

Statistics 410, Fall 2011

Solutions to homework 9, due 11/23/11

12.22 Let μ be the mean thickness of the plastic sheets being manufactured.

- (a) $H_0 : \mu = 1.2\text{mm}$.
- (b) $H_a : \mu \neq 1.22\text{mm}$.
- (c) The significance level is $\alpha = .05$.
- (d) The test statistic is 1.2275, and we expect a t -distribution with 11 degrees of freedom if we standardize. So assuming H_0

$$\bar{t} = \frac{1.2275 - 1.22}{.03864/\sqrt{12}} = .67238.$$

- (e) For a 5% hypothesis test, the critical value is 2.201. Since $|.67238| < 2.201$, we retain H_0 .
There is not sufficient evidence to conclude the mean thickness of the plastic sheets is not equal to 1.22.

12.23 (a) $H_0 : \pi = .9$.

(b) $H_a : \pi < .9$.

(c) $\alpha = .07$.

- (d) To get our test statistic, do 210 trials, and let K be the number of successes. By the normal approximation to the binomial distribution,

$$\bar{z} = \frac{K - 210 \cdot .9}{\sqrt{210 \cdot .9 \cdot .1}} = \frac{K - 189}{4.35}.$$

We can write K in terms of \bar{z} as well:

$$K - 189 = 4.35\bar{z} \text{ so } K = 4.35\bar{z} + 189.$$

- (e) We'll reject H_0 if $P(z \leq \bar{z}) \leq .07$. Consulting our table, we see this corresponds to $\bar{z} \leq -1.475$.
Expressing this in terms of K , this corresponds to

$$K \leq 4.35 \cdot (-1.475) + 189$$

i.e. to $K \leq 182$.

12.27 (a) μ is the mean lifetime of the lightbulbs made under the new process. $H_0 : \mu = 750$ hours, with $\sigma = 50$ hours. $H_a : \mu > 750$ hours.

- (b) To sketch the power curve, we want to examine (for different actual value of μ) the probability of rejecting H_0 . We reject H_0 if $\bar{x} > 761.63$. We'll want to calculate $P(\bar{x} > 761.63)$ for different value of μ .

To do this, consider

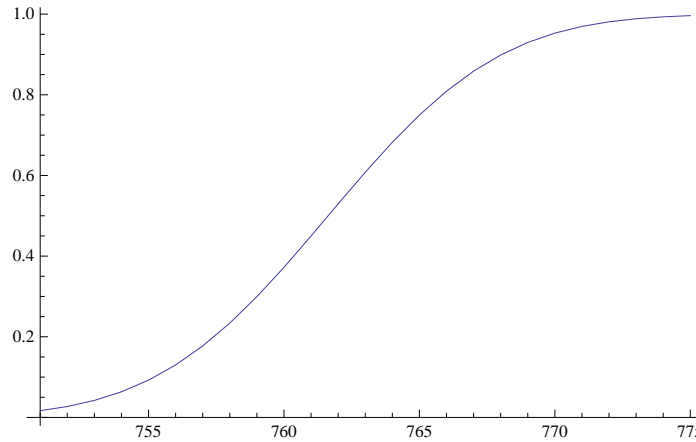
$$\bar{z} = \frac{\bar{x} - \mu}{50/\sqrt{100}} = \frac{\bar{x} - \mu}{5}.$$

$\bar{x} > 761.63$ iff and only if

$$\bar{z} > \frac{761.63 - \mu}{5} = 152.326 - \mu/5.$$

We'll give values for $P(z < 152.326 - \mu/5)$ for $\mu = 751, 753, 755, \dots, 773, 775$.

μ	751	753	755	757	759	761	763	765	767	769	771	773	775
P	.017	.042	.092	.177	.299	.450	.608	.750	.859	.930	.970	.989	.996



(c) This is $P(z < 152.326 - 750/5) = P(z < 2.326) \approx .99$.

(d) This is $P(z > 152.326 - 760/5) = .3722$ and $P(z > 152.326 - 770/5) = .9529$.

13.14 The mean is 24 and $s = 1.8708$. So the standard error is $1.8708/\sqrt{5} = .8366$. We have $t_{.95} = 2.1318$ for a two-sided 90% confidence interval with 4 degrees of freedom. So

$$22.2165 = 24 - 2.1318 \cdot .8366 \leq \mu \leq 24 + 2.1318 \cdot .8366 = 25.7835$$

with 90% confidence, where μ is the mileage.

The 95% confidence interval is $\mu \geq 22.165$.

13.15 (a) The mean is 924.8 hours and the sample standard deviation is 136.606.

(b) $t_{.95} = 1.8331$ for 9 degrees of freedom. The standard error is $136.606/\sqrt{10} = 43.1986$. So if μ is the mean lifetime of the lightbulbs, we get

$$\mu \geq 924.8 - 1.8331 \cdot 43.1986 = 845.613$$

with 95% confidence.

(c) To answer this we test the hypothesis $H_0 : \mu = 1000$ against $H_a : \mu < 1000$ at the .05 significance level. We end up rejecting H_0 if

$$\bar{x} < 1000 - 1.8331 \cdot 43.1986 = 920.813.$$

Based on the data we have ($\bar{x} = 924.8$), we retain H_0 , so we do not reject the manufacturer's claim.

13.16 The sample mean is $\bar{x} = 852.4$ and the sample standard deviation is $s = 79.01$. Then the standard error is $79.01/\sqrt{100} = 7.901$. For a 99% confidence interval, we use $t_{.995} = 2.6259$ for 100 degrees of freedom (we should use 99 df, but that isn't in our table).

So if μ is the speed of light minus 299,000 kilometers per second,
 $831.653 = 852.4 - 2.6259 \cdot 7.901 \leq \mu \leq 852.4 + 2.6259 \cdot 7.901 = 873.147$

with 99% confidence.

13.24 The standard error for the conservative method is $.5/\sqrt{150} = .0408$. For a one-sided 95% confidence interval, we use $z_{.95} = 1.6448$.

So $\mu \geq 66/150 - 1.6448 \cdot .0408 = .44 - .0671 = .3729$. with 95% confidence.

For the bootstrapping method (noting we have at least 5 failures and at least 5 successes) we use standard error

$$\frac{\sqrt{\frac{66}{150} \cdot \frac{84}{150}}}{\sqrt{150}} = .0405.$$

This gives

$$\mu \geq .44 - 1.6448 \cdot .0405 = .3734.$$

13.29 (a) Our critical value is $z_{.95} = 1.6448$. Our sample proportion is $p = .4118$. Let π be the proportion in the theoretical large population of such people who would find improvement from the treatment regimen. Then

$$\pi \geq p - 1.6448 \frac{.5}{\sqrt{34}} = .4118 - .141 = .2708.$$

(b) We ask, if $\pi = .5$, what is the probability that $p \geq .4118$. That is approximately

$$P\left(z \geq \frac{.4118 - .5}{.5/\sqrt{34}}\right) = P(z \geq -1.03) = .84849.$$

13.40 (a) $z_{.95} = 1.6448$. The conservative standard error is $.5/\sqrt{900} = .01667$. The sample proportion is $p = 486/900 = .54$. So our estimate is

$$\mu \geq .54 - 1.6448 \cdot .01667 = .5126$$

with 95% confidence.

(b) Yes.

13.43 (a) $50/\sqrt{9} = 16\frac{2}{3}$ micrograms.

(b) This corresponds to one standard deviation either way. So

$$.1 - 16\frac{2}{3} \cdot 10^{-6} \leq \mu \leq .1 + 16\frac{2}{3} \cdot 10^{-6}$$

grams with 68% confidence.

(c) He should use a standard error of $100/3 = 33\frac{1}{3}$ micrograms, and $t_{.975}$ for 8 degrees of freedom, which is 2.306. This gives

$$.0999231 = .1 - 2.306 \cdot 33\frac{1}{3}10^{-6} \leq \mu \leq .1 + 2.306 \cdot 33\frac{1}{3}10^{-6} = 1.00077$$

with 95% confidence.

(d)

$$P(|x-\mu| < 64) = P(|z| \leq 64/50) = P(|z| \leq 1.28) = 1 - 2\Phi(-1.28) = 1 - 2 \cdot .10027 = .79946.$$

(e) π^3 .

(f) K is a binomial random variable with unknown parameter π . The expected value of K if H_0 is true is $.9 \cdot 10 = 9$. The variance is $n\pi(1 - \pi) = 10 \cdot .9 \cdot .1 = .9$.

(g) A type I error is rejecting H_0 when H_0 is true. So we need to assume H_0 , and calculate

$$P(K \leq 7) = \sum_{k=0}^7 \binom{10}{k} .9^k .1^{10-k} = .0702.$$

Now if $\pi = .8$, we wish to ask about the probability of retaining H_0 . This is

$$P(K \geq 8) = \sum_{k=8}^{10} \binom{10}{k} .8^k .2^{10-k} = .6778.$$

14.3 We have a sample mean of 1.58 hours, with a sample standard deviation of 1.23. The standard error is thus $1.23/\sqrt{16} = .3075$. $t_{.995} = 2.9467$ for 15 degrees of freedom. So if μ is the mean increase in sleeping time, we get

$$.67389 = 1.58 - 2.9467 \cdot .3075 \leq \mu \leq 1.58 + 2.9467 \cdot .3075 = 2.48611$$

with 99% confidence.

14.4 The mean change in time was -6 seconds with a sample standard deviation of 5.06 seconds. The standard error is $5.06/\sqrt{6} = 2.066$. $t_{.995} = 4.0321$ with 5 degrees of freedom. So we get

$$-14.3303 = -6 - 4.0321 \cdot 2.066 \leq \mu \leq -6 + 4.0321 \cdot 2.066 = 2.33032.$$

14.7 (a)

$$SE = \sqrt{\frac{6.4^2}{10} + \frac{5^2}{25}} = 2.2574.$$

(b) (i) $H_0 : \mu_X = \mu_Y$.

(ii) $H_a : \mu_X > \mu_Y$.

(iii) $\alpha = .05$.

(iv) Our test statistic is $\bar{x} - \bar{y} = 4$. We get a t statistic by

$$\bar{t} = \frac{4}{2.2574} = 1.57$$

with 9 degrees of freedom.

(v) Using our table,

$$P(t \geq 1.57) > P(t \geq 1.8331) = .05.$$

(vi) We retain H_0 since our p -value is greater than our significance level.

14.8 (a)

$$SE = \sqrt{\frac{1.8^2}{6} + \frac{1.5^2}{15}} = .8307.$$

(b) (i) $H_0 : \mu_X = \mu_Y$

(ii) $H_a : \mu_X \neq \mu_Y$.

(iii) $\alpha = .05$.

(iv) Our test statistic is $\bar{x} - \bar{y} = 3$. We get a t statistic by

$$\bar{t} = \frac{3}{.8307} = 3.6114$$

with 5 degrees of freedom.

(v)

$$P(|t| \geq 3.6114) < P(|t| \geq 2.7565) = .04 < .05.$$

(vi) We reject H_0 .

14.9 Let μ_X be the mean salary for union workers, and μ_Y be the mean salary for non-union workers.

(a) $H_0 : \mu_X = \mu_Y$.

(b) $H_a : \mu_X \neq \mu_Y$.

(c) Our significance level is .05. We'll use a t -statistic with 80 degrees of freedom. Then $t_{.975} = 1.9901$. If $|\bar{t}| \geq 1.9901$ we reject H_0 , otherwise we retain H_0 .

(d) Our standard error is

$$SE = \sqrt{\frac{1000^2}{100} + \frac{1200^2}{81}} = 500/3.$$

Our t -statistic is

$$\bar{t} = \frac{26240 - 25930}{500/3} = 1.86.$$

So we retain H_0 and find there is no significant difference between the average salaries of union and nonunion workers performing these tasks.

14.12 Let μ_B be the mean the the entire population of boys would receive on the spelling test, and μ_G be the mean that the entire population of girls would receive on the spelling test.

(a) $H_0 : \mu_B = \mu_G$.

(b) $H_a : \mu_G > \mu_B$.

(c) $\alpha = .03$

(d) Our test statistic is

$$\bar{t} = \frac{78.1 - 75.1}{\sqrt{\frac{103.9}{40} + \frac{90.6}{50}}} = 1.4287.$$

with 39 degrees of freedom.

(e) $t_{.97} = 1.9371$. Our test statistic is smaller than this, so we retain H_0 . There is not enough evidence to conclude that the mean for all girls would be higher than the mean for all boys at the 3% significance level.

14.24 Our control mean is $x_1 = 15$. Our treatment mean is $x_2 = 20$. Our sample variances are $s_1^2 = 13$ and $s_2^2 = 9.5$. So using the formula for pooled variance we get

$$s_p^2 = \frac{2 \cdot 13 + 4 \cdot 9.5}{2 + 4} = 10\frac{2}{3},$$

so $s_p = 3.266$. We have a pooled df of $3 - 1 + 5 - 1 = 6$. Finally, $t_{.05} = 1.9432$. μ_1 is the mean weight increase for mice if they were subjected to the conditions of the control group. μ_2 is the mean weight increase for mice if subjected to the conditions of the treatment group.

(a) Our standard error is $\sqrt{\frac{10\frac{2}{3}}{3} + \frac{10\frac{2}{3}}{5}} = 2\frac{2}{3}$.

Our 90% confidence interval is

$$-1.1819 = 5 - 1.9432 \cdot 2\frac{2}{3} \leq \mu_2 - \mu_1 \leq 5 + 1.9432 \cdot 2\frac{2}{3} = 10.1819.$$

- (b) (i) $H_0 : \mu_1 = \mu_2$.
(ii) $H_a : \mu_1 < \mu_2$.
(iii) $\alpha = .05$
(iv)

$$\bar{t} = \frac{x_2 - x_1}{8/3} = \frac{15}{8} = 1.875.$$

(v) $t_{.95} = 1.9432$. Since $1.875 < 1.9432$, we retain H_0 and we don't have sufficient evidence to conclude that the treatment increases weight gain at the 5% significance level.

14.25 Since we assume the population variances are equal, we find the pooled df is $53 = 55 - 2$, and the pooled sample variance is

$$s_p^2 = \frac{24 \cdot 15.32^2 + 29 \cdot 18.23^2}{24 + 29} = 288.123$$

so $s_p = 16.97$. $t_{.995} = 2.6778$ (I'm using 50 df since it is close to 53, and it is in our table).

Let μ_1 be the mean systolic blood pressure from men that age who drink at least 16 ounces of carbonated soft drinks per day, and μ_2 be the mean systolic blood pressure from men that age who don't drink soft drinks.

- (a) $H_0 : \mu_1 - \mu_2 = 0$.
(b) $H_a : \mu_1 - \mu_2 \neq 0$.

(c) $\alpha = .01$.

(d) Our test statistic is

$$\bar{t} = \frac{129.87 - 119.71}{\sqrt{(\frac{1}{25} + \frac{1}{30})288.123}} = 2.210.$$

(e) Since $|\bar{t}| < t_{.995}$, we retain H_0 . There is insufficient evidence to conclude that these two groups have different mean systolic blood pressures at the .01 significance level.