What do Information Frictions do?[*]

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Summary. We present an overlapping generations model in which a labor market friction (moral hazard) coexists and interacts with a credit market friction (costly state verification). Our main results are: (i) while credit market frictions have long- and short-run real effects, labor market frictions typically have only short-run effects unless they also affect the volume of investment per worker, (ii) the frictions amplify each other to produce higher long-run unemployment than would result from only labor market frictions, (iii) these distortions may prolong the effect of temporary shocks, and (iv) the dynamics of economies with both frictions are qualitatively similar to their frictionless counterparts.

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1 Introduction

This paper studies multiple information frictions in a neoclassical growth model with the objective of understanding how credit and labor market frictions interact to contribute to persistent involuntary unemployment and relative income gaps across nations. To that end, an overlapping generations economy with production is analyzed in which a labor market friction (moral hazard) coexists with a credit market friction (costly state verification). The simultaneous presence and interaction of the two frictions is studied. An increase in the severity of credit market distortions is shown to increase long-run unemployment, while worsening labor market friction results in lower income per capita.

Researchers have incorporated labor and credit market imperfections within simple neoclassical models principally to facilitate quick departures from the Arrow-Debreu world thereby opening up a role for institutions and to inject a healthy dose of realism into their models. For instance, the importance of credit frictions in the analysis of growth, development and business cycles has been highlighted by a number of papers including Bernanke and Gertler (1989), Tsiddon (1992), Bencivenga and Smith (1993), Boyd and Smith (1997, 1999), Carlstrom and Fuerst (1997), Williamson (1987b), Azariadis and Chakraborty (1998) and Khan (2001). Concurrently, another branch of the literature, Bencivenga and Smith (1997), Jullien and Picard (1998) for example, has studied the effect of labor market frictions on aggregate outcomes. Almost the entire existing literature, however, treats these frictions in isolation. Yet there is good reason to believe that credit and labor markets are tightly interlinked and that effects in one market often spill over into the other, either dampening or amplifying them. A major focus of this paper is to demonstrate that the "value added" to a joint analysis of these frictions is substantial.

The empirical relevance of these aforementioned frictions, for poor as well as rich countries, is well-documented. For instance, Jappelli (1990) provides systematic evidence of widespread credit rationing in a country as financially evolved as the US. Evans and Jovanovic (1989) and Blanchflower and Oswald (1998) note that the probability of self-employment in the US and UK depends positively upon whether the individual ever received an inheritance, a compelling evidence of imperfect capital markets. About a fifth of respondents in a 1987 UK survey on the self-employed rated *where* to get finance as their biggest difficulty (Blanchflower and Oswald, 1998). In a recent paper, Banerjee, Duflo, and Munshi (2002) find that "the gap between the marginal product of capital and the market interest rate [in India] seems to be at least 70 percentage points, and the gap between the marginal product of capital and the rate paid to savers is even larger." Rajan and Zingales (1998) use industry-level data to show how greater credit frictions in Europe, compared to the US, have subdued growth in industries more dependent on external finance. Similarly, labor market frictions have received ample attention from

researchers analyzing the marked rise of European unemployment during the past three decades (see Rogerson, 2000, for comprehensive and up-to-date evidence).

There is persuasive evidence that credit market conditions have strong effects on employment. The Bureau of Labor Statistics reports that nearly 40% of jobs in the US, for example, are held in small firms (with less than 100 employees) while 58.1% of private sector workers work in firms with less than 500 employees. Small businesses are also the primary source of *new* jobs in the US economy. As Berkowtiz and White (2002) note, "from 1990 to 1995, businesses with fewer than 500 employees accounted for 76.5 percent of net new jobs. But small businesses have a very high turnover rate compared to large businesses. Over 13 percent of U.S. jobs in 1995 were in firms that did not exist before 1990 and over 12 percent of jobs in 1990 were in firms that had ceased to exist by 1995". These small firms, the ones most likely to face the greatest difficulties in raising finance, therefore contribute significantly to job creation and destruction. Similarly, in the case of Europe, Acemoglu (2001) notes how the fraction of employment in the most credit-dependent industries has been lower there than in the US.

Our analysis bridges the gap between the credit and labor friction literatures. On the credit side, we adopt Townsend (1979) and Gale and Hellwig's (1985) costly state verification approach to provide a clear channel through which borrowing constraints and related distortions affect investment decisions. On the labor market side, we adapt the Shapiro and Stiglitz (1984) moral hazard and efficiency wages framework. We prefer this over an adverse selection model partly because the empirical relevance of the efficiency wage model is fairly well-documented (Huang et al., 1998), but also because it facilitates a rich and tractable analysis.

In our model, workers suffer disutility costs of working and of exerting effort. Firms receive a noisy signal about how averse workers are to exerting full effort. Consequently, they hire only those who have 'low enough' disutility costs of working, while others remain *involuntarily* unemployed. Incentive compatible wage payments are tailored to elicit full effort from employees. The higher is the wage rate, the less incentive do workers have to shirk and hence, lower is the equilibrium unemployment needed to 'discipline' the workers. In such a setting, credit frictions lower wages by hurting capital accumulation, thereby aggravating unemployment problems.

Labor frictions, at the same time, resonate in the credit market. Since investment projects are indivisible and subject to idiosyncratic shocks that are observed by lenders at a cost, agency costs determine the number of potential entrepreneurs who obtain loans and produce capital. More severe incentive problems in the labor market lower employment and constrict the pool of potential capitalists. They may also reduce the supply of loanable funds. Relative to a frictionless world, less investment is undertaken and capital production per capita is also lower in the long run. In this context, our analysis suggests that high European unemployment, for example, may be better

understood as the joint outcome of credit and labor market imperfections rather than the latter in isolation.

We also investigate the short-run dynamic effects of frictions. In general, we find that external shocks, for instance to the Solow residual, get amplified when credit and labor markets are distorted. These frictions may also prolong the effects of such shocks. Credit frictions, in particular, slow down steady-state convergence indicating their possible role in explaining the duration and persistence of business cycles. Labor frictions have a similar effect but, interestingly, only when they do not affect investment per worker.[1]

The *interplay* of information frictions in the credit and labor markets is an important new contribution to the literature. Three papers that present an unidirectional connection and relate most closely to our work are Betts and Bhattacharya (1998), Acemoglu (2001) and Wasmer and Weil (2002). Betts and Bhattacharya (1998) study how adverse selection problems in the labor market in the presence of credit market frictions can lead to complex dynamics, cycles and development traps. However, in their setup, since all capitalists remain unemployed and receive a constant "home production output", labor market wages really have no direct effect on the credit market, except through the supply of funds. In contrast, wages here are used as collateral by employed capitalists which determine the terms of credit they receive on bank loans. Wasmer and Weil (2002) model specificity in credit relationships, and assume that credit to potential loan-seekers is rationed due to search frictions. Similarly, search frictions in the labor market produce equilibrium unemployment. They too find that credit frictions exacerbate labor market frictions and further increase unemployment. In contrast, we focus on credit rationing and unemployment problems arising out of informational asymmetries. Acemoglu (2000) uses a simple model in which Europe and the US are identical except for higher credit market frictions in the former. He considers the response of these two economies to a common shock, the arrival of a new set of technologies, and shows how the US economy responds to the arrival of new technologies without an increase in unemployment. In contrast, the technological change can have a persistent adverse effect on European unemployment because, in the absence of efficient credit markets, agents who need funds for startups cannot borrow the necessary amount.

The rest of the paper is organized as follows. We outline the model in the next section. In Section 3, we study the efficiency wage contract and in Section 4, spell out details of the loan contract under costly state verification. General equilibrium analysis in Section 5 studies the short- and long-run effects of the two frictions. Section 6 briefly compares the four models that

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[1]In an extension of the model (see Section 7 below) we show that if heightened distortions in the labor market increased investment per worker through a reallocation of labor across two sectors (one of them unaffected by these frictions), then temporary shocks would be less persistent.

are nested within our two-frictions model. Section 7 studies an extension, and we conclude in Section 8 with ideas for future research.

2   Environment

We analyze a production economy inhabited by an infinite sequence of two-period lived overlapping generations, plus an initial old generation. At each date, $t = 1, 2, 3...$, a new generation is born, consisting of a continuum of agents with mass 1. Thus, the measure of total population at any date is 2.

All agents are *ex ante* identical. They are risk-neutral and care only for their second period consumption (c_2). Specifically, we assume that the utility function is given by $U(c_2) = \beta c_2$, gross of any work-related disutilities (see below). Individuals are endowed with unit labor time only in youth. They may either supply it to the labor market for a wage w_t, or, remain unemployed. Some of the workers subsequently obtain an opportunity to become capitalists and create capital. Letting S_t denote an agent's saving and R_{t+1} the gross return on it, we note that $c_{2,t+1} = R_{t+1}S_t$ and that indirect lifetime utility is simply $\beta R_{t+1}S_t$, gross of work-related disutilities. The initial old generation is endowed with an aggregate capital stock of $K_1 > 0$.

A worker can either work the entire one unit of time or shirk completely, choosing effort level $e \in \{0, 1\}$. Let $v > 0$ and $a > 0$ represent the disutilities from being employed by a firm and from actually working. We assume that v is identical for all young agents but that a differs. In particular, $a \sim F(a)$ with support $[\underline{a}, \infty]$, $\underline{a} > 0$. A worker's utility from full effort $(e = 1)$ is given by $\beta c_2 - (v + a)$, from shirking $(e = 0)$ by $\beta c_2 - v$, and by $\beta c_2 \geq 0$ if unemployed.

The economy produces a perishable good every period which may be consumed or invested. This good is produced by the old capitalists (firms) who have access to a constant returns technology that combines capital, K, with effective labor, $(1-\zeta)L$, to produce output $H(K, (1-\zeta)L)$. Here ζ is the fraction of employees who exert no effort (shirk) and L is the total number of employees. To foreshadow, firms will write efficiency wage contracts in equilibrium that optimally set $\zeta = 0$, that is, only non-shirkers are employed in equilibrium.

Let $k_t \equiv K_t/[(1 - \zeta)L_t]$ denote the capital to effective-labor ratio, and $h(k_t) \equiv H(k_t, 1)$, denote the associated intensive production function where h satisfies $h(0) = 0$, $h' > 0 > h''$, and standard Inada conditions. For much of what we do below, we assume that

$$h(k) = Ak^\alpha, \quad A > 0, \quad \alpha \in (0, 1). \tag{1}$$

Standard factor pricing relationships imply that the gross real return to capital is given by

$$\rho_t \equiv \rho(k_t) = h'(k_t) = \alpha A k_t^{\alpha-1}, \tag{2}$$

and the wage rate by

$$\omega_t \equiv \omega(k_t) = h(k_t) - k_t h'(k_t) = (1 - \alpha)Ak_t^{\alpha}. \tag{3}$$

Employed workers enter a lottery that assigns some of them access to an investment technology. This project is an indivisible and risky technology that converts $q > 0$ units of the final good invested at date t to zq units of new capital at $t+1$, where z is an $i.i.d.$ (across capitalists and dates) random variable realized at the beginning of $t + 1$. We let G denote the probability distribution of z and assume that G has a differentiable density function g with finite support $[0, \bar{z}]$, $\bar{z} < \infty$. We denote by $\hat{z}$ the mean of z.

The technological indivisibility in investment implies that external finance may be needed to operate it. The source of this external finance, as will be made clear below, is savings of workers who did not become capitalists. In an environment with credit rationing, it is possible that some potential capitalists may remain unfunded.

The amount of capital actually produced by a project is private information to the project owner. Following Bernanke and Gertler (1989), we assume that any other agent can perfectly witness the return on that project only by incurring a *fixed* cost of $\gamma > 0$ units of capital. This informational asymmetry embedded in the capital investment technology is the source of credit market frictions. Finally, we assume that once the new capital produced by a project between t and $t + 1$ has been used in the production of final goods at $t + 1$, it depreciates completely.

We summarize our discussion of the environment by providing the reader with the following time line of events. At birth, an agent finds out her disutility of effort, a. Firms (old capitalists) observe a and offer incentive compatible wage-employment contracts that employ young agents with disutility $a < \hat{a}$. The rest are unemployed and earn zero income. All employed agents save their entire income and use them in one of two possible ways. Some of them receive an opportunity to become a capitalist in which case they use their saving as internal finance on their investment projects, pending receipt of external finance. If they get funding, they produce capital and earn a return from this activity. If not, they, along with all employed workers who did not obtain opportunities to become capitalists, deposit their wages with an intermediary which promises them a market-determined return. As will be evident below, the possibility of becoming a capitalist in the future affects worker incentives in the present.

3   Labor Market Contracts

The basic formulation of the labor market closely follows Jullien and Picard (1998). Firms observe a, but receive an imperfectly verifiable signal about a worker's effort in the event he shirks. The signal takes binary values, high

or low. Let θ denote the conditional probability that it takes on a high value even when effort is zero.

As Jullien and Picard (1998) show, firms offer a wage contract $\{\omega_1(a), \omega_0(a)\} \in \Re_+^2$ to each worker of type a, where $\omega_1(a)$ is the wage for a high value of the signal and $\omega_0(a)$ the wage for a low value. Recall R_{t+1} denotes the expected return on a unit of funds between t and $t+1$. Firms want all workers to exert full effort. If a worker exerts high effort, the signal reveals that perfectly and he gets paid the high wage. If he shirks, the signal reports high effort with probability θ and low effort with the remaining probability. The wage contract firms offer will be incentive compatible as long as

$$\beta R_{t+1}\omega_{1t}(a) - v - a > [\theta\omega_{1t}(a) + (1-\theta)\omega_{0t}(a)]\beta R_{t+1} - v$$
$$\Leftrightarrow \beta R_{t+1}(1-\theta)[\omega_{1t}(a) - \omega_{0t}(a)] \geq a \tag{4}$$

Since the unemployed receive nothing, it is rational for a type a worker to sign such a wage contract if

$$\beta R_{t+1}\omega_{1t}(a) - v - a \geq 0 \Leftrightarrow \beta R_{t+1}\omega_{1t}(a) \geq v + a. \tag{5}$$

This is the worker's participation constraint – it is endogenous since it depends upon equilibrium prices.

Assume without loss of generality that $\omega_0(a) = 0$. Then all unemployment is *involuntary* if (4) implies (5) and the latter is not binding.[2] A sufficient condition for this is

Assumption 1
$$\frac{\underline{a}\theta}{(1-\theta)} \geq v > 0,$$

which we henceforth maintain. In equilibrium the incentive constraint (4) will bind and

$$\hat{a}_t = \beta(1-\theta)\omega_{1t}R_{t+1}, \tag{6}$$

determines the cut-off disutility level $\hat{a}_t$, given prices, above which no worker is hired. Since full effort by any worker produces the same output, each employed worker earns the same wage, irrespective of a. Equation (6) shows that, *ceteris paribus*, an increase in θ (noisiness of the output signal) increases unemployment while an increase in wages is associated with higher employment.

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[2]Workers do not self-select into and out of jobs; what they select into is exerting full effort, conditional on finding a job. If a is unobservable, then involuntary unemployment is not possible.

Proposition 1 (Jullien and Picard, 1998) *An optimal labor contract is a triple $\{\omega_0(a_t), \omega_1(a_t), \hat{a}_t\}$ such that (i) $\omega_0(a_t) = 0$ for all $a_t > \hat{a}_t$, (ii) $\omega_1(a_t) = \omega_t$ [see eq. (3)] for all $a_t \leq \hat{a}_t$, and, (iii) all workers with disutility cost of working less than $\hat{a}_t$, determined by eq. (6), are employed and exert full effort.*

It follows that total employment is

$$L_t = F(\hat{a}_t) = F(\beta(1-\theta)\omega_t R_{t+1}), \tag{7}$$

and the unemployment rate

$$u_t \equiv 1 - F(\hat{a}_t). \tag{8}$$

The aggregate capital-labor ratio is given by

$$k_t \equiv \frac{K_t}{L_t} = \frac{K_t}{F(\hat{a}_t)} \tag{9}$$

for an aggregate capital stock K_t, and output per young person by

$$\tilde{y}_t \equiv Y_t = h(k_t)F(\hat{a}_t). \tag{10}$$

4   Credit Markets

We now turn to the credit market. As mentioned earlier, after labor market outcomes are realized and compensation received, employed workers are allocated a production opportunity with probability $\varphi \in (0,1)$ as in Khan (2001). This opportunity enables a young employed worker to become a capitalist and produce capital using the risky, indivisible technology described above. By the law of large numbers, a φ fraction of the employed workers receive this opportunity in any period.[3] Under the terms of the labor contract

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[3]We do not allow unemployed workers to get access to any capital production opportunity. Our assumption captures, albeit in an extreme manner, the notion that people with a high propensity to shirk as workers cannot possibly be great capitalists. There is a technical reason for this assumption. Since unemployed workers in the model earn no income at all, they can provide no internal finance, unlike the employed who can supply their wages as internal finance. Were we to allow the unemployed to also become capitalists, we would have created two kinds of capitalists with two levels of internal finance, and hence two kinds of loan contracts, an unnecessary complication in our opinion.

each worker receives a real income ω_t when young. A measure of workers L_t given by (7) find jobs; the rest remain unemployed and receive zero lifetime utility. Among these employed workers, φL_t have the potential to become capitalists. In an environment with credit rationing, some of these (a fraction μ) potential capitalists receive funding. The rest simply invest their savings in alternative means.

Since employed workers can ultimately become capitalists, the expected return to saving must take account of that possibility. Let R^c denotes the one-period return to becoming a capitalist, R^{fc} denotes the one-period return accruing to a funded capitalist, and R^w denotes the one-period return to saving as a worker. Then, it must be that $R = \varphi R^c + (1 - \varphi) R^w$, and $R^c = \mu R^{fc} + (1 - \mu) R^w$. It follows that

$$R_{t+1} = p_t R^{fc}_{t+1} + (1 - p_t) R^w_{t+1},$$

where $p_t = \varphi \mu_t$ is the probability of becoming a funded capitalist. Equilibrium values of p_t, R^c and R^w will be determined below.

Following Williamson (1986) we assume that all lending is intermediated. In other words, workers who do not receive the capital production opportunity simply deposit their savings with an intermediary which pools these deposits and makes loans on their behalf. Intermediaries perform the role of delegated monitors, verifying project returns as per the terms of loan contracts. We assume that any lender may establish an intermediary. In equilibrium, with unrestricted entry into the market for intermediation services, all intermediaries must offer a common competitively determined deposit return, r_{t+1}, must hold perfectly diversified loan and deposit portfolios, and must earn zero profits. In this context, it is important to note that project returns are *i.i.d.* across a continuum of capitalists and hence there is no aggregate randomness. Consequently, an intermediary with a perfectly diversified loan portfolio earns a non-stochastic return on its assets and need not be monitored by its depositors.

4.1   Loan Contracts

The capital investment technology (project) is an indivisible technology that can only be operated at the scale q. We assume that *all* capitalists need some amount of external financing to operate their individual projects. Equivalently, we assume that

Assumption 2

$$q > \omega_t$$

holds at all dates. All capitalists therefore need external financing of the amount $b_t = q - \omega_t > 0$. Potential borrowers, wishing to obtain external

funding, announce loan contracts that may be accepted or rejected by lenders (banks). Borrowers whose contracts are accepted receive external funding of the amount b_t after which they operate their projects.

Under this costly state verification setup and deterministic monitoring, Gale and Hellwig (1985) and Williamson (1986) show that optimal loan contracts take a simple form. In particular, the state space $[0, \bar{z}]$ is divided into two subsets $\mathcal{A}_t$ and $\mathcal{B}_t$. For all realizations of $z \in \mathcal{A}_t$, intermediaries verify project returns for sure. But if $z \in \mathcal{B}_t \equiv [0, \bar{z}] - \mathcal{A}_t$, no verification occurs. Let $\mathcal{R}_t(z)$ denote the promised real payment per unit borrowed for all $z \in \mathcal{A}_t$. Since no verification occurs on $\mathcal{B}_t$, incentive compatible payments would have to be independent of realizations of z. Let x_t denote this uncontingent payment (per unit borrowed) for all $z \in \mathcal{B}_t$.

Intermediaries take the return to be paid on deposits between t and $t+1$, r_{t+1}, as given and act as if there is a perfectly elastic demand for deposits at this rate. Hence, they agree to the terms of such a loan contract only if the expected return on lending is at least as great as r_{t+1}. Formally, this participation constraint can be written as

$$\int_{\mathcal{A}_t} \left[\mathcal{R}_t(z)b_t - \rho_{t+1}\gamma\right] g(z)dz + x_t b_t \int_{\mathcal{B}_t} g(z)dz \geq r_{t+1}b_t, \qquad (11)$$

where $\rho_{t+1}\gamma \int_{\mathcal{A}_t} g(z)dz$ is the real expected monitoring cost. In addition, all contracts must be incentive compatible, that is,

$$\mathcal{R}_t(z) \leq x_t \text{ for } z \in \mathcal{A}_t \qquad (12)$$

must hold. Finally, it must be feasible for borrowers to repay (limited liability),

$$\mathcal{R}_t(z) \leq \left[\frac{\rho_{t+1}zq}{b_t}\right] \text{ for } z \in \mathcal{A}_t \qquad (13)$$

and

$$x_t \leq \inf_{z \in \mathcal{B}_t} \left[\frac{\rho_{t+1}zq}{b_t}\right]. \qquad (14)$$

Potential capitalists simply maximize their own expected utility by choosing the contractual obligations optimally subject to the constraints (11)-(14). The solution to this problem is a standard debt contract. When the capitalist realizes a return $z \in \mathcal{B}_t$, he pays x_t (principal plus interest) to the lender. When $z \in \mathcal{A}_t$, he defaults; in this case, the intermediary monitors

the project and retains the entire proceeds (rental income from whatever capital is produced) net of monitoring costs.[4]

Proposition 2 (Williamson, 1986, 1987a) *The optimal loan contract satisfies*

$$\mathcal{R}_t(z) = \left[\frac{\rho_{t+1}q}{b_t}\right] z \quad for \ z \in \mathcal{A}_t, \qquad \mathcal{A}_t = [0, \frac{x_t b_t}{q\rho_{t+1}}),$$

and

$$r_{t+1} = \int_{\mathcal{A}_t} \left[\mathcal{R}_t(z) - \frac{\rho_{t+1}\gamma}{b_t}\right] g(z)dz + x_t \int_{\mathcal{B}_t} g(z)dz.$$

Using Proposition 2, it is also easy to show the following:

$$r_{t+1} = x_t - \left(\frac{\rho_{t+1}\gamma}{b_t}\right) G\left(\frac{x_t b_t}{q\rho_{t+1}}\right) - \left(\frac{\rho_{t+1}q}{b_t}\right) \int_0^{\frac{x_t b_t}{q\rho_{t+1}}} G(z)dz \equiv \Phi\left[x_t; \frac{b_t}{\rho_{t+1}}\right],$$

where the function Φ gives the expected return to the lender as a function of the gross loan rate, x_t, the amount borrowed, b_t, and the future relative price of capital, ρ_{t+1}. Following Williamson (1986, 1987), we impose the following assumptions:

Assumption 3

$$g(z) + \left(\frac{\gamma}{q}\right) g'(z) \geq 0 \ for \ z \in [0, \bar{z}]$$

and

Assumption 4

$$\Phi_1\left[0, \left(\frac{b_t}{\rho_{t+1}}\right)\right] \equiv 1 - \frac{\gamma}{q}g(0) - G(0) > 0.$$

Assumption 3 ensures the concavity of Φ, that is, $\Phi_{11} < 0$. In addition, if assumption 4 holds, Φ is inverse U- shaped. Hence there will be a unique

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[4]Such monitoring or auditing costs can be fairly substantial even in developed countries. For example, Del Boca and Lusardi (1998) quote a Bank of Italy estimate that it took 5.5 years on average for an Italian bank to repossess the collateral.

x_t depending upon b_t/ρ_{t+1} which maximizes the expected return to a lender. We denote this value by $\hat{x}\left(b_t/\rho_{t+1}\right)$ where

$$\Phi_1\left[\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right),\left(\frac{b_t}{\rho_{t+1}}\right)\right] \equiv 1-\left(\frac{\gamma}{q}\right)g\left[\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right)\frac{b_t}{q\rho_{t+1}}\right]-G\left[\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right)\frac{b_t}{q\rho_{t+1}}\right]=0$$
(15)

Equation (15) and Assumption 3 imply that

$$\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right)\frac{b_t}{q\rho_{t+1}} \equiv \delta \tag{16}$$

where $\delta > 0$ is a constant satisfying

$$1-(\gamma/q)\,g(\delta)-G(\delta) \equiv 0.$$

The object δ has a straightforward implication: the set $\mathcal{A}_t$ is now defined by $z \in [0,\delta)$. That is, when all potential borrowers are offering the interest rate that maximizes a prospective lender's expected rate of return, δ is the critical project return for which a borrower's project income exactly covers loan principal plus interest.

4.2 Credit Rationing

In the environment specified above, it is quite possible to have an unsatisfied demand for credit as originally noted by Gale and Hellwig (1985) and Williamson (1986, 1987a). If all capitalists want to run their projects at any date t then the total demand for credit is just $\varphi(q-\omega_t)L_t$. Aggregate supply of funds comes from the saving of all employed workers, $(1-\varphi)\omega_t L_t$. Then a necessary condition for credit rationing for all t is

$$(1-\varphi)\omega_t L_t < \varphi\left(q-\omega_t\right)L_t \Leftrightarrow \varphi q > \omega_t. \tag{17}$$

If (17) is satisfied, there will be credit rationing in equilibrium and $x_t \equiv \hat{x}\left(\frac{b_t}{\rho_{t+1}}\right)$ must hold. This implies that under credit rationing, all potential borrowers are offering the interest rate that maximizes a prospective lender's expected rate of return. No borrower can therefore obtain credit by changing the interest rate and other loan terms without reducing the expected return for all lenders. In this setting, some borrowers must remain unsatisfied, or in other words, credit rationing will obtain in equilibrium. Below, we focus on economies where rationing occurs at all dates.

The following lemma summarizes information about the payoffs to lenders and all funded borrowers.

Lemma 1 (Boyd and Smith, 1998) *Suppose*

$$\hat{z} > \left(\frac{\gamma}{q}\right) G(\delta)$$

*holds. Then **(a)** the return on deposits that the intermediary offers, r_{t+1}, is given by*

$$r_{t+1} = \psi \frac{\rho_{t+1}}{b_t} \qquad (18)$$

where

$$\psi \equiv q \left[\delta - \left(\frac{\gamma}{q}\right) G(\delta) - \int_0^\delta G(z)dz \right],$$

*and **(b)** the expected utility of a funded borrower under credit rationing is given by*

$$\rho_{t+1} q \phi - r_{t+1} b_t,$$

where

$$\phi \equiv \hat{z} - \left(\frac{\gamma}{q}\right) G(\delta).$$

The proof of this lemma follows directly from the arguments in Boyd and Smith (1997, 1998). It must also be the case that any potential capitalist actually prefers borrowing and operating his project to simply depositing his income with the intermediary. Boyd and Smith (1998) prove that a sufficient condition for all capitalists to operate their projects using external finance is

Assumption 5

$$(1 - \alpha) \phi q \geq \psi.$$

5 General Equilibrium

We turn to an analysis of the general equilibrium. Equilibrium in the credit market under rationing implies that only a fraction $\mu_t < 1$ of capitalists get external funding at each date t. Each funded borrower borrows an amount b_t. Therefore, the total volume of loans granted is $\mu_t \varphi L_t b_t$. The total supply of savings is the sum of the wage incomes of employed workers who did not receive a production opportunity. These are the natural lenders of our economy. Additionally, unfunded capitalists also deposit their funds with an

intermediary and, in effect, become lenders. The total supply of loanable funds is then given by

$$[(1 - \varphi) + \varphi(1 - \mu_t)] L_t \omega_t. \tag{19}$$

Equality between sources and uses requires that

$$\varphi \mu_t b_t L_t = (1 - \varphi)\omega_t L_t + \varphi(1 - \mu_t)\omega_t L_t \tag{20}$$

holds for all t. This simplifies to

$$\mu_t = \frac{\omega_t}{\varphi q}, \tag{21}$$

which is less than one (from (17) above).

As noted earlier, there is no aggregate randomness in the economy. Consequently, the capital stock at $t + 1$ equals the amount of capital produced, $(\mu_t \varphi L_t)\, \hat{z}q$, less the amount spent on monitoring bankrupt projects. Recall that monitoring occurs for sure when realizations of the idiosyncratic productivity shock z fall below δ. Monitoring costs then amount to $\gamma G\left[x_t b_t/(\rho_{t+1} q)\right] \mu_t \varphi L_t \equiv \gamma G(\delta)\mu_t \varphi L_t$. Hence, the aggregate capital stock at $t + 1$ is

$$K_{t+1} = \mu_t \varphi \phi q L_t, \tag{22}$$

where ϕ is defined in Lemma 1 above. Using (21) and noting that $L_t = F\left(\hat{a}_t\right)$, we eliminate μ_t to obtain

$$K_{t+1} = \phi \omega_t F\left(\hat{a}_t\right). \tag{23}$$

It follows that asset market equilibrium satisfies

$$k_{t+1} \equiv \frac{K_{t+1}}{L_{t+1}} = \frac{\phi \omega(k_t) F\left(\hat{a}_t\right)}{F\left(\hat{a}_{t+1}\right)} = \phi \omega(k_t) \frac{F\left(\hat{a}_t\right)}{F\left(\hat{a}_{t+1}\right)}, \tag{24}$$

a difference equation in $(k_t, \hat{a}_t)$.

The second piece describing general equilibrium comes from the labor market, specifically the worker's incentive constraint (6). First we use equilibrium prices and quantities to simplify this expression. Evidently $p_t = \varphi \mu_t$ and $R_{t+1}^w = r_{t+1}$ as defined in Lemma 1. By Lemma 1(b), an entrepreneur's expected return from investing ω_t of his own funds into the capital production process yields $\rho_{t+1} q \phi - r_{t+1} b_t$, so that $R_{t+1}^{fc} = (\rho_{t+1} q \phi - r_{t+1} b_t)/\omega_t$ is the

expected return to being a funded capitalist. Substituting these expressions into (6) and simplifying gives us

$$\hat{a}_t = \beta(1-\theta)\omega_t[p_t R^{fc}_{t+1} + (1-p_t)R^w_{t+1}] = \beta\phi(1-\theta)\omega(k_t)\rho(k_{t+1}), \quad (25)$$

where the expected gross return on savings, R_{t+1}, equals $\phi\rho_{t+1}$, the marginal product of capital net of monitoring costs incurred per unit invested.

5.1 Stationary and Non-Stationary Equilibria

Using (24) and (25), dynamic equilibria may be expressed as a second-order difference equation in the capital-labor ratio:

$$k_{t+1} = \phi\omega(k_t)\frac{F\left[\beta\phi(1-\theta)\omega(k_t)\rho(k_{t+1})\right]}{F\left[\beta\phi(1-\theta)\omega(k_{t+1})\rho(k_{t+2})\right]} \quad (26)$$

when credit is rationed at all dates. Given an initial value for K_1, equation (26) describes the subsequent evolution of any potentially valid equilibrium sequence $\{k_t\}_{t=2}^{\infty}$. From such a sequence, it is easy to compute the equilibrium sequences for $\{\mu_t\}_{t=1}^{\infty}$, $\{\hat{a}_t\}_{t=1}^{\infty}$, $\{u_t\}_{t=1}^{\infty}$ etc. Stationary equilibria will be time-invariant solutions to $k = \phi\omega(k)$. To examine stationary and non-stationary equilibria more closely it will be more convenient to work with the two-dimensional system:

$$k_{t+1} = \phi\omega(k_t)\frac{F\left(\hat{a}_t\right)}{F\left(\hat{a}_{t+1}\right)}, \quad (27)$$

$$\hat{a}_t = \beta\phi(1-\theta)\omega(k_t)\rho(k_{t+1}). \quad (28)$$

Using (3), we can then write

$$k_{t+1} = \phi(1-\alpha)Ak_t^\alpha\frac{F\left(\hat{a}_t\right)}{F\left(\hat{a}_{t+1}\right)}, \quad (29)$$

$$\hat{a}_t = \alpha\beta\phi(1-\theta)(1-\alpha)A^2 k_t^\alpha k_{t+1}^{\alpha-1}. \quad (30)$$

Rewriting (30) as

$$k_{t+1} = \eta k_t^{\alpha/(1-\alpha)}\hat{a}_t^{-1/(1-\alpha)}, \quad (31)$$

where $\eta \equiv [\alpha\beta\phi(1-\theta)(1-\alpha)A^2]^{1/(1-\alpha)}$, and substituting it into (29) we get

$$F\left(\hat{a}_{t+1}\right) = \sigma k_t^{-\alpha^2/(1-\alpha)}\hat{a}_t^{1/(1-\alpha)}F\left(\hat{a}_t\right), \quad (32)$$

where $\sigma \equiv [\phi(1-\alpha)A]/\eta$. Steady-states $(k^*, \hat{a}^*)$ solve the pair of equations

$$k^* = \eta(k^*)^{\alpha/(1-\alpha)}(\hat{a}^*)^{-1/(1-\alpha)},$$
$$1 = \sigma(k^*)^{-\alpha^2/(1-\alpha)}(\hat{a}^*)^{1/(1-\alpha)}.$$

Proposition 3 *The non-trivial steady-state $(k^*, \hat{a}^*)$ is a saddle.*

Proposition 3 (see Appendix A for a proof) suggests that the unique equilibrium path of $\{k_t, a_t\}$ asymptotically converges to the stationary equilibrium $(k^*, \hat{a}^*)$. Unlike in the standard Diamond (1965) model, where all young agents work in youth, here the initial capital stock per worker (k_1) is not given. What is given is the initial *aggregate* capital stock, K_1. Define $k_1 \equiv K_1/F(\hat{a}_1)$. With perfect foresight, the initial $\hat{a}_1$ is determined such that the configuration $(k_1, \hat{a}_1)$ places the economy on the stable manifold approaching the stationary equilibrium $(k^*, \hat{a}^*)$.

5.2 Comparative Statics

We undertake a bunch of comparative statics exercises to illustrate the effects of the two frictions, parameterized by γ and θ (the agency cost and informativeness of the effort signal) on steady-state capital per worker, output and unemployment. The starting point is the steady-state capital-labor ratio derived for a Cobb-Douglas technology as

$$k^* = [(1-\alpha)\phi A]^{\frac{1}{1-\alpha}} = [(1-\alpha)A]^{\frac{1}{1-\alpha}}\left[\hat{z} - \left(\frac{\gamma}{q}\right)G(\delta)\right]^{\frac{1}{1-\alpha}}, \qquad (33)$$

and the steady-state employment level $L^* = F(\hat{a}^*)$ determined, from (32), by

$$\hat{a}^* = \alpha\beta(1-\theta)(1-\alpha)^{\alpha}A^{1+\alpha}\phi^{\alpha}(k^*)^{\alpha^2}. \qquad (34)$$

Steady-state output per worker, $y^* = A(k^*)^{\alpha}$, depends upon k^*, while output per capita (more accurately, per young person), $\tilde{y}^* = A(k^*)^{\alpha}F(\hat{a}^*)$, also depends on steady-state employment.

Consider first γ, our proxy for agency costs or the costs of acquiring private information. A higher γ has two effects. It implies larger deadweight losses from monitoring. At the same time, higher monitoring costs induce banks to monitor less often, i.e., over a smaller set of bankruptcy states (lower δ). For unit monitoring costs $\gamma G(\delta)/q$ to be increasing in γ, following Azariadis and Chakraborty (1999), we require that

Assumption 6

$$1 - \frac{\gamma}{q} g(\delta) \left[1 + \frac{g(\delta)}{\gamma g'(\delta)/q + g(\delta)} \right] > 0.$$

As long as this is satisfied, the steady-state capital per worker decreases with higher monitoring costs, $\partial k^*/\partial \gamma < 0$.[5] Output per worker is hence lower the higher are agency costs in the credit market. From (21), it is also apparent that credit is more intensely rationed – fewer capitalists are funded because the aggregate cost of monitoring them is greater.

On the employment side, a decline in k^* lowers the steady-state wage rate. *Ceteris paribus*, a lowering of the efficiency wage can be sustained only if the unemployment rate is kept high enough to 'discipline' workers and dissuade shirking. Consequently, u^* rises. With the decline in employment, output per capita falls because each worker is now less productive and secondly, because there are fewer workers earning income.

Turn next to the effect of θ, the conditional probability of receiving a signal that a worker has exerted high effort when he has not. A higher θ makes it harder for firms to judge their workers' actions and this increases the severity of the labor market friction. Somewhat surprisingly, as (33) reveals, labor frictions have no effect on the long-run capital per worker k^*, and therefore, leave steady-state credit rationing μ^* unaffected. At the same time, a higher θ leads to lower long-run employment as is evident from (34).

It is instructive to lay out the adjustment process following a change in θ. From eq. (28), given an aggregate capital stock, an increase in θ increases unemployment, thereby increasing capital per worker. The resulting increase in wages raises the incentive of workers to supply high effort. This induces firms to hire more workers and the initial decline in unemployment is partially offset. The net result of lower employment is to reduce the aggregate flow of savings and loanable funds into the capital market, $\omega_t L_t$. Investment declines as does the aggregate capital stock which via the wage rate elicits more unemployment. Since the size of the workforce (= savers) is constant in steady-state, the long-run effect is to leave the capital-labor ratio unaffected with employment and the aggregate capital stock falling proportionately, by the full increase in θ. Of course this means output per worker y^* remains unchanged. But since the unemployed do not consume anything in our model, a better yardstick of welfare is output per capita, $\widetilde{y}^*$, which clearly declines as labor frictions worsen.

To summarize, while γ has long-run effects on capital and output per worker and on employment, θ affects only employment and the aggregate capital stock in the long-run. But as our preceding discussion indicates, labor frictions have short-run effects as the economy adjusts to its new steady-state.

Appendix A below shows how the Jacobian of the linearized dynamic system depends on both γ and θ via $\hat{a}^*$. The relative effect of the severity of these two frictions along the saddle path is of interest here. The behavior of $(k_t, \hat{a}_t)$ for small departures from the steady-state $(k^*, \hat{a}^*)$ will be governed by the stable eigenvalue of the Jacobian $\mathbf{Q}$ for the linearized system. If λ_1 denotes this eigenvalue, we have

$$\tilde{k}_t = \lambda_1^t \tilde{k}_0 \text{ and } \tilde{a}_t = \lambda_1^t \tilde{a}_0,$$

---

[5]See Carlstrom and Fuerst (1997) and Azariadis and Chakraborty (1999) for two examples of $g(.)$ that satisfy this assumption.

where $\tilde{z}_t \equiv (z_t - z^*)/z^*$ denotes deviation from steady-state. The smaller is λ_1, the faster is convergence since the more rapidly does z_t approach z^*. As we prove near the end of Appendix A, more severe credit and labor market frictions (increases in γ and θ), are both associated with higher values of λ_1. Hence, convergence to steady-state is slower the greater are frictions in the two markets.

These results have interesting implications for low-frequency movements. A number of researchers, Barro and Sala-i-Martin (1992) for example, have deliberated on the speed of convergence in the context of development. Whether or not this speed is high determines the relative emphasis we should place on the dynamics of convergence versus the properties of the steady-state in analyzing the sources of long-run growth. The slower speed of convergence in our model is one possible explanation why so many countries, particularly those in Africa, have lagged behind even as a few made enormous strides in the past half-century.

6   The Underlying Nested Models

The model described thus far in fact nests three different models, one where only the labor market friction is active (as in, say, Jullien and Picard, 1998), one where only the credit market friction is active (as in, Boyd and Smith, 1998, for example), and of course, the full-information frictionless Diamond (1965) model. We briefly sketch each of the nested models with the purpose of uncovering the exact influence of each friction on the models' deeper features.

To facilitate comparisons, we set $\hat{z} = 1$ and restrict ourselves to $\phi \in [0, 1]$. When $\gamma = 0$ we have $\phi = 1$, the case where all information is effectively public and credit markets work frictionless. Similarly, setting $\theta = 0$ gives us a frictionless labor market since firms can perfectly deduce their employees' work efforts. In order that our model behaves exactly like the full-information Diamond model when $\gamma = \theta = 0$, we require that

$$A \geq \max \left\{ \frac{(\varphi q)^{1-\alpha}}{1 - \alpha}, \left(\frac{\bar{a}}{\alpha \beta} \right)^{1-\alpha} \left(\frac{1}{1 - \alpha} \right)^{\alpha} \right\},$$

which guarantees that all borrowers are funded and all workers employed when credit and labor markets are free of frictions.[6]

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[6]The frictionless ($\gamma = \theta = 0$) version of our model does not reduce to the textbook Diamond model because we have assumed fixed project size (q) and worker disutility from exerting effort (a). Even without frictions, it is possible that workers with very high a do not find exerting effort to be in their best interest. In that case wages have to be sufficiently high to align these workers' incentives with the firms', or else these workers would remain unemployed. Similarly, wages have to be high enough for aggregate savings to fund all entrepreneurs. A high enough value of A is sufficient to obtain high wages.

6.1 Nested models

1. **Model with Only Labor Friction:** In most respects this is the classic Diamond model except that workers are of heterogenous types and firms issue efficiency wage contracts that keep shirkers away at the cost of some unemployment. With employed agents saving their entire wage income, we have $K_{t+1} = L_t \omega_t$ and hence the equilibrium consists of sequences of $\{k_t, \hat{a}_t\}$ that satisfy

$$k_{t+1} = \omega(k_t) \frac{F(\hat{a}_t)}{F(\hat{a}_{t+1})}, \qquad \hat{a}_t = \beta(1-\theta)\omega(k_t)f'(k_{t+1}).$$

The positive stationary equilibrium can be shown to be unique and saddle-path stable.

2. **Model with Only Credit Friction:** There is full employment, $L_t = 1$ for all t, in this case. The active CSV friction affects returns on loans and deposits as defined in Lemma 1: $R_{t+1} = \psi f'(k_{t+1})/\{q - \omega(k_t)\}$ and also results in credit rationing. Under our assumption of zero first period consumption, using (23), it follows that $k_{t+1} = \phi\omega(k_t)$. Standard arguments establish the existence of a unique asymptotically stable steady state (see Boyd and Smith, 1998).

3. **Full-information Diamond model:** The law-of-motion for capital per worker will be $k_{t+1} = \omega(k_t)$ here, which possesses a unique and asymptotically stable positive steady-state.

It is clear that adding the two frictions does not fundamentally alter the stability properties of the steady-states, nor does it contribute to multiple equilibria of the kind that Betts and Bhattacharya (1998) and Acemoglu (2001) note.

6.2 Model comparisons

Define k_D^* to be the steady state capital-labor ratio in the full-information Diamond model, k_L^* to be the same in the model with the labor market friction, k_C^* for the model with just the credit market friction, and k_{LC}^*, the stationary capital-labor ratio for the model with both frictions.

Lemma 2 *Under the assumption of Cobb-Douglas technology,*

$$k_D^* = [A(1-\alpha)]^{\frac{1}{1-\alpha}}, \quad k_L^* = [A(1-\alpha)]^{\frac{1}{1-\alpha}},$$
$$k_C^* = [\phi A(1-\alpha)]^{\frac{1}{1-\alpha}}, \quad k_{LC}^* = [\phi A(1-\alpha)]^{\frac{1}{1-\alpha}},$$

where ϕ is defined in Lemma 1. The corresponding employment rate in the presence of labor frictions is given by equation (34).

Recalling equation (21), the fraction of borrowers funded in steady-state is $\mu^* = \omega(k^*)/(\varphi q)$ as long as $\gamma > 0$ (all borrowers are funded for $\gamma = 0$). Since only employed workers obtain production opportunities, we may also consider economy-wide access to credit, that is, the fraction of young agents who obtain loans, $\tilde{\mu}^* = \varphi \mu^* F(a(k^*))$.

Comparing steady-states across these four nested economies we note that:

Proposition 4 *(a)* $k_{LC}^* = k_C^* < k_L^* = k_D^*$*: the steady-state capital per worker is lowest for economies with credit market frictions and moreover does not depend upon frictions in the labor market.*
(b) $\mu_{LC}^* = \mu_C^* < \mu_L^* = \mu_D^* = 1$*: the extent of credit rationing is identical in the economy with both frictions and in the economy with only credit market frictions. However, the fraction of the population with access to credit is higher in the former:* $\tilde{\mu}_{LC}^* < \tilde{\mu}_C^*$*.*
(c) $\hat{a}^* < \hat{a}^*$*: long-run unemployment is higher in the economy with both frictions than in the economy with only the labor market friction.*

Proposition 4 (a) and (b) follow directly from our earlier discussions. For (c), note that credit frictions affect worker's incentives through the wage rate. In the presence of these frictions, capital accumulation is lower and so is the wage that workers are offered in equilibrium. The lower wage increases worker incentives to shirk; employers respond by cutting back on employment.

Proposition 4 carries an important implication. It suggests that instead of cancelling out each other's effects, frictions are reinforced and weigh down on real activity along the growth path. In particular, the presence of the credit market friction worsens the severity of the labor market friction and raises long-run unemployment. Several caveats notwithstanding, our analysis suggests that employment policies designed to reduce unemployment may perform better if other policies concurrently lower the severity of the credit market friction. There is evidence to suggest that both credit and labor market frictions are more acute in Europe relative to the US. The model indicates that institutional imperfections in both markets may be vital for explaining Europe's deeper unemployment problems.[7]

7   Extensions

As we noted in Section 5, a surprising feature of our setup is that labor market frictions do not affect long-run real activity per worker. As will be evident below, this result hinges on an assumption that precludes the unemployment rate from affecting saving and investment per worker. In this section, we discuss a simple modification to this assumption.

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[7]Wasmer and Weil (2002) also argue that Europe's unemployment problems have persisted and economic activity remained sluggish even after European labor markets eliminated many of the rigidities widely blamed for its problems.

Suppose we allow unemployed workers to receive an alternate means of income. In particular, suppose that the unemployed can ultimately engage in household production, converting their youthful labor time into τ units of the consumption good without experiencing any disutility of effort.[8] We assume $\tau < \omega(k)$ for all "relevant" values of k. Just as before, these workers do not receive any other production opportunity, but can now deposit their savings with the same intermediary that the employed workers access. Since savings of the unemployed flow into the credit market, effective savings per worker will now depend on labor market frictions via the size of the unemployed pool.[9]

Without presenting a full reworking of the model, we highlight some of the basic changes to the analysis. The specifics of the loan contracts remain as in Section 4, but credit market outcomes are affected. Total demand for credit, when all capitalists want to operate their projects, is $\varphi(q - \omega_t)L_t$. Aggregate supply of funds now comes from the savings of employed workers who did not receive a production opportunity, $(1 - \varphi)\omega_t L_t$, *and* of the unemployed, $\tau(1 - L_t)$. Then a necessary condition for credit rationing to hold for all t is

$$(1 - \varphi)\omega_t L_t + \tau(1 - L_t) < \varphi\left(q - \omega_t\right) L_t \Leftrightarrow \varphi q > \omega_t + l_t \tau,$$

where $l \equiv (1 - L)/L$ is the fraction of workforce engaged in household production. Equating supply and demand for loanable funds under rationing, we find the fraction of funded capitalists to be

$$\mu_t = \frac{\omega_t + l_t \tau}{\varphi q}.$$

Compared to (21), *ceteris paribus*, credit rationing will now be less pervasive since the unemployed have positive savings invested in the capital market. What is key, however, is that labor frictions now *directly* affect credit rationing through l_t.

Turning to the labor market, we reconsider a worker's incentive compatibility constraint. We normalize the wage that firms pay to shirking workers as $\omega_{0t} = 0$ and to non-shirkers as $\omega_{1t} = \omega_t$, as before. Workers exert full

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[8] The income τ may be interpreted as home production, as in the real business cycle literature. Alternatively, we can view it as legal informal sector activity, as in the development economics literature.

[9] As an alternative, consider a model where people potentially work in both periods of life (wage contracts are still one period long). If agents can become unemployed when old, then their saving behavior would depend upon that possibility. It is possible to have labor market frictions affect steady-state real activity in this environment. This reformulation would require some risk aversion on the part of agents which, unfortunately, would preclude the possibility of deriving a standard debt contract in the credit market.

effort iff

$$\beta R_{t+1} (1 - \theta) \omega_t (a) \geq a, \qquad (35)$$

a condition similar to (4). Since unemployed workers earn τ from household production, it is rational for a type a worker to sign such a contract if

$$\beta R_{t+1} \omega_t (a) - v - a \geq \beta R^w_{t+1} \tau.$$

All unemployment is involuntary if the first constraint implies the second and the latter is not binding, that is, as long as

$$a \geq (1 - \theta) \left[v + a - \beta \tau R^w_{t+1} \right] > 0.$$

A sufficient condition for this is

$$\frac{\theta}{1 - \theta} \underline{a} > v - \beta \tau R^w (k_1)$$

anticipating that starting from a initial $k_1 < k^*$, return to savings will monotonically decline as the economy approaches its steady-state. We assume parameter values, including the initial capital stock k_1, which satisfy this condition.[10]

Equilibrium employment is determined by (35) holding as an equality. Again, since expected utility of a funded borrower under credit rationing is $\rho_{t+1} q\phi - r_{t+1} b_t$, his expected return on savings is $R^{fc}_{t+1} = [\rho_{t+1} q\phi - r_{t+1} b_t]/\omega_t$. Given the probability of becoming a capitalist $p_t = \varphi \mu_t$, expected return to savings is given by

$$
\begin{aligned}
R_{t+1} &= [p_t R^{fc}_{t+1} + (1 - p_t) R^w_{t+1}]\omega_t \\
&= \phi \rho_{t+1} + l_t \left(\frac{\tau}{\omega_t} \right) (\phi \rho_{t+1} - r_{t+1}),
\end{aligned}
$$

which now depends on the allocation of labor between the formal and household activities (l_t) and on returns to household production relative to formal sector earnings (τ/ω_t). Equilibrium employment is determined by $\hat{a}_t$ according to

$$
\begin{aligned}
\hat{a}_t &= \beta(1 - \theta)[\phi \rho_{t+1} \omega_t + l_t \tau (\phi \rho_{t+1} - r_{t+1})] \\
&= \beta(1 - \theta) h'(k_{t+1}) \left[\phi \omega(k_t) + \tau \left(\frac{1 - F(\hat{a}_t)}{F(\hat{a}_t)} \right) \left\{ \phi - \frac{\psi}{q - \omega(k_t)} \right\} \right] (36)
\end{aligned}
$$

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[10] Alternatively, we can set $v = 0$ and automatically satisfy the involuntary unemployment condition for all $a \geq 0$. We could not have done so in section 2, because $v > 0$ is required to ensure that all unemployed workers are involuntarily unemployed.

In the capital market, aggregate flow of loanable funds now comprises of the savings of the employed workers, $\omega_t L_t$, and of the unemployed, $\tau(1 - L_t)$. The capital market clearing condition becomes:

$$k_{t+1} = \phi \omega_t \frac{F(\hat{a}_t)}{F(\hat{a}_{t+1})} + \phi\tau \frac{1 - F(\hat{a}_t)}{F(\hat{a}_{t+1})}. \tag{37}$$

Equations (36) and (37) comprise the new dynamical system whose steady states satisfy

$$
\begin{aligned}
k &= \phi\omega(k) + \phi\tau \left[\frac{1 - F(\hat{a})}{F(\hat{a})} \right], \\
\hat{a} &= \beta(1-\theta)h'(k) \left[\phi\omega(k) + \tau\left(\frac{1 - F(\hat{a})}{F(\hat{a})} \right) \left\{ \phi - \frac{\psi}{q - \omega(k)} \right\} \right].
\end{aligned}
$$

This dynamical system is a lot more complicated than the one in (24) and (25). Since it is harder to establish stability and comparative statics results analytically, we opt for numerical simulations. We report results for one parameterization, but qualitatively similar results were obtained for other sets of parameter values. We choose $h(k) = 6k^{1/3}, \phi = 0.9, \tau = 1, \psi = 2, \varphi = 0.75, \theta = 0.2, q = 20$, and $f \sim$ uniform on $[0.2, 3.2]$. Directly assuming a value for ϕ and ψ is convenient especially since it does not require us to specify a distribution function g; it also allows a more direct interpretation of the credit market distortion.

For these parameter values, the non-trivial steady state is $(k^*, \hat{a}^*) = (6.9, 3.0)$, with an associated unemployment rate of about 6.9%, $\omega(k^*)/\tau \simeq 7.6$, and roughly half of the set of potential capitalists are funded. To examine stability, we log-linearized equations (36) and (37) around this steady-state and obtained eigenvalues of 7.55 and 0.45 indicating that $(k^*, \hat{a}^*)$ is a saddle-point.

We are interested in how the steady-state adjusts (and how fast) to changes in the severity of the two frictions. Raising ϕ (improving credit market efficiency), raises k^*, reduces unemployment and increases the availability of finance to capitalists, similar to results obtained analytically in Section 5 above. However, unlike before, labor frictions now have long-run effects. Raising θ (worsening labor market conditions) increases unemployment, and interestingly, raises k^* as well as the fraction of capitalists who receive funding. What is at work here are the disparate effects of labor frictions on the demand and supply of loanable funds. When θ goes up, fewer workers are employed, receive production opportunities, and subsequently, apply to banks for loans. At the same time, a decline in employment reduces the flow of savings from the formal sector. The decline in the pool of applicants and formal sector savings are proportional, for a given capital-labor ratio. What

mitigates the situation, a channel absent before, is the additional flow of savings from a larger pool of unemployed households. This flow increases the overall supply of credit relative to demand. Hence a greater proportion of the potential capitalists now get funded, increasing the availability of capital per worker.[11]

Although this means output per worker y^* is higher when labor markets are more distorted, output per capita need not be. Specifically, per capita income derives from the formal sector as well from household production, $\tilde{y}^* = y^* F(\hat{a}^*) + \tau[1 - F(\hat{a}^*)]$. More severe labor market frictions tilt economic activity in favor of lower paying household (informal) activities. Under plausible assumptions regarding the low-productivity of such activities, income per capita could decline.

An increase in τ reduces unemployment and raises the steady-state capital per worker as well as the fraction of capitalists receiving funding. Higher τ makes home production more attractive and reduces the cutoff $\hat{a}$ for firms. Hence, fewer are unemployed, and the supply of loanable funds goes up. This reduces the severity of credit rationing, and increases capital production.[12]

Finally, consider the dynamic effects of the two frictions, in particular how ϕ and θ affect the non-explosive eigenvalue of the Jacobian. With an increase in ϕ, the stable eigenvalue falls in absolute value. Not only is the steady-state now higher, the speed of convergence is faster too. All else being equal, two economies with different credit market distortions would respond differently to exogenous one-time shocks – the more distorted one will take longer to bounce back. Higher values of θ, in contrast to what we obtained earlier, are now associated with smaller eigenvalues: an economy with more distorted labor markets actually adjusts to its steady-state faster. Recall that changes in θ lead to proportional changes in the demand for credit and the supply of formal sector savings. With more distorted labor markets, a greater fraction of the population is involved in household production. On

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[11]A similar effect would be present in the alternative framework proposed in footnote 9. When labor frictions worsen and steady-state unemployment rises, each risk-averse worker would save more, leading to a higher steady-state capital per worker. Of course the nature of this relationship would also depend on the availability of unemployment insurance and whether jobless young agents can borrow against an uncertain future income.

[12]It is interesting to note that the traditional development economics literature has portrayed the informal sector as a drag on development and actively discouraged its presence and growth. In contrast, our analysis finds redeeming aspects to actively encouraging it. Note however that we treat τ as strictly exogenous, i.e., unconnected with the state of the economy.

a per worker basis, their supply of savings to the credit market is higher. Consequently, there is more investment per worker, and the associated rate of capital accumulation is more rapid, leading to faster convergence.

Increases in τ, likewise, reduce the stable eigenvalue and speed up convergence. The mechanism here is similar to that of an increase in θ – higher savings from household production allows for greater investment which is instrumental in faster convergence to the steady-state.

8   Concluding Remarks

In this paper, we have studied credit and labor market frictions in a two-period overlapping generations model. We embedded a costly state verification problem in the credit market and in the labor market an agency problem where worker effort is imperfectly observed. Compared to the no-frictions overlapping generations model, these frictions give rise to equilibrium credit rationing and involuntary unemployment. We examined how the two frictions interact and affect stationary and non-stationary equilibria.

The economy possesses a unique asymptotically stable positive steady-state to which equilibrium sequences of the capital-labor ratio and employment monotonically converge. The speed of convergence and the steady-state itself are affected by market imperfections. Higher credit frictions lead to greater deadweight losses and more pervasive credit rationing, discouraging capital accumulation and worsening unemployment problems. Distortions in the labor market, on the other hand, increase unemployment but affect steady-state capital and output per worker only when the unemployed have alternative sources of income. Comparative dynamics exercises suggest that credit frictions prolong convergence to the stationary equilibrium, while the effect of labor frictions depend upon whether or not unemployment affects the flow of savings per worker.

The simplicity of our model with strong microfoundations of credit and labor market frictions and its attractive dynamic properties make it particularly suitable for quantitative studies. One avenue we would like to explore in future work is how well the two frictions explain cross-country unemployment problems, especially between Europe and the US. Another research question is the importance of information frictions in explaining business cycle movements (Greenwald and Stiglitz, 1994, Arnold, 2002). Obviously our overlapping generations framework would have to be abandoned in favor of

the infinite-horizon model. As Carlstrom and Fuerst (1997) have demonstrated, the credit friction margin is easily extended to such an environment. Similarly Gomme (1999) offers a way to incorporate the efficiency wage model into an infinite-horizon economy. It is conceivable that these settings can be profitably married to study the joint influence of the two frictions on variables at business cycle frequencies.

Appendix

A   Proof of Proposition 3

To investigate stability of the steady-states, we log-linearize the system described by equations (31) and (32) using the approximation $(z_t - z^*)/z^* = (z_t/z^*) - 1 \simeq \ln(z_t/z^*) \equiv \tilde{z}_t$.

Begin by linearizing equation (31):

$$
\begin{aligned}
k_{t+1} - k^* &= \frac{\alpha}{1-\alpha} \eta (k^*)^{\alpha/(1-\alpha)} (\hat{a}^*)^{-1/(1-\alpha)} \left[\frac{k_t - k^*}{k^*} \right] \\
&\quad - \frac{1}{1-\alpha} \eta (k^*)^{\alpha/(1-\alpha)} (\hat{a}^*)^{-1/(1-\alpha)} \left[\frac{\hat{a}_t - \hat{a}^*}{\hat{a}^*} \right]
\end{aligned}
$$

which, using the expression for k^*, becomes

$$
\begin{aligned}
\frac{k_{t+1} - k^*}{k^*} &= \frac{\alpha}{1-\alpha} \left[\frac{k_t - k^*}{k^*} \right] - \frac{1}{1-\alpha} \left[\frac{\hat{a}_t - \hat{a}^*}{\hat{a}^*} \right] \\
&\Rightarrow \tilde{k}_{t+1} = \frac{\alpha}{1-\alpha} \tilde{k}_t - \frac{1}{1-\alpha} \tilde{a}_t
\end{aligned} \tag{38}
$$

Similarly, linearizing (32) we get

$$
\begin{aligned}
f(\hat{a}^*)(\hat{a}_{t+1} - \hat{a}^*) &= -\frac{\alpha^2}{1-\alpha} [\sigma (k^*)^{-\alpha^2/(1-\alpha)} (\hat{a}^*)^{1/(1-\alpha)} F(\hat{a}^*)] \left[\frac{k_t - k^*}{k^*} \right] \\
&\quad + \frac{1}{1-\alpha} [\sigma (k^*)^{-\alpha^2/(1-\alpha)} (\hat{a}^*)^{1/(1-\alpha)} F(\hat{a}^*)] \left[\frac{\hat{a}_t - \hat{a}^*}{\hat{a}^*} \right] \\
&\quad + [\sigma (k^*)^{-\alpha^2/(1-\alpha)} (\hat{a}^*)^{1/(1-\alpha)} F(\hat{a}^*)] \frac{\hat{a}^* f(\hat{a}^*)}{F(\hat{a}^*)} \left[\frac{\hat{a}_t - \hat{a}^*}{\hat{a}^*} \right],
\end{aligned}
$$

or,

$$
f(\hat{a}^*)(\hat{a}_{t+1} - \hat{a}^*) = -\frac{\alpha^2}{1-\alpha} F(\hat{a}^*) \tilde{k}_t + \frac{1}{1-\alpha} F(\hat{a}^*) \tilde{a}_t + F(\hat{a}^*) \frac{\hat{a}^* f(\hat{a}^*)}{F(\hat{a}^*)} \tilde{a}_t
$$

or,

$$
\tilde{a}_{t+1} = \frac{F(\hat{a}^*)}{\hat{a}^* f(\hat{a}^*)} \left[-\frac{\alpha^2}{1-\alpha} \tilde{k}_t + \left\{ \frac{1}{1-\alpha} + \frac{\hat{a}^* f(\hat{a}^*)}{F(\hat{a}^*)} \right\} \tilde{a}_t \right] \tag{39}
$$

Equations (38) and (39) can be written in matrix notation as

$$\mathbf{x}_{t+1} = \mathbf{Q}\mathbf{x}_t,$$

where $\mathbf{x}_t = (k_t \quad a_t)'$ and the elements of the Jacobian $\mathbf{Q}$ are

$$q_{11} = \frac{\alpha}{1-\alpha}, \qquad q_{12} = -\frac{1}{1-\alpha},$$
$$q_{21} = -\left(\frac{\alpha^2}{1-\alpha}\right)\left(\frac{F}{\hat{a}^*f}\right), \quad q_{22} = 1 + \frac{1}{1-\alpha}\left(\frac{F}{\hat{a}^*f}\right).$$

Eigenvalues of $\mathbf{Q}$ satisfy the characteristic equation

$$p(\lambda) \equiv \lambda^2 - T\lambda + D = 0,$$

where

$$T = q_{11} + q_{22} = \frac{1}{1-\alpha}\left[1 + \left(\frac{F}{\hat{a}^*f}\right)\right] > 0,$$

$$D = q_{11}q_{22} - q_{12}q_{21} = \frac{\alpha}{1-\alpha}\left[1 + \left(\frac{F}{\hat{a}^*f}\right)\right] > 0.$$

Therefore,

$$p(0) = D > 0$$

indicating that the two roots (λ_1, λ_2) fall on the same side of 0. Moreover,

$$p(1) = 1 - T + D = -\left(\frac{F}{\hat{a}^*f}\right)\left[\frac{1}{1-\alpha} - \frac{\alpha}{1-\alpha}\right] = -\left(\frac{F}{\hat{a}^*f}\right) < 0$$

which implies that (λ_1, λ_2) fall on either sides of 1. Hence, we have $\lambda_1 > 1$ and $0 < \lambda_2 < 1$ so that the steady-state $(k^*, \hat{a}^*)$ is a saddle-point.

Now define $m(\hat{a}^*) \equiv 1 + F/(\hat{a}^*f)$ so that $T = m/(1-\alpha)$ and $D = \alpha m/(1-\alpha)$. The smaller (stable) eigenvalue of $\mathbf{Q}$ is given by

$$\lambda_1 = \frac{T - \sqrt{T^2 - D}}{2}.$$

Straightforward differentiation shows that $\partial\lambda_1/\partial m < 0$. We are interested in $\partial\lambda_1/\partial\gamma$ and $\partial\lambda_1/\partial\theta$. This depends upon whether $\partial m/\partial\gamma$ and $\partial m/\partial\theta \gtrless 0$. For a uniform distribution $a \sim U[\underline{a}, \overline{a}]$, $F/(\hat{a}^*f) = 1 - \underline{a}/\hat{a}^*$ is increasing in $\hat{a}^*$. Hence, anything that decreases $\hat{a}^*$ would decrease m, in turn increasing λ_1. From Section 5.2, we know that $\partial\hat{a}^*/\partial\gamma < 0$ and $\partial\hat{a}^*/\partial\theta < 0$. Therefore, higher credit and labor market frictions both decrease the speed of convergence via λ_1. ∎

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