





## LAB 10: SIMPLE HARMONIC MOTION

### OBJECTIVES

---

- To understand two ways to measure the spring constant of a spring.
- To understand simple harmonic motion.
- To be able to describe the forces involved in the motion of a pendulum.

### OVERVIEW

---

Simple harmonic motion (SHM) is a type of motion of a body that is periodic: it repeats itself. The term SHM describes various kinds of motions. In a mechanical systems SHM occurs when the force on a body is (1) proportional to the displacement of the body from its equilibrium position and (2) acts in a direction opposite to that of the displacement. SHM motion is a model that describes the oscillation of a mass hanging from a spring as long as the mass of the spring is negligible compared to the hanging mass. The motion of a simple pendulum—e.g., a mass swinging at the end of a string—is also approximately described as SHM under certain circumstances.

The oscillation of a mass hanging from a spring is SHM since it is subject to the linear elastic restoring force as given by Hooke's law. The motion is sinusoidal in time. In the case of the simple pendulum, the restoring force on the bob—the object that is swinging—is the gravitational force.

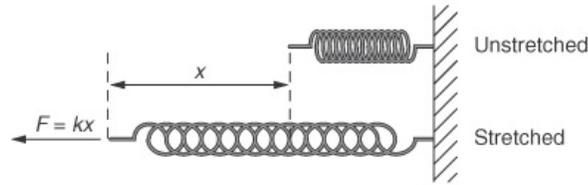
In this lab you will begin by exploring SHM with a spring, and the dependence of the period on the spring constant of the spring and the mass. Next, you will explore a pendulum system and the dependence of the period of pendulum on the length of the string and the mass of the bob. You will explore whether or not mechanical energy is still conserved in such systems.

### INVESTIGATION 1: SHM AND ELASTIC POTENTIAL ENERGY

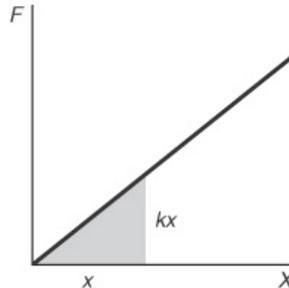
---

You have seen in Lab 6 that the magnitude of the force applied by most springs is proportional to the amount the spring is stretched or compressed beyond its unstretched length. This is usually written:  $F_{spring} = -kx$ , where  $k$  is called the spring constant.

The spring constant can be measured by applying measured forces to the spring and measuring its extension. In the diagram below, the applied force is shown. By the third law, the force applied is  $F = kx$  as is shown in the diagram.



You also saw in Lab 6 that the work done by a force can be calculated from the area under the force vs. position graph. Shown below is a force vs. position graph for a spring. Note that  $k$  is the *slope* of this graph, i.e., it is how much the force increases (in newtons) for a 1 meter increase in the amount the spring is stretched.



**Question 1-1:** How much work is done in stretching a spring of spring constant  $k$  from its unstretched length by a distance  $x$ ? (**Hint:** Look at the triangle on the force vs. position graph above and remember how you calculated the work done by a changing force in Lab 6.)

To measure the work done in stretching a spring in the following activity, you will need:

- IOLab and software
- spring, unstretched about 7 cm long
- smooth board or other level surface at least 0.5 m long that can be inclined
- meter stick or measuring tape

### Activity 1-1: Spring Constant

In this Activity, you will use the wheel and the force sensor to measure the spring constant. By moving the IOLab from the essentially un-stretched (just taut) length of the spring and zero force, you can make a graph of force vs. position (the amount the spring was stretched).

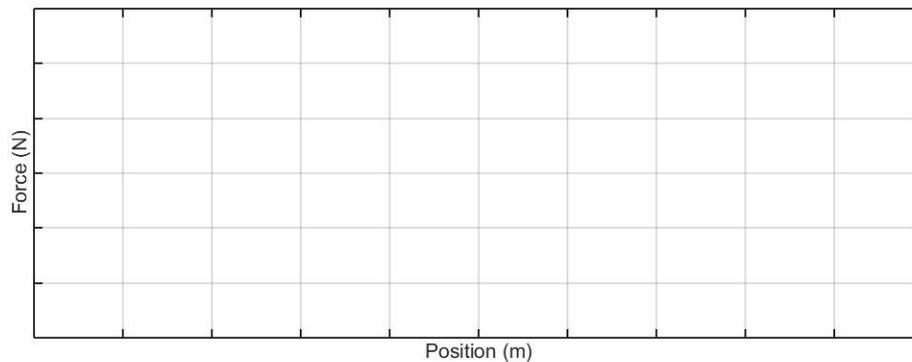
1. Use the IOLab on a level surface, calibrate the force sensor, then select wheel (only  $R_y$ ) and force, and re-zero the force sensor. There are two ways to find the spring constant.
2. Measure the force as a function of the position as the IOLab is moved such that the spring gradually stretches from 0 to 0.2 m, and fill in the table below. Do not exceed a force larger than 15 N.
3. Save or print your graphs.

4. Find the spring constant by plotting a graph of force vs. position on the axes below, or by using the parametric plot feature in the software that you used previously.

Force (N)	Position (m)

5. For the graph, calculate the spring constant from the graph below as the slope of a straight line fitting your data points. You can also use the Excel (see Appendix).

k= \_\_\_\_\_ N/m



6. For the parametric plot, do a plot of force vs. position using the software. (When the spring is unstretched, the force should read zero.) The slope should give you the spring constant.

**Question 1-2:** Was the force exerted by the spring proportional to the displacement of the spring?

**Question 1-3:** If two springs are stretched different amounts by the same mass hung from them, which spring has the larger spring constant, the one that stretches most or the one that stretches least? Explain.



**Prediction 1-1:** Suppose the IOLab is hanging from the spring and the force sensor is re-zeroed at the equilibrium position. As the IOLab oscillates up and down, what would the direction of the force be as measured by the force sensor (a) when the mass is above the equilibrium position and (b) when it is below the equilibrium position? (Note that since the mass oscillates up and down around its new equilibrium position, you don't need to include gravitational force.)

**Prediction 1-2:** As the IOLab oscillates up and down, how will the period change as the amplitude is changed?

To test your predictions in the following activities, you will need:

- IOLab and software
- spring and large binder clip
- plank or board setup almost vertically



### Activity 1-2: Determine the spring constant again

Attach the spring to the eyebolt and screw the eyebolt into the force sensor. Set up the plank such that it is practically vertical against a wall or table. (See diagram to the left.) Affix the spring securely to the top of the ramp. A large binder clip will work. The wheels should be in touch with ramp. You will first need to find the new equilibrium position of the spring with the IOLab hanging from the spring resting on its wheels against the almost vertical ramp.

1. Suspend the IOLab from the spring and be sure it is at rest.
2. Start the software and measure Force and the Wheel (position, velocity and acceleration).
3. Be sure to select **reverse y-axis for both the Force and Wheel.**
4. Re-zero the force sensor. Start the IOLab oscillating with an amplitude of about 5 cm. Take care that the wheels stay in touch with the ramp and that the motion of the IOLab

is up and down and not sideways. Save or print your graph, and sketch them on the axes below.

5. Determine the period of the oscillations by counting the time for a known number of oscillations. From the graph, find the time one complete oscillation, starting from one peak to the next is one oscillation. You can find the time it took. This is not that accurate, so better approach is to count say ten oscillations and find the time for them. The period is then the total time divided by ten. Show your calculations.

$$T_1 = \underline{\hspace{2cm}} \text{ sec}$$

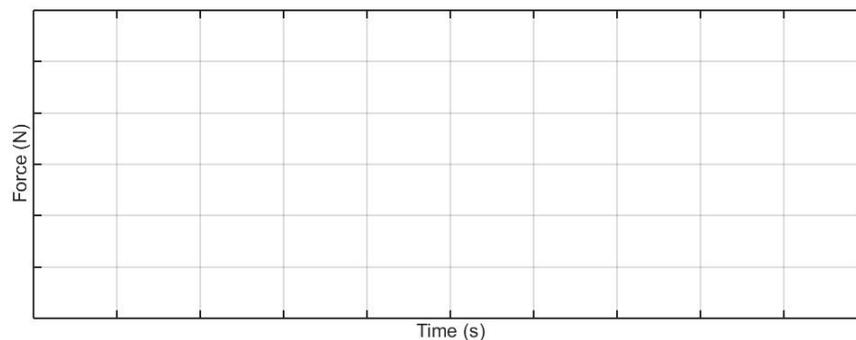
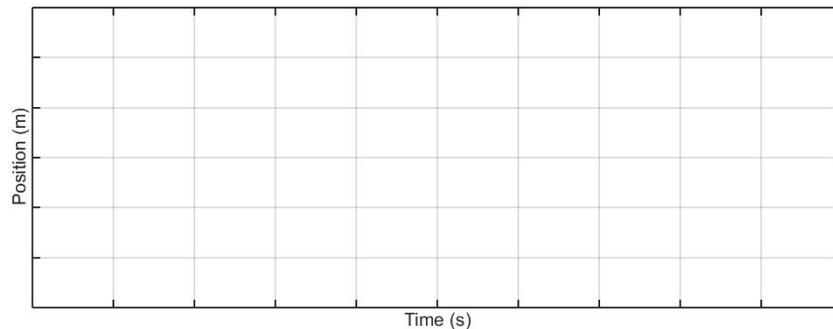
6. Repeat with a larger amplitude, and again determine the period.

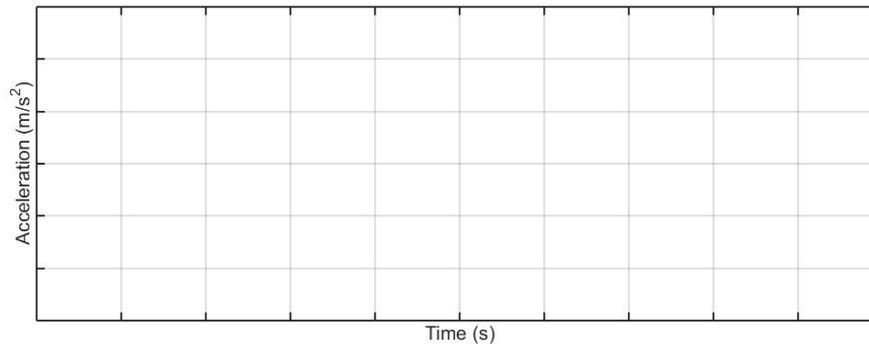
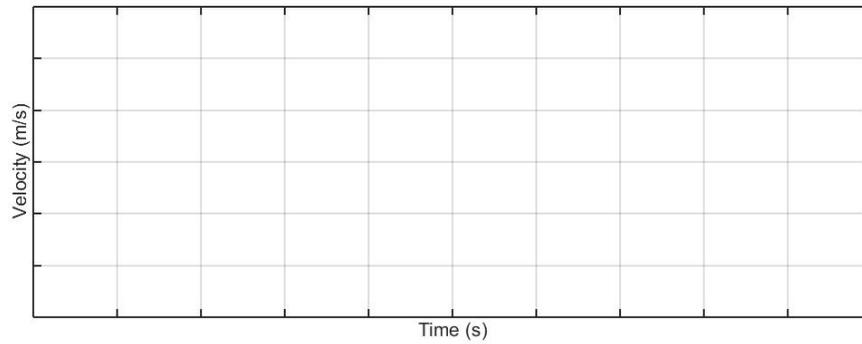
$$T_2 = \underline{\hspace{2cm}} \text{ sec}$$

**Question 1-4:** For simple harmonic motion of a mass on a spring, the period can be calculated from  $T = 2\pi\sqrt{\frac{m}{k}}$ . Using the spring constant you measured previously and the mass of the IOLab you also measured previously, calculate the period of oscillations. Show your calculation.

$$T = \underline{\hspace{2cm}} \text{ sec}$$

How does this calculated period compare to the experimentally determined one?





**Question 1-5:** Does the period appear to depend on the amplitude of the oscillations? Explain.

**Question 1-6:** How do the directions of the force and acceleration compare? Can you explain why?

**Question 1-7:** How do the magnitudes of the force and acceleration compare? Can you explain why?

## **INVESTIGATION 2: THE SIMPLE PENDULUM: ANOTHER EXAMPLE OF SHM**

---

In this Investigation you will explore the motion of a simple pendulum—a mass hanging from a string—which is SHM under certain conditions. You will explore what the period of a simple pendulum depends on.

To complete the next activity you will need:

- IOLab
- a long string about 1.5 m in length

### Activity 2-1: Determining the Period of a Simple Pendulum



1. Set up the IOLab and string as a pendulum as shown in the diagram on the left, with the string looped around and taped to either side of the IOLab and hung from a rod such that the  $y$ -direction is vertical. (The IOLab can swing in and out of the page.)

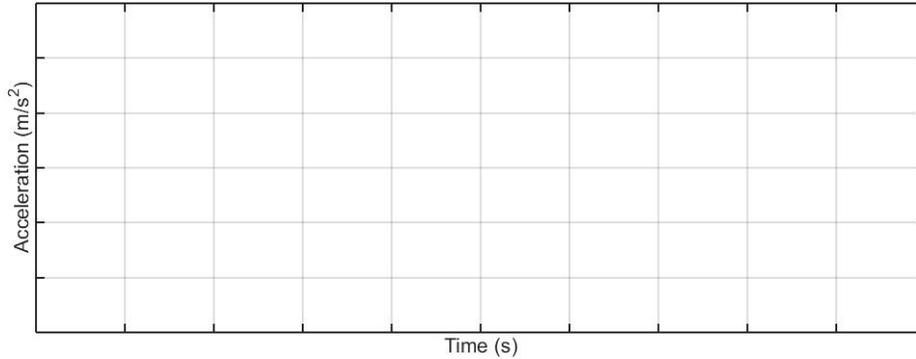
**Prediction 2-1:** What would be the acceleration, as measured by the accelerometer, for the IOLab hung in this way *at rest*? Remember that the acceleration is a vector quantity. Explain.

**Prediction 2-2:** What would the acceleration, as measured by the accelerometer, be for the IOLab swinging back and forth—(1) at one end of the swing, (2) at the other end of the swing and (3) at the bottom of the swing.

Test your predictions.

2. Start the software and measure the Acceleration only and then start the pendulum swinging such that it moves about 5 cm forwards and backwards.

3. Save or print the Acceleration graph, and sketch it on the axes below.



**Question 2-1:** What does the accelerometer measure in the  $y$ -direction when the pendulum is at rest? How does this compare to your prediction? Explain.

**Question 2-2:** What does the accelerometer measure when the pendulum reaches one end of its swing (maximum displacement from equilibrium)? What does it measure when it reaches the other end? Do these values make sense in terms of your predictions? Explain.

**Question 2-3:** What does the accelerometer measure when the pendulum reaches the bottom of its swing (zero displacement from equilibrium)? Does this value make sense in terms of your predictions? Explain.

4. Use your graph and the IOLab software to determine the period of the motion of this pendulum.

$$T_1 = \text{_____ sec}$$

**Question 2-3:** Also measure the period of the pendulum with a stopwatch. . Do the values agree?

**Prediction 2-3:** How would the period of a simple pendulum change as the length of the pendulum is increased?

**Prediction 2-4:** How would the period of a simple pendulum change as the amplitude of the oscillations is increased?

Test your predictions.

5. Adjust the length of string from which the IOLab is hanging, and use the IOLab software to determine the period of the pendulum for a number of different lengths. Fill in the table below.

Case	Length (m)	Period (S)
1		
2		
3		
4		
5		

6. Use the IOLab software to measure the period of the IOLab pendulum for a number of different amplitudes of oscillation. Be sure that the angular amplitude is under  $10^\circ$  in each case. Fill in the table below.

Case	Amplitudes (m)	Period (S)
1		
2		
3		
4		
5		

**Question 2-4:** How does the period of a simple pendulum appear to depend on the length of the pendulum? How does this compare to your prediction?

**Question 2-5:** Does the dependence of period on the amplitude support the formula  $T = 2\pi \sqrt{\frac{L}{g}}$ ? Justify your answer using your data.

6. Use the IOLab software to measure the period of the IOLab pendulum for a number of different amplitudes of oscillation. Fill in the table below.

Case	Amplitude (cm)	Period (S)
1	5 cm	
2		
3		
4		
5		

**Question 2-6:** Does the dependence of period on the length support the formula  $T =$

$2\pi \sqrt{\frac{L}{g}}$ ? Justify your answer using your data.

