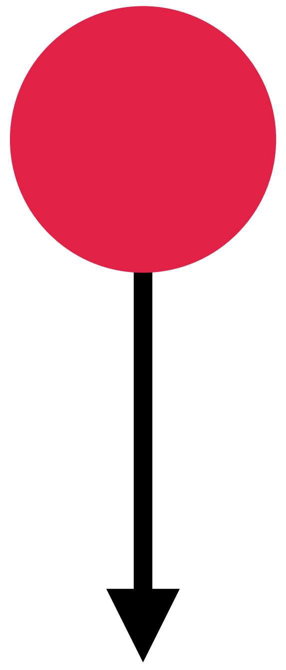
If I allowed P to depend on particle mass, would have several possible powers of the variables (graphical argument shows EM radiation doesn't depend on mass)

As we'll see, if particle is moving at relativistic speed, formula can depend on speed of particle, so this is really only valid for speeds small compared to the speed of light

Dimensional Analysis Examples to Show/Give to Students



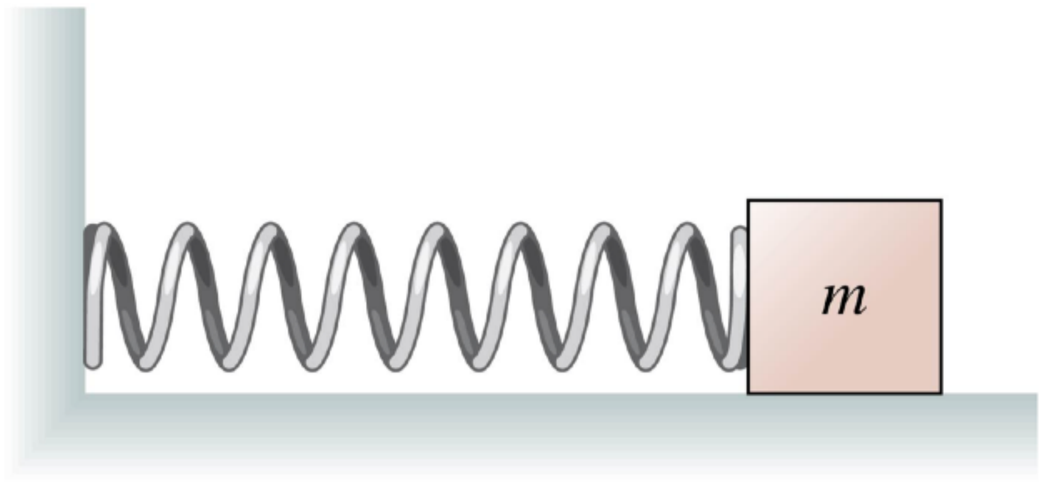
Find a formula for period T in terms of m, l, g



Find the drag force for a sphere of radius R moving at speed v through air of density ρ.

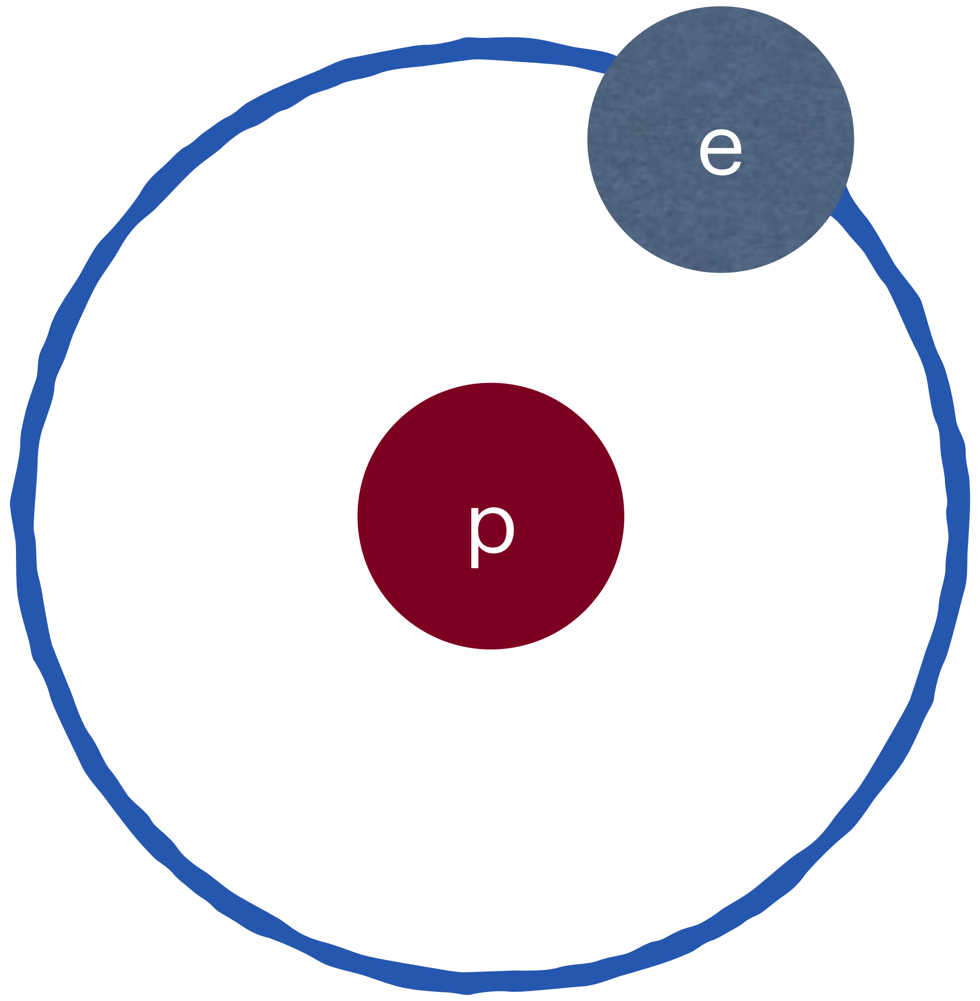
Then find terminal velocity.

Find a formula for escape velocity terms of G, M, R



Find a formula for potential energy of spring in terms of k, x

Application 1: Classical Instability of Atoms



Electron in circular orbit around nucleus has centripetal acceleration and thus must lose energy from emitting EM radiation which will cause it to fall in towards the nucleus

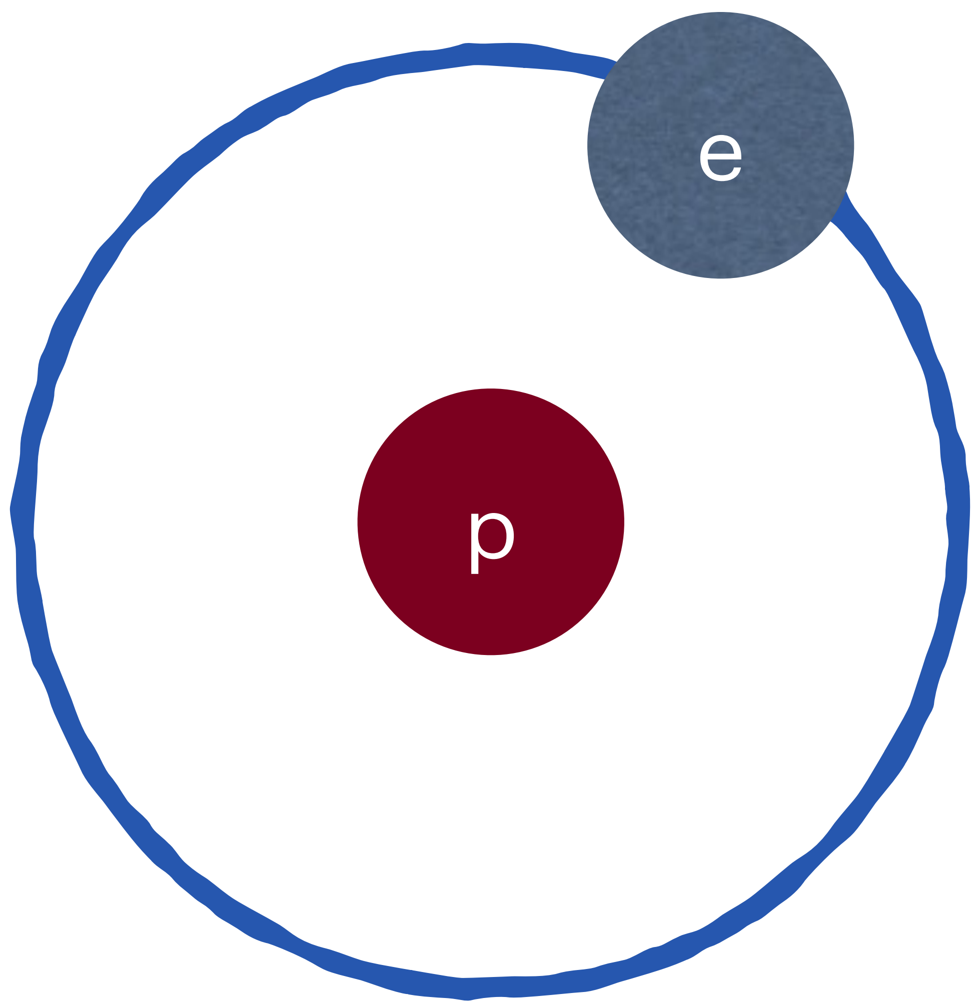
Lets estimate the time scale for this to happen for Hydrogen

For a circular orbit of radius R, we have $|\vec{F}| = \dfrac{q^2}{4\pi\epsilon_0 R^2} = m_e \dfrac{v^2}{R}$

so $\dfrac{1}{2}m_e v^2 = \dfrac{q^2}{8\pi\epsilon_0 R} = -\dfrac{1}{2}\left(-\dfrac{q^2}{4\pi\epsilon_0 R}\right) = -\dfrac{1}{2}U$

Thus $E = \dfrac{1}{2}m_e v^2 + U = \dfrac{1}{2}U$

Application 1: Classical Instability of Atoms



Estimate of instability time for atom is

$$t_{\text{unstable}} = |E|/P = \left(\frac{q^2}{8\pi\epsilon_0 R}\right) / \left(\frac{q^2}{6\pi\epsilon_0 c^3} a^2\right) = \frac{3}{4}\frac{c^3}{Ra^2}$$

$$\sim \frac{c^3}{R(v^2/R)^2} = \frac{Rc^3}{v^4}$$

Size of electron orbits are ~ Angstrom = 10^{-10} m2, with velocity $v \sim 0.01c = 3 \times 10^6$ m/s

$$t_{\text{unstable}} \approx \frac{10^{-10}\,\text{m}(3 \times 10^8\,\text{m/s})^3}{(3 \times 10^6\,\text{m/s})^4} \approx 10^{-10}\,\text{s}$$

Quantum Solution to Instability

Due to uncertainty principle, not possible for electron to be confined to arbitrarily small region around proton without increasing its kinetic energy

This leads to a minimum energy solution called the ground state

Hydrogen atoms excited energy levels can lose energy, but not the ground state!

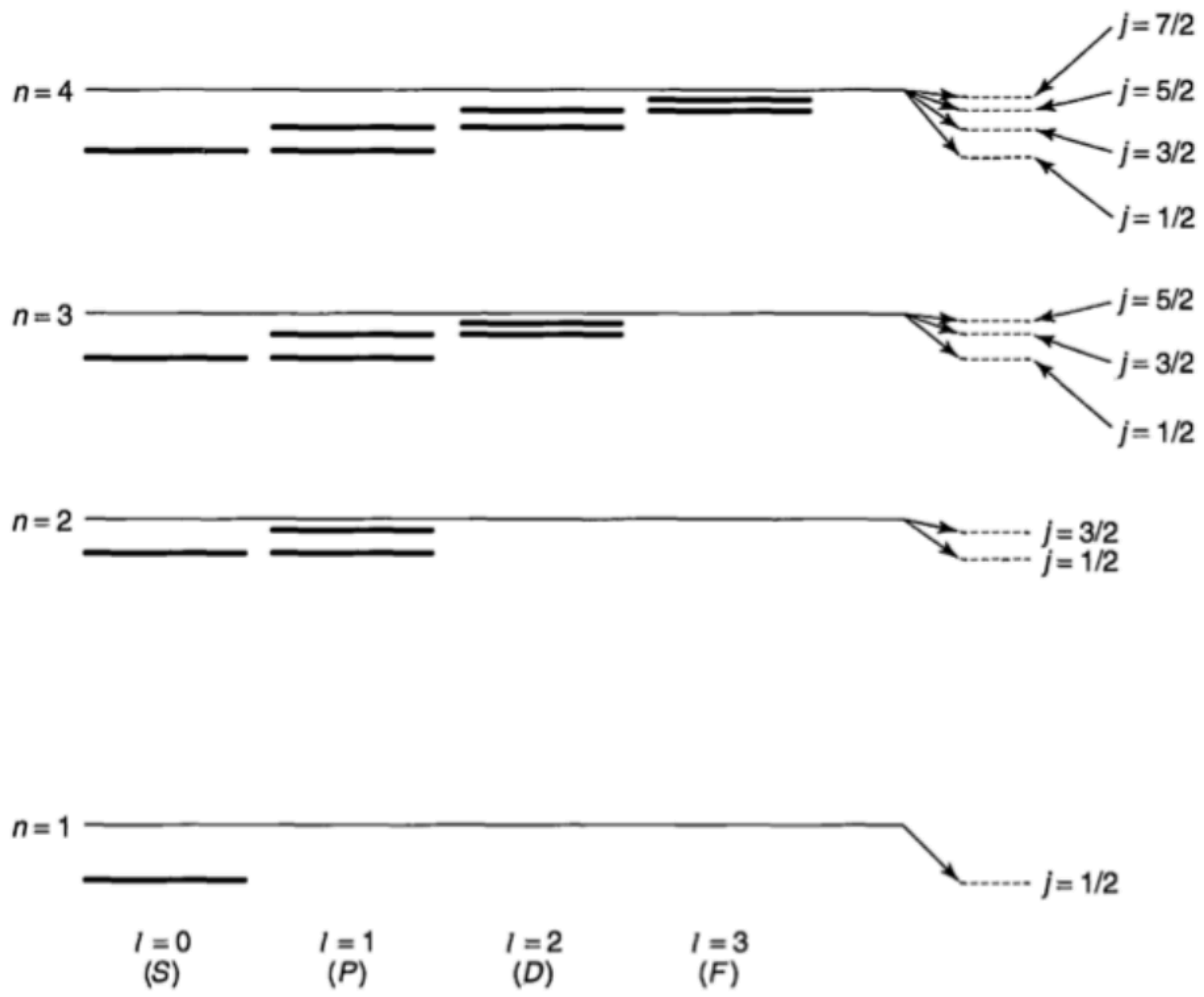
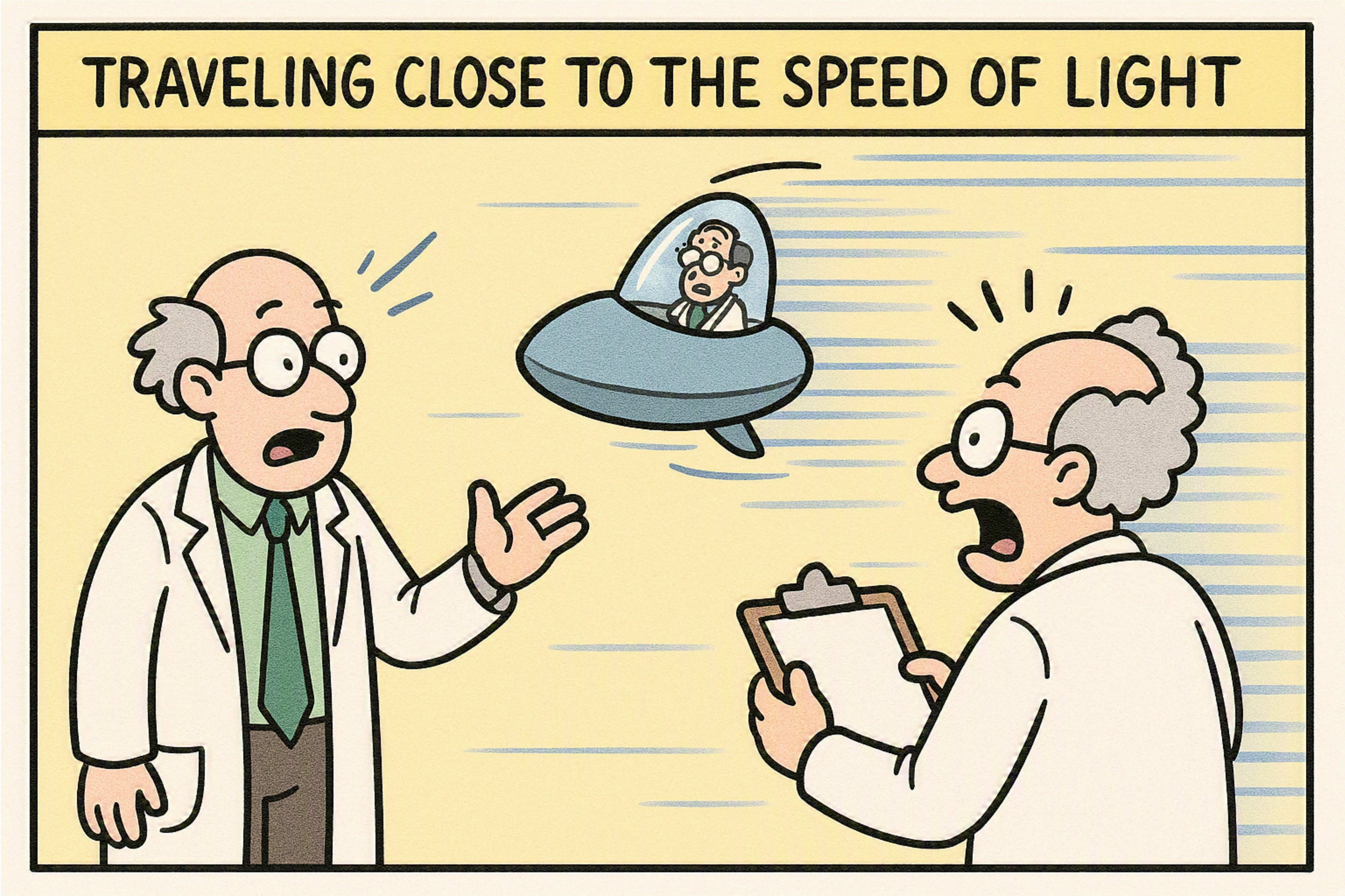


FIGURE 6.9: Energy levels of hydrogen, including fine structure (not to scale).

Credit: Griffiths

Larmor Formula for Particles Moving Close to the Speed of Light



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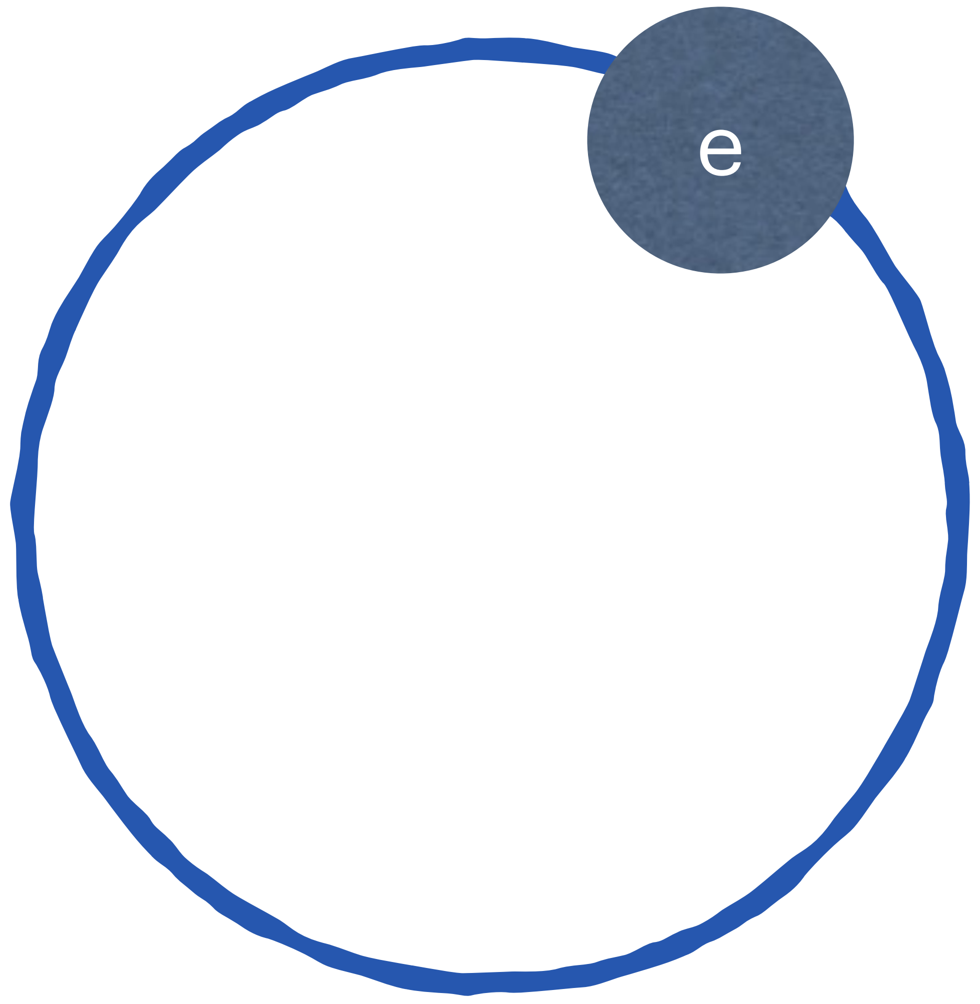


Relativistic Circular Orbits

$$P = \frac{q^2}{6\pi\epsilon_0 c^3} a^2 = \frac{q^2 v^4}{6\pi\epsilon_0 c^3 R^2}$$

What happens as v gets close to c?

Here, we have multiple possibilities. The correct choice is to write
v= mv/m = p/m

The relativistic generalization of momentum is
$p = m\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}$ which approximates mv for
v small compared to c

$$\boxed{P = \frac{q^2}{6\pi\epsilon_0 c^3 R^2}\left[\frac{p}{m}\right]^4}$$

Power Constraints of Colliders

For a relativistic particle

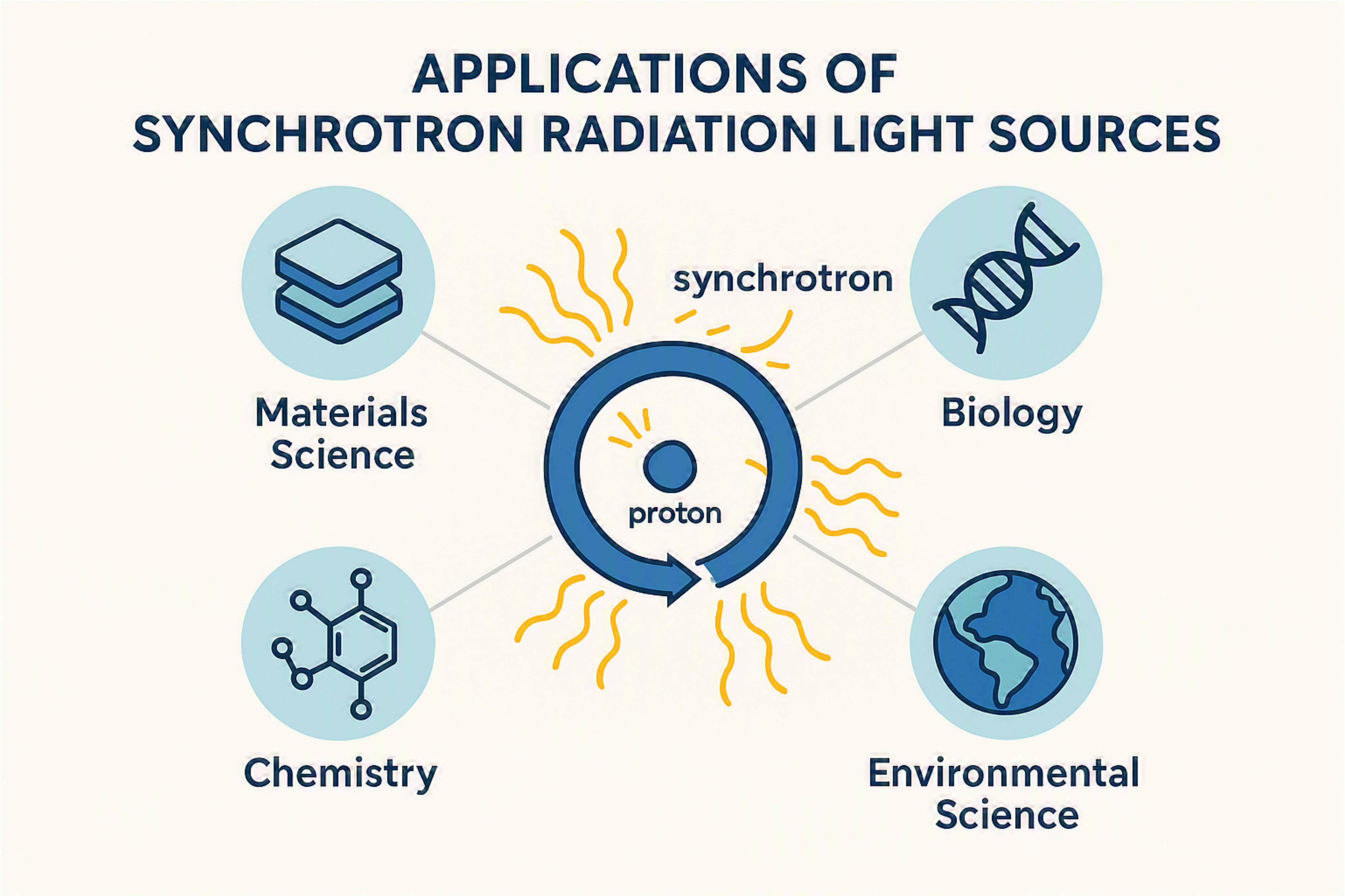
$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \approx pc \qquad P = \frac{q^2 c}{6\pi\epsilon_0 R^2}\left[\frac{E}{mc^2}\right]^4$$

Collider	Particle	m	E	$(E/mc^2)^4$
LEP2	e^+, e^-	0.5 MeV/c^2	100 GeV	10^{21}
LHC	p	GeV/c^2	7 TeV	10^{15}
LHC 100 TeV	p	GeV/c^2	50 TeV	10^{19}
LEP TeV	e^+, e^-	0.5 MeV/c^2	500 GeV	10^{24}
Muon Collider	μ^+, μ^-	0.1 GeV/c^2	5 TeV	10^{19}

Takeaways:

1) LEP2 beams lost 3% of energy each orbit
2) 100 TeV LHC is ok power wise, but magnets not strong enough
3) LHC magnets could run an electron, positron collider at 1 TeV, but energy costs go up 1000! Need a linear collider for electron, positron
4) Muon collider power, magnets aren't the issue

Synchrotron Radiation Applications



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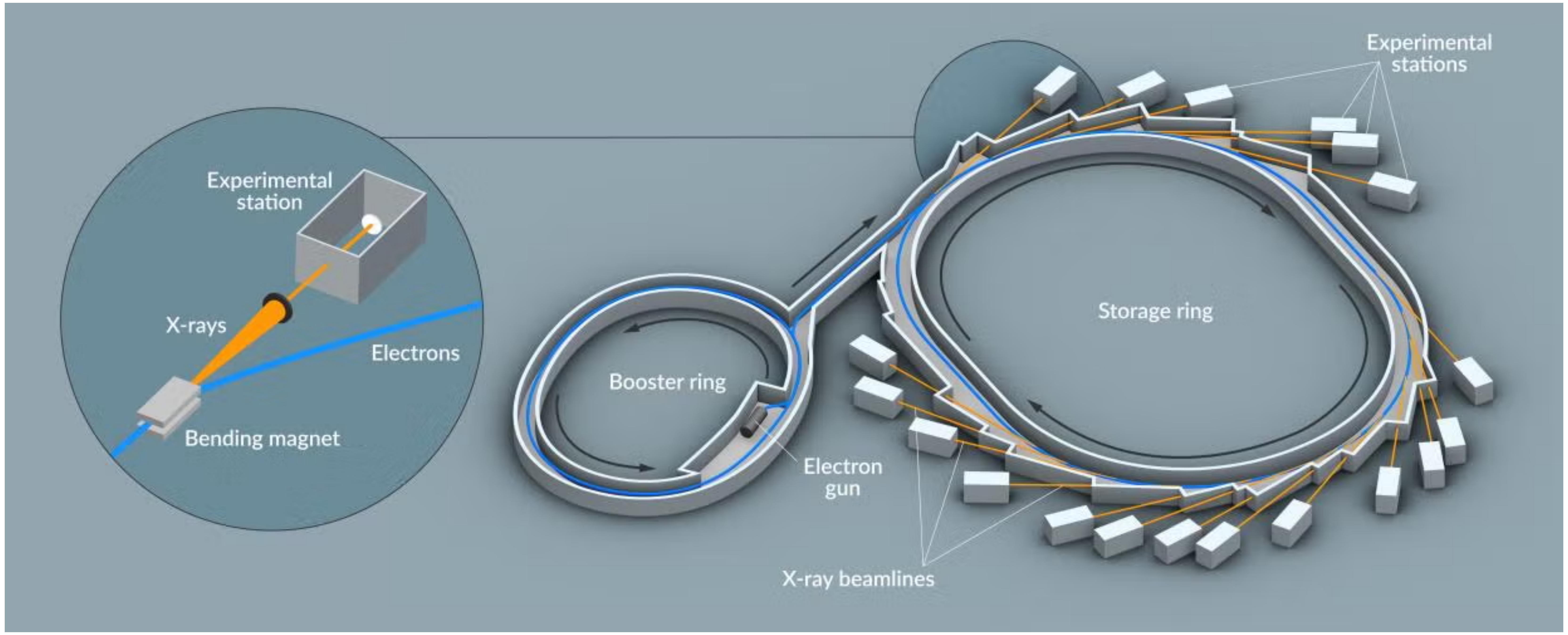
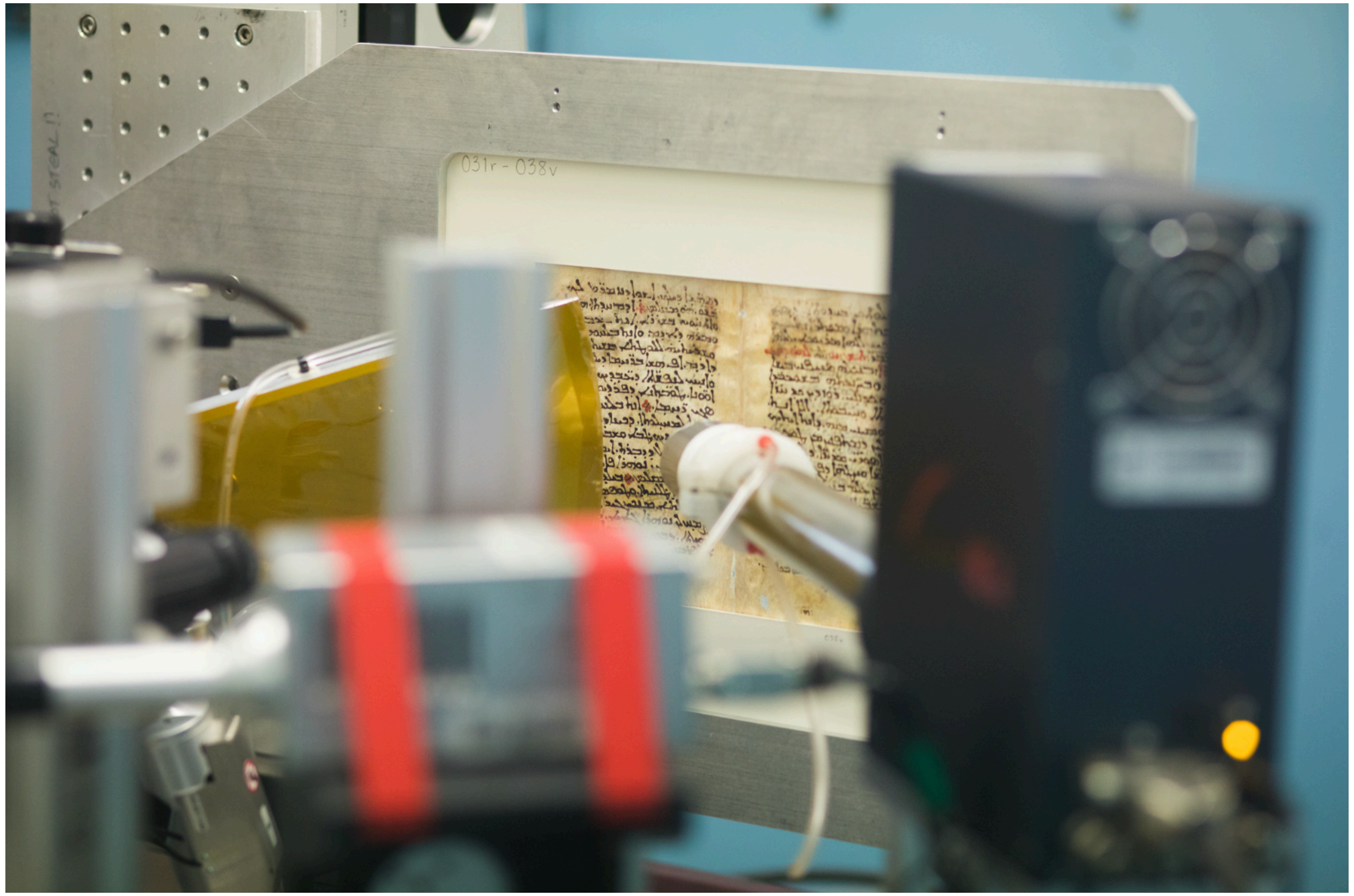
Radiation of Circulating Electrons

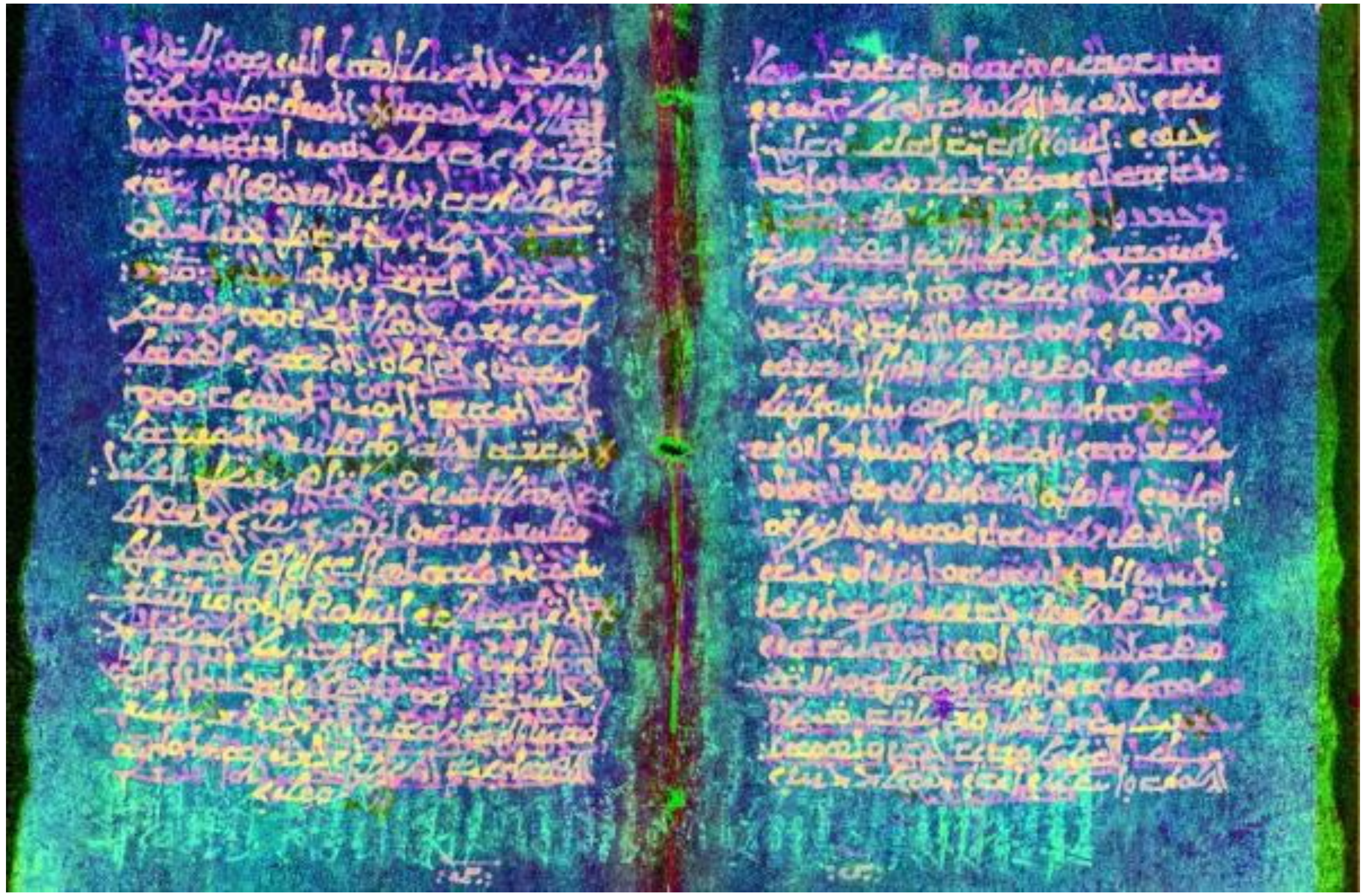
The radiation emitted by circulating electrons has been harnessed for important research outside particle physics



Many national labs (like SLAC national accelerator laboratory) have synchrotron sources producing intense x-ray sources with impressive imaging capabilities for proteins, battery design, and archaeology

Example from SLAC

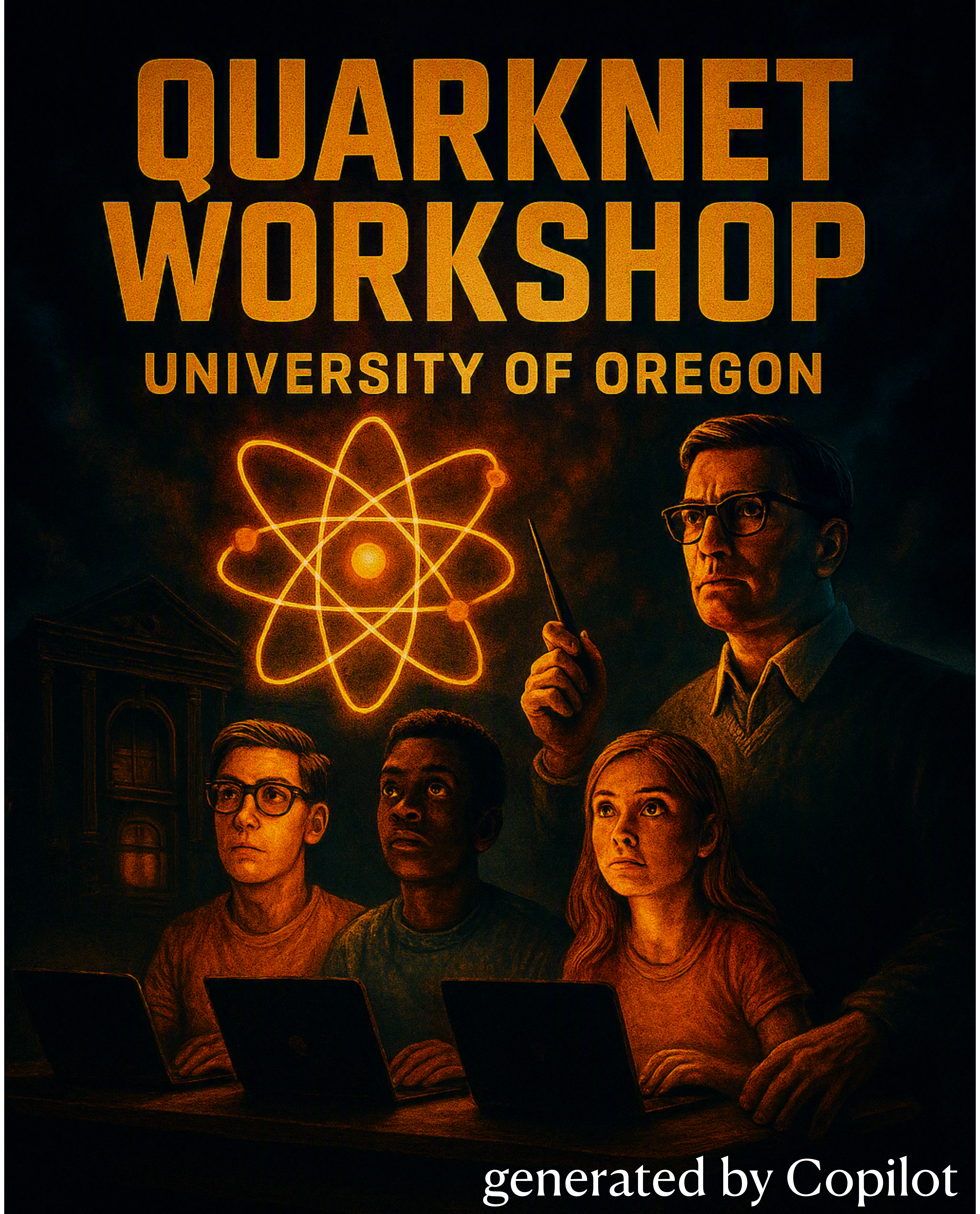




6th century translation
revealed by Xray

Conclusions

- Radiation of accelerating particles is a fascinating subject
  - Teachable Moments
    - Graphical argument - Intuition for technical result
    - Dimensional analysis - Ability to get dependence on variables
  - Theoretical Puzzles for classical models of matter
    - Instability of atoms
    - (Didn’t discuss) Radiation Reaction
  - Practical applications
    - Constraints on collider design
    - Synchrotron sources as a spinoff application of national labs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Thank You!

Radiation Reaction

Larmor formula suggests that an accelerating charged particle must lose energy, which leads to a force (Abraham-Lorentz)

$$m\vec{a} = \overrightarrow{F}_{ext} + \frac{q^2}{6\pi\epsilon_0 c^3}\frac{d\vec{a}}{dt}$$

Lots of pathologies occur due to this term!

If there are no external forces, we get a new exponentially growing term in solutions!

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \vec{a}_0 e^{t/\tau}, \quad \tau = \frac{q^2}{6\pi\epsilon_0 mc^3}$$

Radiation Reaction

Larmor formula suggests that an accelerating charged particle must lose energy, which leads to a force (Abraham-Lorentz)

$$m\vec{a} = \overrightarrow{F}_{ext} + \frac{q^2}{6\pi\epsilon_0 c^3}\frac{d\vec{a}}{dt}$$

If there are external forces, we get a solution, which knows about the entire future behavior of the external force!

$$\vec{a}(t) = \frac{1}{m\tau}\int_t^{\infty} e^{-(t'-t)/\tau}\overrightarrow{F}_{ext}(t')dt'$$

Suffice to say that classical behavior of electrons has lots of paradoxes...