Math 691, Spectral Sequences, Homework 1, due February 3rd.

(1) Let (X, A) be a pair of spaces. Filter $C^*(X)$ by
$$F^0C^*(X) = \ker(C^*(X) \to 0)$$
$$F^1C^*(X) = \ker(C^*(X) \to C^*(A))$$
$$F^2C^*(X) = 0.$$
Describe the remaining $F^iC^*(X)$, and give a complete description of the spectral sequence (E_r and d_r) for $r \geq 1$, including E_∞.

(2) Let X be a CW complex. Define
$$F^iC^*(X) = \ker(C^*(X) \to C^*(X^{(i-1)})).$$
Give a complete description of the spectral sequence $(E_r^{*,*}, d_r^{*,*})$ starting at $r = 1$.

(3) Let $s \geq 1$ be some fixed integer. Give a chain complex and a filtration so that in the associated spectral sequence, $d^r = 0$ except when $r = s$ in which case $d^s \neq 0$.

(4) Use the homology Serre Spectral Sequence to calculate $H_*(\Omega S^n)$ for $n \geq 2$.

(5) Use the cohomology Serre Spectral Sequence to calculate $H^*(\Omega S^n)$ for $n \geq 2$. You can use the result of the previous excercise to identify the groups, but I want you to give the cup product structure also.

(6) Use the Serre Spectral seqeuence to calculate $H^*(\Omega(S^3 \vee S^3))$.

(7) Let n be odd. Calculate $H^*(\Omega^k S^n; \mathbf{Q})$ for $k < n$ using induction on k. (You'll want to iterate the cohomology Serre Spectral Sequence).