Practice Final Exam
**Statistical Methods and Models** - Math 410, Fall 2011
December 4, 2011

You may use a calculator, and you may bring in one sheet (8.5" by 11" or A4) of notes. Otherwise closed book.

The real test will have about 10 questions.

Be sure when you do a hypothesis testing problem that you describe your parameters in terms of the problem when stating $H_0$ and $H_a$. Also be sure you rephrase you conclusion in terms of the original question being asked in the problem.

(1) You have a set of three dice.
   (a) Calculate the probability of getting 0, 1, 2 and 3 sixes if the dice are fair.
   (b) Suppose you roll the three dice 100 times. The results are

| # of 6s | # of times |
|---|---|
| 0 | 47 |
| 1 | 35 |
| 2 | 15 |
| 3 | 3 |

   Test the null hypthesis that the dice are fair at $\alpha = .01$ significance level.

(2) Males and females were asked about what they would do if they received a $100 bill by mail, addressed to their neighbor, but wrongly delivered to them. Would they return it to their neighbor? 69 men were sampled, of whom 52 said yes. 131 females were sampled, of whom 120 yes. Does the data indicate that the proportions that said yes are different for male and female at a 5% level of significance?

(3) Ten students have first and second exam scores as follows:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| first | 23 | 33 | 44 | 33 | 84 | 76 | 65 | 79 | 85 | 47 |
| second | 27 | 38 | 52 | 38 | 98 | 90 | 77 | 93 | 100 | 55. |

Use this data to give a 95% confidence interval for the improvement in test scores from the first to the second exams.

(4) Nineteen female rats are fed a specific diet between their 28th day and their 84th day. The groups were chosen randomly and 12 were fed a high protein diet while 7 were fed a low proteien diet.

The mean weight gain for the 12 rats on a high protein diet was 120 grams. For the rats on the low protein diet it was 101 grams. Construct a 95% confidence interval for the difference between weight gain on the high protein diet and weight gain on the low protein diet.

(5) In a 1974 study, 862 Marine recruits were assigned randomly to either take daily 2 gram vitamin C supplements, or a placebos for

eight weeks. During the study some recruits were removed from their platoons, and others were excluded from analysis because they didn't take their pills for the entire 8 weeks. At the end of the study there were 331 recruits in the vitamin C group, and 343 in the placebo group.

The vitamin C group spend a mean of 20.3 days with a cold, with standard deviation .879 days. The placebo group spent a mean of 20.7 days with a cold, with standard deviation .642.

Is there evidence at the 5% significance level that the vitamin C treatment reduces the length of cold symptoms from this study?

(6) Use a one-sided $\chi^2$ contingency table at the $\alpha = .05$ level to test whether acupuncture is more successful than placebo using the following table of data:

| | Success | Failure | Total |
|---|---|---|---|
| Acupuncture | 7 | 6 | 13 |
| Placebo | 4 | 13 | 17 |
| Total | 11 | 19 | 30 |

(7) Suppose independent random samples from two normal populations gave the following results:

$$n_X = 25, \overline{x} = 16, s_x = 4.7$$
$$n_Y = 45, \overline{y} = 20, s_y = 2.3.$$

Find a conservative .95 confidence interval for the difference $\mu_X - \mu_Y$.

(8) How large a sample is needed to obtain an estimate of the mean IQ of college students to within a 95% margin of error of 1 point? (Use $\sigma = 15$.)

(9) A professional basketball player used to make 55% of his free throws. His agent claims he has improved, and offers as evidence that he has made 40 out of his last 60 free throws. Test the hypothesis at the .05 significance level that his free throw percentage has improved from 55%.

(10) Assume an airline flies a plane whose safe passenger load is 16000 pounds. Assume the passengers weigh 190 pounds on average (overall) with a standard deviation of 40 pounds, and that on each flight we have a random sample of 80 passengers. How often will the plane be overloaded?

(11) Compute $\mu$, $E(X^2)$ and $\sigma$ for the random variable $X$ with pdf

$$f(x) = \begin{cases} |x| & -1 \leq x \leq 1 \\ 0 & \text{else.} \end{cases}$$

(12) Compute $\mu$, $E(X^2)$ and $\sigma$ for the random variable $X$ with pdf

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{else.} \end{cases}$$

We assume $a < b$ here.

(13) The length of time necessary to perform a specific manufacturing operation is a random variable $T$ with mean 12 and standard deviation 2 minutes. Use Chebyshev's Inequality to find a lower bound for $P(6 < T < 18)$.

(14) An urn contains 50 black balls and 10 white balls. You pick 10 balls at random. Give a formula for the probability of picking all black balls.

(15) Give the definition of $E$ being independent of $F$. Prove that if $E$ is independent of $F$ then $F$ is independent of $E$.

(16) State Bayes's Theorem. A test for a particular cancer has sensitivity 2/3, specificity .91. Prevalence among the relevant population is .015. Calculate the predictive value positive.

(17) Suppose $\pi = .55$ (a population proportion). But you don't know $\pi$ and you are testing the hypothesis $\pi = .5$ against the hypothesis $\pi > .5$. Your procedure will be to pick a random sample of 20 individuals and do a $z$-test at the .02 significance level. What is the power of this test?