

What math do students need for college success?

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Goals for high school math education?

1. What is the math that everyone “should” know in order to function well as an adult, a citizen and in the workforce?
2. What is the math that the CCSS requires high school students to master?
3. What is the math students need to succeed in a (four-year) college?

I'll concentrate on 3 and relationship to 2.

The first question can't really be answered, so of course it's the most interesting. one

Is algebra necessary?

Thought provoking essay by Andrew Hacker (retired Political Science professor) in *New York Times* on July 29 called “Is algebra necessary?”

He really means “is it important for every person to learn algebra?”

<http://www.nytimes.com/2012/07/29/opinion/sunday/is-algebra-necessary.html>

- ▶ Small number of jobs actually require proficiency in algebra “or above.”
- ▶ Effort teaching algebra might be better used teaching “quantitative literacy” to more students.

Different majors

Returning to our question 3.

- I. Humanities and arts, many social sciences demand no mathematics.
- II. Business, some social sciences demand some mathematics but often not so much.
- III. Engineering, mathematics, computer science, physics, other natural science.

Group II. should come to college ready for our Math 111 (College Algebra).

Group III. should come to college ready for a science based calculus class.

Statistics in entering college level mathematics

At University of Oregon

- ▶ About 65% of students will take Math 111.
- ▶ About 30% of those will “fail” (D, F, N, W), many of the rest will get Cs.
- ▶ Of the remaining 35%, some are better math students, some are worse.
- ▶ Entering UO students are stronger than average high school graduates, though not the strongest.

Tentative conclusion: we are not very effective at preparing high school students for college level mathematics.

College level math

- ▶ Most students taking math courses are not math majors.
- ▶ They need to take away the ability to use what we are teaching them in non-math courses.
- ▶ This means they need to leave our courses with modeling skills.

College level math

Examples:

- ▶ Solve quadratic equations? No. Site a factory to minimize shipping expenses from suppliers and to customers? Yes.
- ▶ Table of values of 2^x ? No. Is 8% compounded quarterly more or less than 7.95% compounded continuously? Yes.
- ▶ $\log_5(24)$? No. If a population of bacteria increases 5% per hour, how long does it take to double? Yes.
- ▶ $\tan^{-1}(1/12)$? No. Sketch a design for an ADA compliant ramp that gains 10 inches of altitude? Yes.

Requisites for College Readiness in Mathematics

Summary:

1. Fluency and facility in algebra and algebraic skills is required for almost all college level mathematics, and most college level courses using mathematics.
2. Developing facility in algebra and algebraic skill depends on arithmetic fluency and facility and the ability to use the standard algorithms of arithmetic correctly.
3. The ability to model using algebraic skills is paramount both for the transition to college and for doing college level work.

Although we think we are teaching these skills to students in high school, they have a surprising level of trouble with even basic skills when called on to do modeling with them. **I can't emphasize point 3 too strongly!!**

Example questions that expose student weaknesses

1. Write an equation, using the variables S and P to represent the following statement: "At this university there are six times as many students as professors." Use S for the number of students and P for the number of professors.
2. Write an equation using the variable C and S to represent the following statement: "At Mindy's restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel." Let C represent the number of cheesecakes and S represent the number of strudels ordered.
3. You have a pile of pennies and another pile of dimes. The value of the pile of pennies is equal to the value of the pile of dimes. If P is the number of pennies in the pile, and D is the number of dimes in the pile, then write an equation expressing a relationship between P and D .

Depending on the group of students, on the order of half cannot do the first problem. Students do worse on the succeeding problems.

The problems above are from, and discussed in:

- ▶ Clement, John. “Algebra Word Problem Solutions.” *J. Res. in Math. Ed.*, **13** (1962):16–30.
- ▶ Philipp, Randolph. “The Many Uses of Algebraic Variables.” *Math. Teacher*, **85**,7,(1992):557–561.

In college math classes we are typically asking students to do word problems where the *first step* might be analogous to the problems on the previous page. If they can't do that, they can't even get started on applying the mathematics we are trying to teach.

High School Math Standards and College transition at UO

- ▶ CCSS are an improvement over current Oregon HS standards.
- ▶ K-8 *Grade level standards* are clear that every (say) 5th grader should learn this.
- ▶ *High school standards less clear.* Does every high school student need to learn the statistics standards? Or just those who take statistics? Etc.
- ▶ If all our 18 year-olds were fluent in all the CCSS high school standards we would be achieving a *very* high level of minimum mathematical education.
- ▶ What is necessary for college level math? Some part of the high school CCSS.
- ▶ Currently, most of our incoming freshmen at UO are well *below* this smaller part of the CCSS standards.
- ▶ If our incoming students met CCSS 8th grade standards, they would be ready for Math 111.

CCSS Number and Quantity (High School)

There are four areas: real numbers, quantities, complex numbers, vector and matrix quantities. As mathematics, all are very worthwhile. For college readiness, fluency with the real numbers topics and quantities is essential. The other topics will be retaught in the contexts in which they are needed, although sufficiently advanced students may skip those courses.

N-RN-1,2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

For $x, y > 0$ write $\sqrt{9y^5/x^4}$ in the form Cx^ay^b .

N-Q Reason quantitatively and use units to solve problems.

Use units when modeling. Use units to check answers for sense. Use units in graphical displays of information. Consider levels of accuracy appropriate to measurements and calculations when reporting answers.

N-CN and N-VM are good mathematics that is worth learning. But they are not essential for college readiness. Most of the number sense needed for algebra is covered in earlier grades in the NCSS. Unfortunately this doesn't mean students will really know this when they get to high school. If they don't know it when they get to college, they will require remedial mathematics there.

Algebra Overview

Fluency in parts of A-CED and A-REI is essential for college readiness. Other parts of the algebra standards are valuable but not essential for college readiness.

A-CED Create equations (to model) and solve problems. Graph those equations, interpret solutions. Usually one or two variables.

A-REI:1-4 Solve equations in one variable. Explain steps of solutions using properties of numbers. Especially, but not only, linear and quadratic.

A-REI:5-7 Solve sufficiently simple systems of equations.

Solutions by matrices (linear algebra) not important for college readiness.

Functions Overview

- F-IF:1-6 For college readiness, functions should be thought of with domains and ranges both sets of *numbers*. Recursive functions not needed for college readiness. Rates of change are not important except when they are velocities.
- F-IF7 a. is critically important. b.-d. are desirable. e. not needed for college readiness.
- F-IF8 a. is important for college readiness. b. is not.
- F-BF 1a. is important for college readiness. The rest of F-BF is not.
- F-LE For college readiness, important that student be able to recognize and model linear phenomena.
- F-TF 1-5 useful but not essential for college readiness. 6-9 not needed for college readiness.

Modeling

Modeling is probably the most important skill for college readiness. For every algebraic or arithmetic skill a student masters, the student should also practice and learn numerous modeling techniques and exercises that use that skill.

This simply can't be overemphasized. Being able to solve an abstract equation but being unable to solve a word problem that reduces to the same equation will *not* be useful to the student later on.

Geometry

Geometry is very important historically, and the practice of proof in geometry is good practice for those students who are going to learn more proof in higher mathematics. But most college students who study math will take courses emphasizing applications rather than proof. So while facility with geometric proof is a nice thing, it is not required for college readiness.

What *is* required is use of similar triangles, geometric measurement, and use of geometric properties for modeling. The particular items of most importance then are G-SRT5-8, G-GMD3, G-MG1-3.

Statistics and Probability

This is very useful, and in particular statistics is one of the most useful areas (beyond algebra) that students going into natural or social science (or who just want to be well-informed citizens) can learn.

However, it is not a requirement for college readiness.