Statistics 410, Fall 2011
Solutions to homework 1, due 9/28/11

1.13 a. Let $f(x) = x(1-x) = x - x^2$. This is a quadratic with negative x^2 coefficient, so its graph is a parabola open down. It has its (global) maximum when the derivative is 0, which is when $0 = 1 - 2x$, so when $x = 1/2$.

b. Consider $g(\pi) = \sqrt{\frac{\pi(1-\pi)}{n}}$. g has its maximum when g^2 has its maximum (g and hence g^2 are always positive). This will be at the value of π where $\pi(1-\pi)$ has its maximum, so when $\pi = 1/2$. So

$$\sqrt{\frac{\pi(1-\pi)}{n}} \leq \sqrt{\frac{.5(1-.5)}{n}} = \sqrt{\frac{1}{4n}} = \frac{1}{2\sqrt{n}}.$$

1.14 Since our coin is fair, $p = .5$. $n = 100$, so we get

$$SE_p = \frac{1}{2\sqrt{100}} = \frac{1}{20} = .05$$

400 tosses changes n to 400, so we get $\sqrt{400}$, so

$$SE_p = \frac{1}{40} = .025.$$

1.17 a. For two bags, our standard error will be $\sqrt{1^2 + 1^2} = \sqrt{2}$ ounces.

b. For 16 bags, it will be $\sqrt{1^2 + \cdots + 1^2}$ (16 1s) ounces, which is 4 ounces.

1.19 Because the number of polio cases overall is quite small, if there had been a much smaller number of subjecs, one would be likely to end up with an analysis like the analysis of the fatal cases of polio. That is, with a smaller number of subjects, even if most cases of polio were in the vaccinated group, we might still not have strong evidence that the vaccine was effective.

2.10 The grouped mean is

$$(2 \cdot 1 + 6 \cdot 4 + 8 \cdot 3 + 10 \cdot 2 + 13 \cdot 2)/12 = \frac{96}{12} = 8.$$

2.15 The grouped mean is

$$(1 \cdot 0 + 2 \cdot 2 + 4 \cdot 3.5 + 1 \cdot 6)/8 = 24/8 = 3.$$

So to get the grouped sample variance, we use formula 2.2 on p. 48:

$$s^2 = \frac{1 \cdot (-3)^2 + 2 \cdot (-1)^2 + 4 \cdot (.5)^2 + 1 \cdot 3^2}{7} = \frac{21}{7} = 3$$

so of course $s = \sqrt{3}$.

Finally, the estimated standard error (using the formula on p. 50) is

$$SE_{\overline{x}} = \frac{s}{\sqrt{n}} = \frac{\sqrt{3}}{\sqrt{8}} \approx .612$$

2.19 I used Excel to calculate these by entering the weights in a column and using the functions average and stdev. This gives $\overline{x} = 6.994$, $s = 2.139$, and $SE_{\overline{x}} = .535$.

3.3 In this problem, we will think of the set of all possible outcomes by labeling the tosses in order. So, for example, $HHTH$ stands for heads on the first and second tosses, tails on the third, and heads on the fourth.

(a) There are 4 ways this can happen, $THHH, HTHH, HHTH, HHHT$. Since there are $2^4 = 16$ possibilities, and all are equally likely, these 4 outcomes represent a probability of .25.

(b) The probability of the third toss being heads is .5. The probability of the 4th toss being heads is also .5. Since these are independent, the "multiplication rule" applies, and the probability of both being heads is .25.

(c) This is .5 as discussed above.

(d) There are two ways for this to happen, out of the 16 possiblities. So the probability is $1/8 = .125$.

3.4 In this problem we think about the the outcomes by labelling the results of the tosses in order. So for example, we represent by $(2, 5, 6)$ the result of getting a 2 on the first through, 5 on the second throw and 6 on the third.

(a) For each roll, the probability of the result being even is .5. Since the three rolls are independent, the "multiplication rule" applies, and the probability of all three being even is $(.5)^3 = .125$.

(b) This is similar to the above, but is $(.5)^2 = .25$.

(c) This happens half the time, so has probability .5.

(d) For any individual roll, the probability of a single dot is $1/6$. Since the three rolls are independent, the probability is $(1/6)^3 \approx .005$.

(e) There are 6 ways for this to happen ($(1, 1, 1), (2, 2, 2)$ etc.) out of a total number of possible outcomes of $6^3 = 216$. Since each outcome is equally likely, the probability of this is $6/216 = 1/36 \approx .028$.

3.8 (a) We use $P(E \cap F) = P(F|E)P(E) = .2 \cdot .55 = .11$.

(b) We use $(E \cap F)' = E' \cup F'$, so

$$P(E' \cup F') = 1 - P(E \cap F) = .89.$$

(c) We use $E' \cap F' = (E \cup F)'$, so

$$P(E' \cap F') = 1 - P(E \cup F) = 1 - [P(E) + P(F) - P(E \cap F)] = 1 - .5 - .2 + .11 = .41.$$

(d) We use that $P(E|F) = P(E \cap F)/P(F) = .11/.4 = .275$.

3.9 We use the basic multiplication rule which tells us that

$$P(E \cap F) = P(E)P(F|E) = .5 \cdot .3 = .15$$

We also use that

$$P(E \cup F) = P(E) + P(F) - P(E \cap F) = .7 - .15 = .55$$

so $P(E \cup F) = .55$

By deMorgan's laws,

$$E' \cap F' = (E \cup F)'$$

so $P(E' \cap F') = 1 - P(E \cup F) = 1 - .55 = .45$.

Finally, using the formula for conditional probabilities,

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{.15}{.2} = .75$$

3.10 Let E be the event the husband watched and F the event the wife watched. So $P(E) = .2$, $P(F) = .08$ and $P(F|E) = .25$

(a) $P(E \cap F) = P(F|E) \cdot P(E) = .25 \cdot .2 = .05$.

(b) $P(E \cup F) = P(E) + P(F) - P(E \cap F) = .2 + .08 - .05 = .23$.

(c) This is the complement of the previous situation, so the probability is $1 - .23 = .77$.

(d) $P(E|F) = P(E \cap F)/P(F) = .05/.08 = .625$.

3.12 We want $P(E_1 \cup E_2)$. One way to do this is:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = .03 + .03 - P(E_2|E_1)P(E_1) = .06 - \frac{2}{99}.03 = .059.$$

3.14 The probability of no aces in 4 rolls is $(5/6)^4 \approx .482$, so the probability of at least one ace is $1 - .482 = .518$.

The probability of no pair of aces in 24 rolls is $(35/36)^{24} = .509$. So the probability of at least one pair of aces is $1 - .509 = .491$.

The first probability is higher than the second one!