

Practice Midterm Exam  
**Statistical Methods and Models** - Math 410, Fall 2011  
 October 21, 2011

You may use a calculator, and you may bring in one sheet (8.5" by 11" or A4) of notes. Otherwise closed book.

- (1) (a) You roll a fair (six sided) die twice. The total of the two rolls is 9. Use Bayes' method to determine the probability that the first roll was a 4.
- (b) You roll a fair (six sided) die twice. Let  $F$  be the event that the first roll was a 1. Let  $E$  be the event that the total shown is 5. Calculate  $P(E|F')$ . Is  $E$  independent of  $F$ ?
- (2) Let  $E$  and  $F$  be possible outcomes of an experiment. Prove that if  $E$  is independent of  $F$  then  $F$  is independent of  $E$ . That is, if  $P(E) = P(E|F)$  then  $P(F) = P(F|E)$ .
- (3) Suppose you have a screening test for the Lurgy virus. Suppose the prevalence of Lurgy is 1%, the sensitivity of the test is .98 and the specificity is .95. What is the predictive value positive in this example? What is the predictive value negative?
- (4) 15 statisticians are giving a training to increase their social competence. They are scored on a 7 point scale before and afterwards with the following results:

<i>Subject</i> :	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
<i>Before</i> :	5	3	4	2	1	6	7	3	2	3	5	1	4	4	3
<i>After</i> :	6	2	4	4	3	6	7	5	3	5	5	3	4	5	2

Use a sign test to determine if there sufficient evidence at the .05 significance level to conclude that this training increases social competence.

- (5) You wish to test whether your cat prefers one paw over the other. You dangle a ribbon in front of your cat 10 times, and he bats it with his right paw 8 of those time, and with his left paw two of those times.  
 Use a binomial exact test to determine if this is sufficient evidence at the .05 significance level to conclude that your cat prefers one paw over the other.
- (6) (This is a slight over-simplification of a real study.) In the FUTURE II study of the HPV vaccine, 10,565 women were divided randomly into two groups. One group was given Gardasil, and the other group was given a placebo. After three years, 43 women received a diagnosis related to cervical cancer. 1 was in the vaccine group, 42 were in the placebo group.

Is this sufficient evidence to conclude that Gardasil protects against cervical cancer at the .01 significance level? (Use the binomial exact test.)

- (7) Let  $K$  be a binomial variable with  $n = 3$ ,  $\pi = .4$ . Calculate the expected value, variance and standard deviation of  $K$ .
- (8) Let  $X$  be a continuous random variable, uniform on  $[1, 5]$ . Give the pdf and cdf for  $X$  and graph them both.
- (9) One of the 4 functions below is the pdf of a random variable. Which one? (Say briefly what is wrong with the others.)

(a)

$$f(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x \leq 1 \\ 1 & x \geq 1. \end{cases}$$

(b)

$$f(x) = \begin{cases} 0 & x < 0 \\ 1 & 0 \leq x \leq 2 \\ 0 & x \geq 2 \end{cases}$$

(c)

$$f(x) = \begin{cases} 0 & x < -1 \\ x & -1 \leq x \leq \sqrt{3} \\ 0 & x \geq \sqrt{3} \end{cases}$$

(d)

$$f(x) = \begin{cases} 0 & x < 0 \\ 1/2 & 0 \leq x \leq 2 \\ 0 & x \geq 2 \end{cases}$$