Statistics 410, Fall 2011
Solutions to homework 6, due 11/2/11

7.17
$$\mu = E(X) = \int_0^1 \frac{6x^2 - 3x^3}{2}\ dx = x^3 - \frac{3}{8}x^4|_0^1 = 5/8.$$
$$E(X^2) = \int_0^1 \frac{6x^3 - 3x^4}{2}\ dx = \frac{3}{4}x^4 - \frac{3}{10}x^5|_0^1 = .45.$$
$$\sigma = \sqrt{.45 - .625^2} = .24367.$$

7.18
$$\mu = E(X) = \int_{-1}^1 x|x|\ dx = \int_{-1}^0 -x^2\ dx + \int_0^1 x^2\ dx = -\frac{1}{3} + \frac{1}{3}.$$
$$E(X^2) = \int_{-1}^1 x^2|x|\ dx = \int_{-1}^0 -x^3\ dx + \int_0^1 x^3\ dx = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$
$$\sigma = \sqrt{1/2 = 0^2} = \sqrt{1/2} = .707107.$$

7.26 (a) The common rules say that this range contains about 68% of women's weights.
Since this is one standard deviation, we'd apply Chebyshev with $r = 1$. So we would learn that this range contains at least 0% of women's weights.
(b) This is 2 standard deviations, so the common rules say this range contains about 95% of women's weights.
This corresponds to $r = 2$ under Chebyshev, so Chebyshev's rule says this contains at least $1 - \frac{1}{4} = 75\%$ of women's weights.
(c) This is 3 standard deviations. So the common rules say this contains about 99.7% of women's weights.
Chebyshev says this contains at least $1 - \frac{1}{9} \approx 89\%$ of women's weights.

7.27 (a) About 68% and at least 0% respectively.
(b) About 95% and at least 75% respectively.
(c) About 99.7% and at least 89% respectively.

7.32
$$F(u) = \int_{-\infty}^u f(t)\ dt.$$
So if $u \leq a$, then $f(t) = 0$ for all $t < u$, thus the integral is 0. Therefore $F(u) = 0$ if $u \leq a$.

For $a \leq u \leq b$,
$$F(u) = \int_{-\infty}^a f(t)\ dt + \int_a^u f(t)\ dt = 0 + \int_a^u \frac{1}{b-a} = \frac{u-a}{b-a}.$$
Finally, if $u \geq b$,
$$F(u) = \int_{-\infty}^a f(t)\ dt + \int_a^b f(t)\ dt + \int_b^u f(t)\ dt = 0 + \frac{b-a}{b-a} + 0 = 1.$$

7.33 We solve the equation

$$\frac{q}{100} = F(u_q) = \frac{u_q - a}{b - a}.$$

so $u_q - a = \frac{q(b-a)}{100}$, so $u_q = a + \frac{q}{100}(b - a)$.

7.34

$$\mu = E(U) = \int_a^b \frac{1}{b-a} x \, dx = \frac{x^2}{2(b-1)}|_a^b = \frac{b^2 - a^2}{2(b-a)} = \frac{b+a}{2}.$$

$$E(U^2) = \int_a^b \frac{x^2}{b-a} \, dx = \frac{x^3}{3(b-a)}|_a^b = \frac{b^3 - a^3}{3(b-a)} = \frac{1}{3}(b^2 + ba + a^2).$$

So

$$Var(U) = E(U^2) - E(U)^2 = \frac{1}{3}(a^2 + ab + b^2) - (\frac{a+b}{2})^2 =$$

$$\frac{1}{12}(4a^2 + 4ab + 4b^2) - \frac{1}{12}(3a^2 + 6ab + 3b^2) = \frac{1}{12}(a^2 - 2ab + b^2) = \frac{1}{12}(a-b)^2.$$