Statistics 410, Fall 2011
Solutions to homework 7, due 11/9/11

8.2 $(3.7-1.6)/2.1 = 1$. $(1.6-1.6)/2.1 = 0$. $(-1.3-1.6)/2.1 = 1.38$. $(1.8-1.6)/2.1 = .095$. $(0-1.6)/2.1 = -.762$.

8.3 -56, -50, -42.8, -65, -23.

8.4 3.8, 3.08, 5.6, 7.04, 9.2.

8.5 (a) $\Phi(1.25) - \Phi(0) = .39435$
(b) $\Phi(1.25) - \Phi(.25) = .89435 - .59871 = .29564$
(c) $\Phi(-.25) - \Phi(-1.25) = \Phi(1.25) - \Phi(.25) = .29564$
(d) $\Phi(0) - \Phi(-1.25) = \Phi(1.25) - \Phi(0) = .39435$

8.6 (a) $P(X < 40) = P(Z < .25) = .29564$.
(b) $P(X > 45) = 1 - P(X < 45) = 1 - P(Z < .5) = 1 - .69146 = .30584$.
(c) $P(0 < X < 35) = P(-1.75Z < 0) = .5 - .04006 = .45994$.
(d) $P(35 < X < 45) = P(0 < Z < .5) = .69146 - .5 = .19146$.
(e) $P(-5 < X < 45) = P(-2 < Z < .5) = .69146 - .02275 = .66871$.
(f) $P(40 < X < 45) = P(.25 < Z < .5) = .69146 - .29564 = .39582$

8.13 (a) This corresponds to $P(Z < -1/3) = .37070$, or 37%.
(b) This is the complementary group, so is about 63%.
(c) By symmetry, this is the same as the first questions, so is about 37%.
(d) $P(X > 220) = P(Z > (220-176)/30) = P(Z > 1.467) = 1 - .92922 = .07078$.
(e) $P(X > 260) = P(Z > (260-176)/30) = P(Z > 2.8) = 1 - .99744 = .00256$.

8.14 (a) $P(X < 258) = P(Z < -.5) = .30854$.
(b) $P(X > 274) = P(Z > .5) = .30854$.
(c) $P(X < 255) = P(Z < -11/16) = .24510$.
(d) $P(X > 286) = P(Z > 1.25) = 1 - .89435 = .10565$.
(e) $P(X > 310) = P(Z > 2.75) = .00298$.

8.19 Aras's score when normalized is 2. Rimas's score when normalized is 1. So Aras's score is higher.

8.23 We think of $P(K_{100} = 50)$ as $P(49.5 < K_{100} < 50.5)$ This is approximately equal to

$$P\left(\frac{49.5-50}{\sqrt{100\cdot .5^2}} < Z < \frac{50.5-50}{\sqrt{100\cdot .5^2}}\right) = P(-.1 < Z < .1) = .07966$$

(The exact binomial probability is .0795892.)

8.24 This is $P(K_{120} = 20) = P(19.5 < K_{120} < 20.5)$. This is approximately

$$P\left(\frac{19.5-20}{\sqrt{120\cdot(1/6)(5/6)}} < Z < \frac{20.5-20}{\sqrt{120\cdot(1/6)(5/6)}}\right) = P(-.122464 < Z < .122474) = .09552.$$

(The exact binomial probability is .0973007.)

8.25 The approximation is

$$P(\frac{7.5-7.5}{\sqrt{15\cdot .5^2}} < Z\frac{10.5-7.5}{\sqrt{15\cdot .5^2}}) = P(0 < Z < 1.54919) = .93943 - .5 = .43943.$$

The actual binomial probabiliy is .440765.

8.28 (a) Using the normal approximation, we get

$$P(\frac{4.5-1.6}{\sqrt{40\cdot .04\cdot .96}} < Z < \frac{40.5-1.6}{\sqrt{40\cdot .04\cdot .95}}) = P(2.33993 < Z < 31.3873) = 1 - .99036 = .00964.$$

(b) Using the binomial distribution we get

$$\sum_{i=5}^{40}\binom{40}{i}.04^i.96^{40-i} = .0210223.$$

(c) This isn't a good situation to use tbe normal approximation in. In particular, n is not large enough to compensate for the fact that $\pi = .04$ is too far away from .5. In particular, $n\cdot\pi = 1.6$ which violates our guideline that $n\cdot\pi$ and $n\cdot(1-\pi)$ both be ≥ 5.

8.30 Using the normal approximation:

$$P(K_{650} > 480) = P(Z > \frac{480.5 - 650\cdot .7}{\sqrt{650\cdot .7\cdot .3}}) = P(Z > 2.1826) = 1 - .98537 = .01463.$$

(The actual binomial probability is .01369.)

8.33 (a) $P(X > 3.3) = P(Z > 1.5) = .06681$.

(b) $P(2.9 < X < 3.1) = P(-.5 < Z < .5) = .38292$.

(c) The first percentile for the standard normal variable is -2.325. Translating this to our random variable X, we get

$$3 - 2.325\cdot .2 = 2.535$$

(in minutes).

8.36 (a) We continue to assume the standard deviation is .1 ounce. So we are being asked what percentage is 2 standard deviations short. By the common rule for normal distributions, about 5% is outside of 2 standard deviations. Half is above, and half below. So about 2.5% are below.

(b) Again we assume that standard deviation is still .1 ounce. Consulting our table of Z-values, we see that the second percentile corresponds to $z = -2.05$. So we wish to have 11.9 ounces be -2.05 standard deviations below the setting. So the setting should be

$$11.9 + .1\cdot(-2.05) = 12.105$$

ounces.

8.41 (a) Using the normal approximation, $P(-.5 < X < 119.5)$ is approximately

$$P(\frac{-.5-150}{\sqrt{1000 \cdot .15 \cdot .85}} < Z < \frac{119.5-150}{\sqrt{1000 \cdot .15 \cdot .85}}) = P(-13.3285 < Z < -2.70113) = .00347.$$

(b) The 90th percentile corresponds to $Z > 1.28$. Since the mean is 150, and the standard deviation is $\sqrt{1000 \cdot .15 \cdot .85} = 11.2916$, this correspons to $X > 150 + 11.2916 \cdot 1.28 = 164.452$.

8.42 (a) The precise probability is $.9^{100} = .00003$. If we were to use the normal approximation, it would be

$$P(\frac{99.5-90}{\sqrt{100 \cdot .1 \cdot .9}} < Z < \frac{100.5-90}{\sqrt{100 \cdot .1 \cdot .9}}) = P(3.1667 < Z < 3.5).$$

This probability is too tiny to use our tables.

(b) This is the probability that 100 or fewer people come. Using the normal approximation, and that $.9 \cdot 105 = 94.5$.

$$P(X < 101) = P(Z < \frac{100.5-94.5}{\sqrt{105 \cdot .09}}) = P(Z < 1.9518) = .97441.$$

(c) We do the same calculation as before, but with $n = 106$. Then $n \cdot .9 = 106 \cdot .9 = 95.4$.

$$P(X < 101) = P(Z < \frac{100.5-95.4}{\sqrt{106 \cdot .09}}) = P(Z < 1.65) = .95053.$$

So, the airline can be 95% sure that everyone who shows up will get a set.

8.48 (a) We are asking what percent of readings will be $\geq .10\%$. Since the mean of reading is .09% and the standard deviation is .004%, having a reading $\geq .10\%$ is having a reading at least 2.5 standard deviations above the mean.

If X is the random variable associated to a reading, we are asking for

$$P(X \geq .10\%) = P(X \geq 2.5) = .00621.$$

So this is 6 tenths of 1 percent.

(b) μ is the actual blood alcohol percentage (the mean of the readings). If $\mu = .09\%$, then a reading of .105% is 3.75 standard deviations above the mean. The probability of a reading at least that far above the mean is .00009. So we can be quite certain that the blood alcohol is above .09% (99.991% certain).

(c) 99% of readings will be below 2.33 standard deviations above the mean. So if a reading is above 2.33 standard deviations above .09%, we can be 99% certain the suspect is legally drunk. That corresponds to a reading $\geq$

$$.09 + 2.33 \cdot .004 = .09932 \text{ percent}.$$

11.1 Two cards, one 0 and one 1. We expect approximately .5 heads give or take .5 heads!

11.2 10 cards, 6 with a 1, 4 with a 0. We expect .6 females on a draw, give or take .49 females.

11.3 Expect 200 heads, give or take $\sqrt{400 \cdot .5 \cdot .5} = 10$ heads. About 68% of the time we'll be between 190 and 210 heads.

11.7 The sum of 19 weights will have expected value $19 \cdot 180 = 3420$ with standard deviation $\sqrt{19} \cdot 40 = 174.4$ pounds. 3800 is 2.18 standard deviations above the expected value. The probability of being this far above the mean is about 1.463%. So the plane will be overloaded about 1 out of eery 68 times.

11.8 Let X be the R.V. for total weight. $E(X) = 80 \cdot 180 = 14400$ pounds. The standard deviation is $40\sqrt{80} = 357.77$ pounds. 16000 is 1600 above 14400. This is 4.47 standard deviations above the expected value. The probability of that is less than 1/1000 of 1%, and too small to be evaluated by our table.