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## Lab 10: <br> Two-Dimensional Motion (Projectile Motion)



The essential fact is that all the pictures which science now draws of nature . . .
are mathematical pictures.
—Sir James Jeans

## OBJECTIVES

- To explore the type of motion that results when an object falls close to the Earth's surface.
- To review how two-dimensional vector quantities such as velocities and accelerations can be represented by components that can be treated independently.
- To understand the experimental and theoretical basis for describing projectile motion as the superposition of two independent motions: (1) that of a body falling in the vertical direction under the influence of a constant force, and (2) that of a body moving in the horizontal direction with no applied forces.


## OVERVIEW

So far we have been dealing separately with motion along a horizontal line and motion along a vertical line. The focus of this lab is to describe the motion close to the surface of the Earth that occurs when an object is allowed to move in both the vertical and horizontal directions. Examples are the motion of a baseball or tennis ball after being hit. This type of motion is commonly called projectile motion. To understand this motion, it is helpful to review vertical and horizontal motions separately and then consider how they might be combined.

This lab begins with a review of the classic kinematic equations that describe the relationships between instantaneous position, velocity, and acceleration of objects that move in one dimension. In some cases an object moves with a constant velocity (zero acceleration). In others-such as the motion of a cart pushed along by the constant force of a fan unit as in Lab 2 or the falling motion of a ball pulled

## INVESTIGATION 2: EXAMINATION OF TWO-DIMENSIONAL MOTION

The world is full of phenomena that we can know of directly through our senses-objects moving, pushes and pulls, sights and sounds, winds and waterfalls. A vector is a mathematical concept-a mere figment of the mathematician's fancy. But vectors can be used to describe aspects of "real" phenomena such as positions, velocities, accelerations, and forces. Vectors are abstract entities that follow certain rules. For example, in the figure below, the velocity vector of an object is represented by the vector $\vec{v}$, drawn relative to different coordinate axes.


As discussed in Labs 1 and 2, a vector has two key attributes-magnitude and direction. The magnitude of a vector can be represented by the length of the arrow and its direction can be represented by the angle, $\theta$, between the arrow and the coordinate axes chosen to help describe the vector.

In earlier labs you drew vectors in only one dimension. But vectors are especially useful in representing two-dimensional motion because they can be resolved into components. In the middle figure above, the velocity vector is resolved into $x$ and $y$ components. Added together, these components are equivalent to the original vector, but they can be analyzed independently of each other. This is one reason it is convenient to use vectors.

If a cannonball is shot off a cliff with a
 certain initial velocity in the $x$ direction, the two-

- dimensional motion that results is known as projectile motion. But will it continue to move forward in that direction at the same velocity and at the same time fall in the $y$ direction as a result of the gravitational attraction between the Earth and the ball?

In this investigation you will examine the motion of a tennis ball that is tossed into the air so that it is moving in both the $x$ and $y$ direction. The toss of the ball and its trajectory are shown in the photos below.


Because the motion of the ball is in two dimensions, it is not easy to make measurements using a motion detector. Instead, you will use the method of video analysis to examine the motion and to determine the mathematical representations of the horizontal and vertical components of the motion. First some predictions.

Prediction 2-1: On the axes below, sketch your predictions for how the $x$ coordinate of the ball and the $x$ component of the velocity will vary with time. (Assume that positive $x$ direction in the photos is to the right.)


Prediction 2-2: On the axes below, predict how the $y$ coordinate of the ball and the $y$ component of the velocity will vary with time.


To test your predictions, you will need

- video analysis software
- RealTime Physics experiment configuration files


## Activity 2-1: Horizontal Motion of a Projectile

1. Open the experiment configuration file Projectile Motion (L10A2-1). This will open the video analysis software with a movie called Ball Toss.
2. Play the movie and observe the motion of the ball.

To access and play the movie, click HERE
Question 2-1: Describe the shape of the trajectory of the ball.
Note: Steps 3-5 have been done for you. To continue with this Activity and Activity 2-2, click HERE to access the measured graphs for $x$ and $y$, and click HERE for $v_{x}$ and $v_{y}$.
3. Search the Help menu in Video Analysis to find the "Video Analysis How To." This will give you directions on how to record the position of the ball, frame by frame.
4. Follow the directions to set the scale of the measurements, using the height given for the pile of books in frame 0 .
5. Record the positions of the ball for all frames.
6. Find the graph for $x$ vs. time for the ball, and either sketch it below or print the graph and attach it to these sheets.


Question 2-2: Does the graph for $x$ vs. time agree with your Prediction 2-1? Explain.

Question 2-3: Does the graph for $x$ vs. time represent motion at a constant velocity or constant acceleration? How do you know? Refer back to your observations in Investigation 1, if necessary.
7. Choose the kinematic equation that describes $x$ vs. time for this motion, and model it in the software to find the values of the parameters in that equation, e.g., $v_{0}$ and $x_{0}$. (That is, use the modeling feature in the software to find the equation that best represents the data, and find the parameters from this model. Note: Make intelligent guesses for $v_{0}$ and $x_{0}$, and then adjust the values to get a mathematical equation that fits the measured data the best.)

Question 2-4: What is the kinematic equation for $x$ vs. $t$ ? Give the measured values (from your mathematical model) of all parameters.
8. Display the graph of $v_{x}$ vs. time. Sketch the graph on the axes below, or print the graph and attach it to these sheets.


Question 2-5: Does the graph for $v_{x}$ vs. time agree with your Prediction 2-1? Explain.

Question 2-6: Does the graph for $v_{x}$ vs. time represent motion at a constant velocity or constant acceleration? How do you know? Refer back to your observations in Investigation 1, if necessary.
9. Choose the kinematic equation that describes $v_{x}$ vs. time for this motion, and model it to find the values of the parameters in that equation, e.g., $v_{0}$. (Again, use the modeling feature in the software to find the equation that best represents the data, and find the parameter(s) from this model. Once again, make an intelligent guess for $\nu_{0}$, and then adjust the value to get a mathematical equation that fits the measured data the best.)

Question 2-7: What is the kinematic equation? Give the values of all parameters.

## Activity 2-2: Vertical Motion of a Projectile

1. Find the graph of $y$ vs. time. Sketch the graph on the axes below, or print the graph and attach it to these sheets.


Question 2-8: Does the graph for $y$ vs. time agree with your Prediction 2-2? Explain.

Question 2-9: Does the graph for $y$ vs. time represent motion at a constant velocity or constant acceleration? How do you know? Refer back to your observations in Investigation 1, if necessary.
2. Choose the kinematic equation that describes $y$ vs. time for this motion, and model it to find the values of the parameters in that equation, e.g., $v_{0 y}, y_{0}$, and $a_{y}$. (Again, as in Activity 2-1, use the modeling feature in the software to find the equation that best represents the data, and find the parameters from this model.)

Question 2-10: What is the kinematic equation? Give the values of all parameters.
3. Display the graph of $v_{y}$ vs. time. Sketch the graph on the axes below, or print the graph and attach it to these sheets.


Question 2-11: Does the graph for $v_{y}$ vs. time agree with your Prediction 2-2? Explain.

Question 2-12: Does the graph for $v_{y}$ vs. $t$ represent motion at a constant velocity or constant acceleration? How do you know? Refer back to your observations in Investigation 1, if necessary.
4. Choose the kinematic equation that describes $v_{y}$ vs. time for this motion, and model it to find the values of the parameters in that equation, e.g., $v_{0 y}, y_{0}$, and $a_{y}$. (Again, as in Activity 2-1, use the modeling feature in the software to find the equation that best represents the data, and find the parameters from this model.)

Question 2-13: What is the kinematic equation? Give the values of all parameters.
5. Find the value of the $y$ component of the acceleration of the tennis ball.

Question 2-14: Is this value for the vertical component of acceleration what you expected? Explain.

Question 2-15: Use your observations in the two investigations of this lab to justify the statement that projectile motion is a combination of horizontal motion at a constant velocity (zero acceleration) and vertical motion with a constant (gravitational) acceleration.

