Structural Equation Modeling (SEM)

A Workshop Presented to the College of Education, University of Oregon, May 29, 2009

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Overview and resources

- Overview
- Listserv: http://www2.gsu.edu/~mkteer/semnet.html
- Web site and links:
  http://www.uoregon.edu/~stevensj/EDLD607/
- Software:
  AMOS EQS LISREL Mplus SAS R WinBugs

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Workshop Overview

- How statistical tools influence scientific behavior; SEM can facilitate better scientific practice
- Path Analysis
- Model Specification
- Model Estimation
- Testing and Evaluating Model Fit
- Kinds of SEM models
  - Regression models
  - Measurement models, Confirmatory Factor Analysis (CFA)
  - Hybrid models or Full LISREL models
- Invariance testing

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Rationale and Overview of SEM

- Flexible, comprehensive statistical analysis system
- More confirmatory than exploratory (continuum)
- Allows both manifest and latent variables
- Subsumes many other statistical techniques
- Analysis of variances and covariances rather than raw data
- Usually a large sample technique (N>200)
- Allows researcher to test entire models as well as individual parameters
- Known by several names including analysis of covariance structures, LISREL, SEM, "causal" modeling

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SEM and Scientific Practice

- Statistical Tools: Hammers and nails
- Some Important Features of Scientific Method and Practice:
  - Explicit representation of theory and hypotheses
  - Testing GOF of theory to data
  - Testing competing models
  - Cross-validation and replication
  - Making data publicly available and testable

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History and Background: Path Analysis

- Path analysis: precursor of SEM
  - Specifies relations among observed or manifest variables
  - Uses system of simultaneous equations to estimate unknown parameters based on observed correlations
  - Developed by biometrician Sewell Wright, 1918-1922

Path Analysis

- Wright's work on relative influence of heredity and environment in guinea pig coloration
- Developed analytic system and first path diagrams
- Path analysis characterized by three components:
  - a path diagram,
  - equations relating correlations to unknown parameters,
  - the decomposition of effects

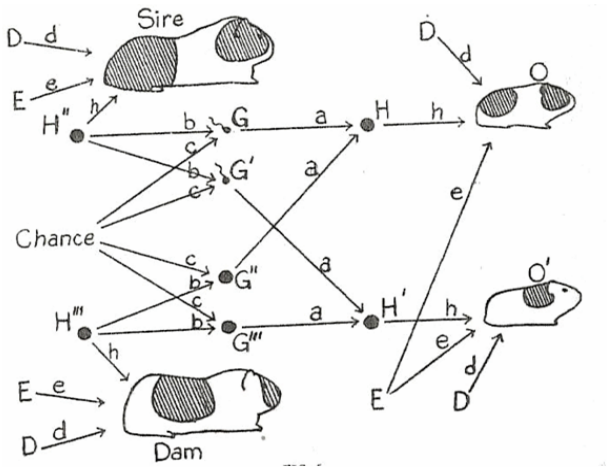


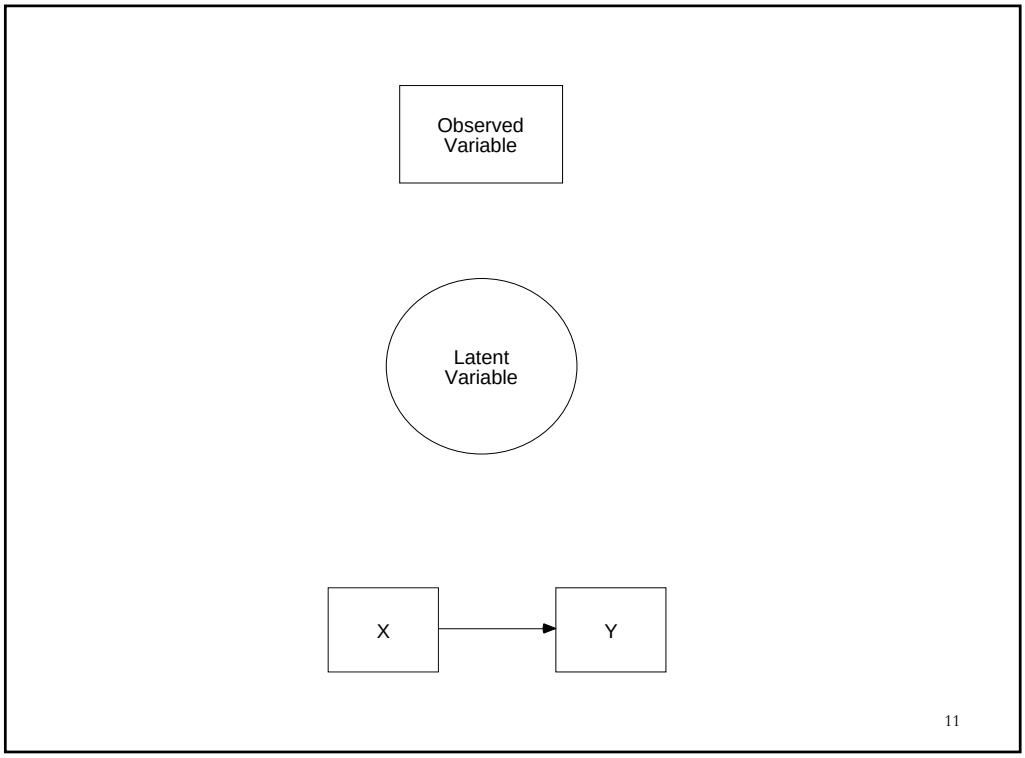
Figure I. Wright's First Path Diagram showing the influence of heredity and environment on the inheritance of color in the guinea pig (reproduced from Wright, 1920, p.328).

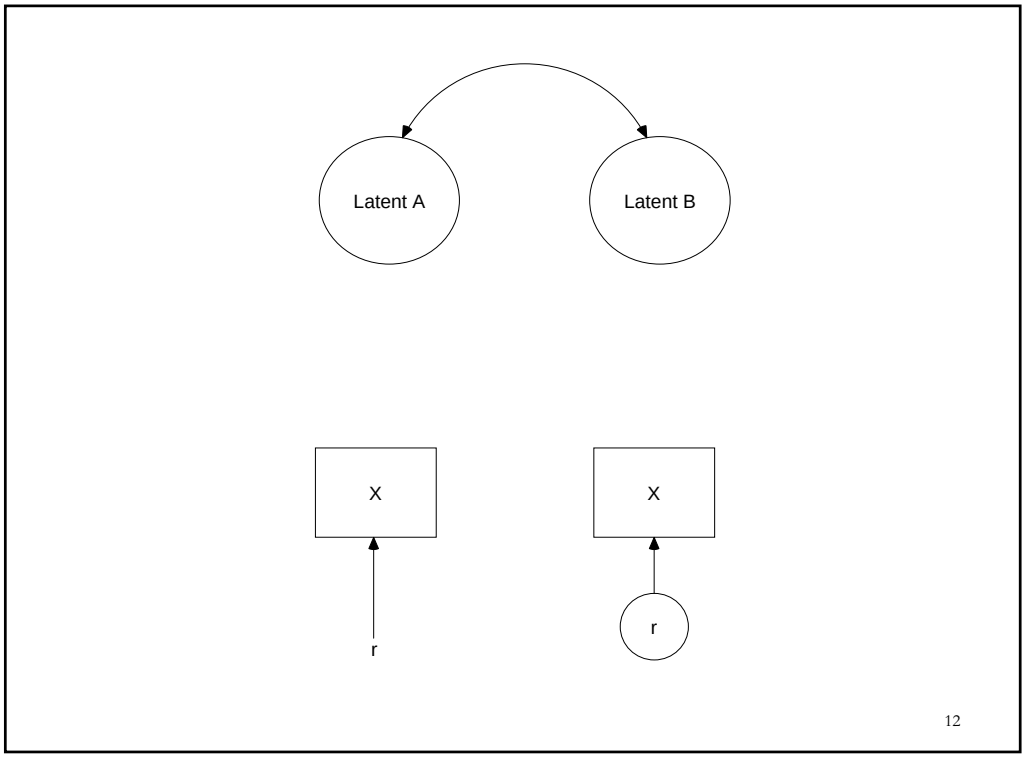
Path Analysis

- Little application or interest in path analysis following Wright until sociologists Hubert Blalock and O. D. Duncan in 1960's
- Major developments occurred in early 1970's through simultaneous work of Jöreskog, Keesing, and Wiley (JKW model)
- LISREL and expansion of path analysis

Path Diagramming

- A pictorial representation of a system of simultaneous equations (n.b. importance of explicit model representation)
- A box represents an observed or manifest variable
- A circle or ellipse represents an unobserved or latent variable
- A single headed straight arrow represents the influence ("cause") of one variable on another
- A double-headed curved arrow represents a covariance or correlation between two variables
- In some diagrams, an arrow by itself represents a residual or disturbance term


Observed
Variable
Latent
Variable
X
Y
11


Latent A
Latent B
X
X
r
r
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Definitions and Assumptions

- The "causes" of exogenous variables are outside of the posited model
- Any variable that has a straight arrow coming to it is an endogenous variable
- All endogenous variables have a residual term (unless strong assumption made)
- Total effects can be decomposed into direct and indirect effects

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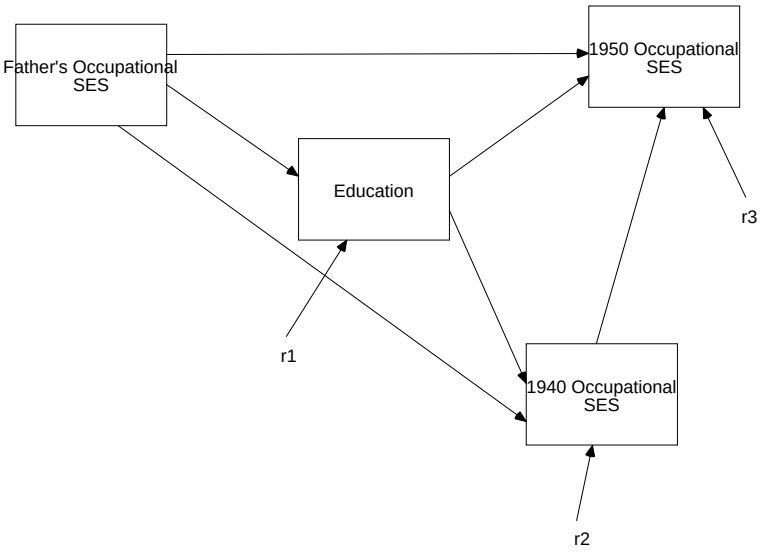
Duncan & Hodge (1963)



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AMOS Example

First law of path analysis: Decomposition of correlations

$$r_{yz} = \Sigma\, \beta_{yx}\, r_{xz}$$

where:

r_{yz} = observed correlation of Y and Z

β_{yx} = any path coefficient from X to Y

r_{xz} = any correlational path from X to Z

Note that Y must be endogenous, the correlations of exogenous variables are unanalyzable and cannot be decomposed. X's represent all "causes" of Y.

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Wright's Tracing Rules (system for writing equations)

Any observed correlation can be represented as the sum of the product of all paths obtained from each of the possible tracings between the two correlated variables.

- No loops (same variable not entered more than once)
- No going forward, then backward ("causal" flow)
- Maximum of one curved arrow per path

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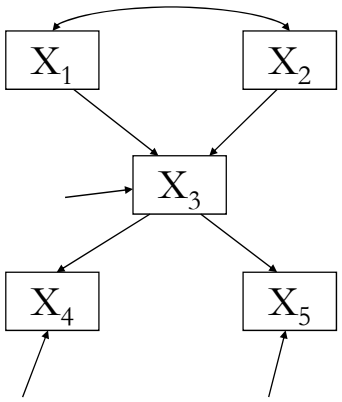
Examples: No loops



Path can't go through same variable twice. Path 435 is OK for r_{45}, but 431235 is not

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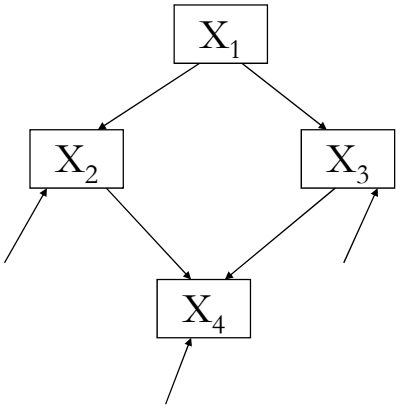
Examples: No forward then back

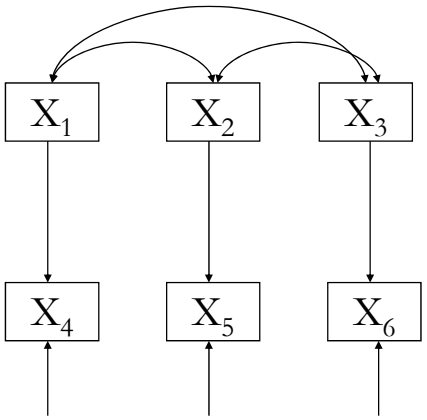


Once you've gone down-stream on a path you can't go upstream.

For r_{23}, 213 is OK, 243 is not.

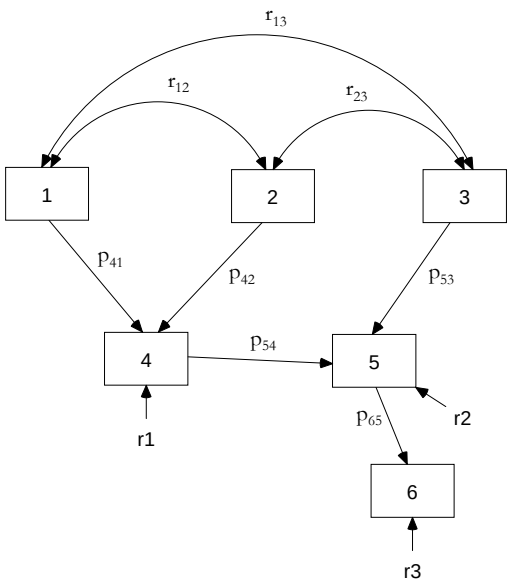
Events can be connected by common causes, but not by common consequences

Examples: Only one curved arrow per path



For r_{46}, 4136 is OK, 41236 is not

Example of tracing paths



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Specification of the model

- How many variables are exogenous, how many endogenous?
- Each arrow is represented by a coefficient
- The numerical value of a compound path is equal to the summed product of the values of the constituent tracings:

  For example, $r_{14} = p_{41} + (p_{42})(r_{12})$

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Wright's rules for calculating variance explained

- Same tracing approach but traces from a variable back to the same variable, total variance of a variable accounted for by the path model (R^2)
- For example for variable 4:

$$R^2 = (p_{41})^2 + (p_{42})^2 + 2[(p_{41})(r_{12})(p_{42})]$$

- Also means that the residual for each variable can be calculated as $1 - R^2$

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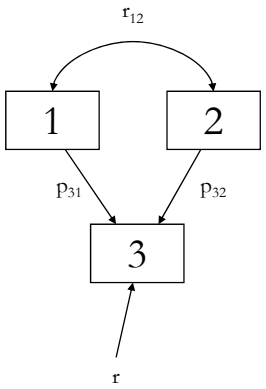
Numerical example



	1	2	3
1	1.00		
2	0.50	1.00	
3	0.40	.35	1.00

Three simultaneous equations with two unknowns

$r_{12} = r_{12} = .50$

$r_{13} = p_{31} + (r_{12})(p_{32})$

$r_{23} = p_{32} + (r_{12})(p_{31})$

$r_{13} = .40 = p_{31} + (.50)(p_{32})$

$r_{23} = .35 = p_{32} + (.50)(p_{31})$

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Doubling the second equation and subtracting from the first:

$$.70 = p_{31} + (2.0)(p_{32})$$

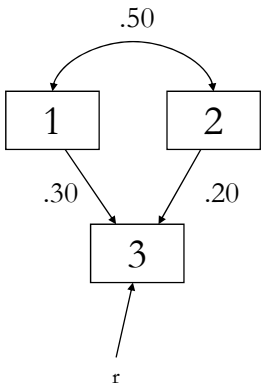
$$-\ .40 = p_{31} + (0.5)(p_{32})$$

$$.30 = \qquad (1.5)(p_{32})$$

so $p_{32} = .30 / 1.5 = .20$,

and $p_{31} = .30$

Numerical example



	1	2	3
1	1.00		
2	0.50	1.00	
3	0.40	.35	1.00

Path coefficients are equivalent to standardized partial regression coefficients so the same solution can be obtained using regression formulas*:

$$\beta_{31.2} = r_{13} - (r_{23})(r_{12}) / (1 - r_{12}^2)$$

$$= (.40) - (.35)(.50) / (1- .25)$$

$$= .30$$

$$\beta_{32.1} = r_{23} - (r_{13})(r_{12}) / (1 - r_{12}^2)$$

$$= (.35) - (.40)(.50) / (1 - .25)$$

$$= .20$$

*These formulas only appropriate with one endogenous variable in model

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$R^2 = \beta_{31.2}\, r_{13} + \beta_{32.1}\, r_{23}$

$= (.30)(.40) + (.20)(.35)$

$= .19$

Can also be computed using Wright's rules for calculating a variance explained:

$R^2 = (p_{31})^2 + (p_{32})^2 + 2[(p_{31})(r_{12})(p_{32})]$

$= (.30)^2 + (.20)^2 + 2(.30)(.50)(.20)$

$= .19$

So, 19% of the variance of variable 3 is accounted for by the path model, 81% is residual variance.

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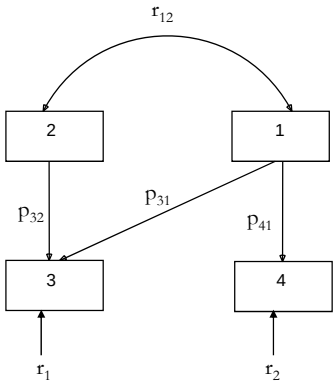
Second Numerical example



	2	3	4
2	1.00		
3	0.70	1.00	
4	0.30	.48	1.00

Given that $p_{31} = p_{41}$

Three simultaneous equations with two unknowns

$r_{12} = .50$

$r_{23} = p_{32} + (r_{12})(p_{31}) = .70$

$r_{24} = (r_{12})(p_{41}) = .30$

$r_{34} = (p_{31})(p_{41}) + (p_{32})(r_{12})(p_{41}) = .48$

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Given that $p_{31} = p_{41}$ and subtracting the third equation from the second:

$$.70 = p_{32} + (r_{12})(p_{31})$$

$$-\ .30 = \quad (r_{12})(p_{31})$$

$$.40 = p_{32}$$

so $r_{23} = .40 + (r_{12})(p_{31}) = .70$

$$(r_{12})(p_{31}) = .30$$

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$$(p_{31})^2 + (p_{32})(r_{12})(p_{41}) = .48$$

$$(p_{31})^2 + p_{32} + .30 = .48$$

$$(p_{31})^2 + (.40)(.30) = .48$$

$$(p_{31})^2 + .12 = .48$$

$$(p_{31})^2 = .36$$

$$p_{31} = .60$$

$$(r_{12})(p_{31}) = .30$$

$$(r_{12})(.60) = .30$$

$$r_{12} = .50$$

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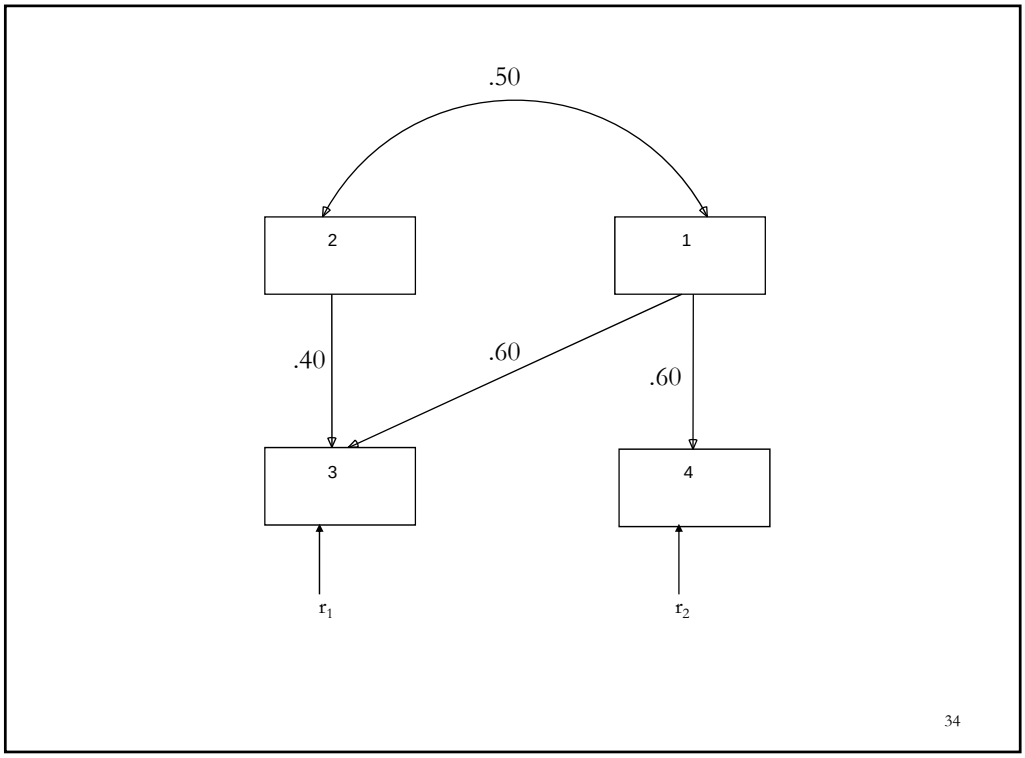
.50
2
1
.40
.60
.60
3
4
r1
r2
34

Can compute variance of variable 3 explained by the model using Wright's rules:

$R^2 = (p_{31})^2 + (p_{32})^2 + 2[(p_{31})(r_{12})(p_{32})]$

$= (.40)^2 + (.50)^2 + 2(.40)(.50)(.50)$

$= .61$

So, 61% of the variance of variable 3 is accounted for by the path model, 39% is residual variance.

Can compute variance of variable 1 explained directly as $r^2 = .60^2 = .36$ explained by the model

So, residual variance for variable 1 is 1 - .36 = .64

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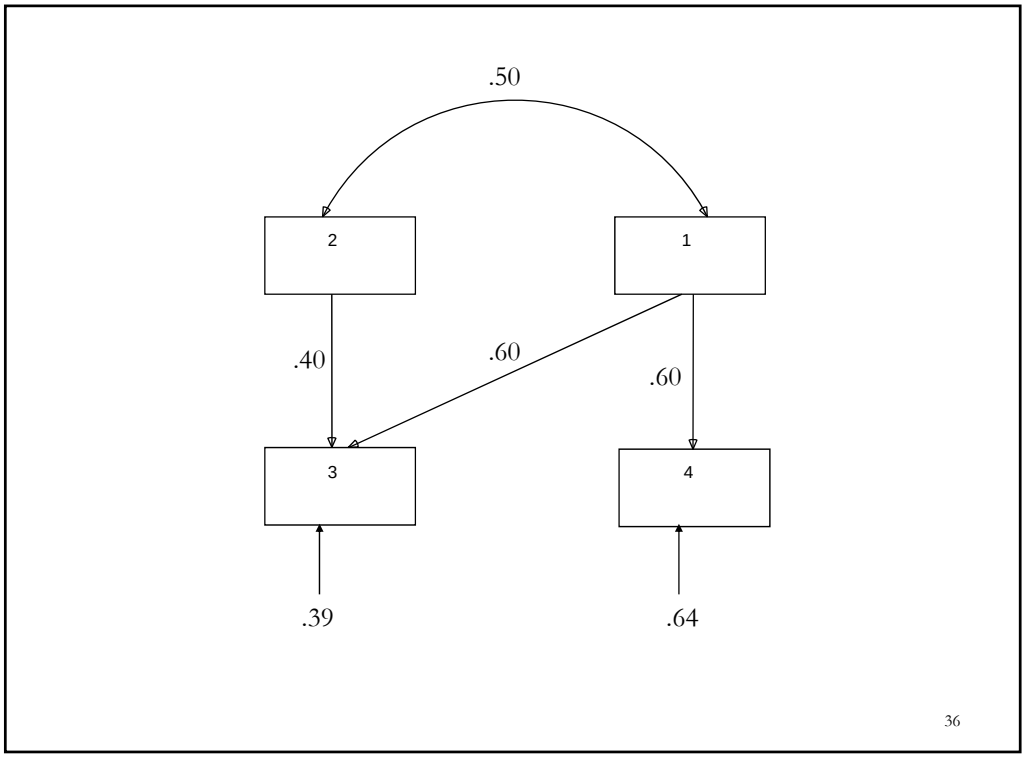
.50
2
1
.40
.60
.60
3
4
.39
.64
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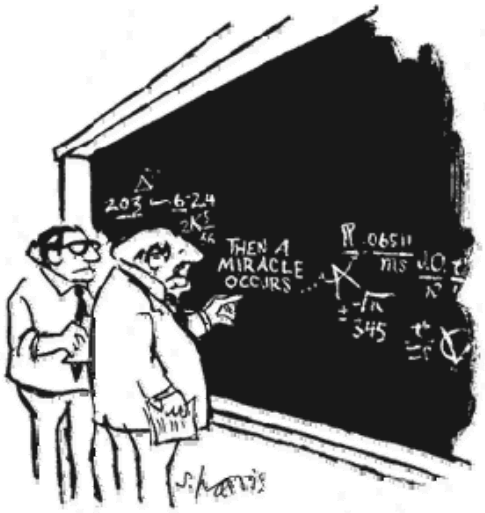
Model Specification and Identification

- Process of formally stating the variables of interest and the nature of relations among them
- In GLM statistics, specification is often implicit (e.g., correlation vs. regression; independent residuals) or emergent (as in EFA)
- In SEM, model specification more explicit, especially through path diagramming and resultant specification of equations
- In SEM, can specify Latent Variables (LV), manifest variables (MV), direct effects, indirect effects, and unanalyzed associations



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

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Model Specification

- In addition to specification of variables and their relations, parameters are specified as:
  - Fixed
  - Free
  - Constrained
- Goodness Of Fit (GOF) tests examine the way in which the fixed portions of the model fit the observed data (observed v-c matrix)

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Model Specification

- All models wrong to some degree vs. perfect models
- Debate over close fit versus exact fit
- Can only conclude a close-fitting model is plausible not a correct model
- There are always alternative models that will fit to a similar degree. Distinctions in this case depend on substantive or theoretical considerations

"All models are wrong,

some are useful"

– *George Box*

Model Specification

- In an SEM model, can have reflective or formative measurement
  - Reflective – constructs "cause" manifest variables
  - Formative – construct is formed by manifest variables (e.g., SES)
- For reflective, desirable to have 3 or more manifest variables for each latent variable (but more may not always be better, c.f. Marsh, et al)
- When there are not multiple indicators, a latent variable is omitted and represented by an error perturbed manifest variable

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Model Specification

- Specification requires the researcher to describe the pattern of directional and nondirectional relationships among the variables
  - Directional effects are regression coefficients
  - Nondirectional effects are covariances
  - Along with variances these three types of coefficients represent the model parameters
  - For formative measurement, construct automatically becomes an endogenous latent variable with a residual

Model Specification: Parameters in SEM models

- Every exogenous variable (MV, LV, residual) has a variance defined as a model parameter
- Variances of endogenous variables are not parameters but are implied by influences on the variable; that is, their variance is an algebraic function of the "causes" of the variable hence not parameters to be estimated
- All covariances are parameters
- Nondirectional associations among endogenous variables are not allowed
- All directional effects are parameters (LV on LV, LV on MV, residual on MV, etc.)

AMOS Example

Model Specification: Parameters in SEM models

- Fixed parameters, often based on requirements to make model identifiable and testable
- Two choices to establish a scale for each latent variable including residuals:
  - Can fix variance of latent variable
  - Can fix one regression coefficient for manifest indicator of latent (sets scale of latent to scale of manifest)
- Free parameters—in essence an unspecified aspect of model, more exploratory than confirmatory and not represented in the GOF test

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Model Specification: Parameters in SEM models

- Free parameters are usually tested individually for statistical significance. Most commonly, test is whether parameter significantly differs from zero
- Constrained parameters – parameters may also be constrained to a range of values in some software or constrained to be equal to another parameter
- The number of estimated parameters is equal to the total number of parameters minus the number of fixed parameters

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Model Identification

- For each free parameter, it is necessary that at least one algebraic solution is possible expressing that parameter as a function of the observed variances and covariances
- If at least one solution is available, the parameter is identified
- If not, the parameter is unidentified
- To correct this, the model must be changed or the parameter changed to a fixed value

Model Identification

- In addition to the identification of individual parameters (and the definition of latent variables through the fixing of a variance or a regression coefficient), model as a whole must be identified
- Model identification requires that the number of estimated parameters must be equal to or less than the number of observed variances and covariances for the model as a whole
- Number of observed variances-covariances minus number of parameters estimated equals model degrees of freedom

Model Identification

- Number of observed variances-covariances =
  [k(k + 1)] / 2,
  where k = the number of manifest variables in the model
- If df are negative (more estimated parameters than observations), the model is underidentified and no solution is possible
- If df = 0, the model is just identified, a unique solution to equations is possible, parameters can be estimated, but no testing of goodness of fit is possible

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Model Identification

- If df > 0, the model is overidentified (more equations than unknown parameters) and there is no exact solution, more than one set of parameter estimates is possible
- This is actually beneficial in that it is now possible to explore which parameter estimates provide the best fit to the data

Simple example

$$x + y = 5 \quad (1)$$
$$2x + y = 8 \quad (2)$$
$$x + 2y = 9 \quad (3)$$

- With only equation 1, there are an infinite number of solutions, x can be any value and y must be (5 – x). Therefore there is a linear dependency of y on x and there is underidentification
- With two equations (1 and 2), a unique solution for x and y can be obtained and the system is just identified.
- With all three equations there is overidentification and there is no exact solution, multiple values of x and y can be found that satisfy the equations; which values are "best"?

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Model Identification

- Disconfirmability – the more overidentified (the greater the degrees of freedom) the model, the more opportunity for a model to be inconsistent with the data
- The fewer the degrees of freedom, the more "overparameterized" the model, the less parsimonious

Model Identification

- Also important to recognize possibility of empirically equivalent models (see Lee & Hershberger, 1990)
- Two models are equivalent if they fit any set of data equally well
- Often possible to replace one path with another with no impact on empirical model fit (e.g., A → B versus A ← B versus A ↔ B)

Model Identification

- Researchers should construct and consider alternative equivalent models and consider the substantive meaning of each
- Existence of multiple equivalent models is analogous to the presence of confounding variables in research design
- Distinctions among equivalent models must be made on the basis of theoretical and conceptual grounds

Model Estimation

- Unlike the least squares methods common to ANOVA and regression, SEM methods usually use iterative estimation methods
- Most common method is Maximum Likelihood Estimation (ML)
- Iterative methods involve repeated attempts to obtain estimates of parameters that result in the "best fit" of the model to the data

Model Estimation

- Fit of an SEM model to the data is evaluated by estimating all free parameters and then recomputing a variance-covariance matrix of the observed variables that would occur given the specified model
- This model implied v-c matrix, $\Sigma(\hat{\theta})$, can be compared to the observed v-c matrix, S, to evaluate fit
- The difference between each element of the implied and observed v-c matrix is a model residual
- This approach leads to Goodness of Fit (GOF) testing in SEM (more on this later)

Model Estimation

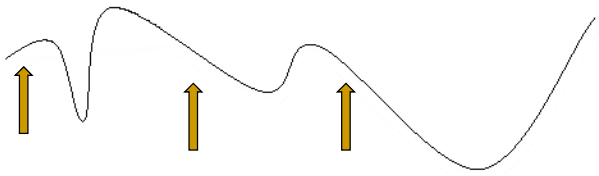
- Iterative estimation methods usually begin with a set of start values
- Start values are tentative values for the free parameters in a model
- Although start values can be supplied by the user, in modern software a procedure like two-stage least squares (2SLS) is usually used to compute start values
  - 2SLS is non-iterative and computationally efficient
  - Stage 1 creates a set of all possible predictors
  - Stage 2 applies ordinary multiple regression to predict each endogenous variable
  - Resulting coefficients are used as initial values for estimating the SEM model

Model Estimation

- Start values are used to solve model equations on first iteration
- This solution is used to compute the initial model implied variance-covariance matrix
- The implied v-c matrix is compared to the observed v-c matrix; the criterion for the estimation step of the process is minimizing the model residuals
- A revised set of estimates is then created to produce a new model implied v-c matrix which is compared to the previous model implied v-c matrix (sigma-theta step 2 is compared to sigma theta step 1) to see if residuals have been reduced
- This iterative process is continued until no set of new estimates can be found which improves on the previous set of estimates

Model Estimation

- The definition of lack of improvement is called the convergence criterion
- Fit landscapes
- Problem of local minima



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Model Estimation

- Lack of convergence can indicate either a problem in the data, a misspecified model or both
- Heywood cases
- Note that convergence and model fit are very different issues

Model Estimation: Kinds of errors in model fitting

- Definitions:
  - Σ = the population variance-covariance matrix
  - S = the sample variance-covariance matrix
  - $\Sigma(\theta)$ = the population, model implied v-c matrix
  - $\Sigma(\hat{\theta})$ = the sample estimated, model implied v-c matrix
- Overall error: $\Sigma - \Sigma(\hat{\theta})$
- Errors of Approximation: $\Sigma - \Sigma(\theta)$
- Errors of Estimation: $\Sigma(\theta) - \Sigma(\hat{\theta})$

Estimation Methods

- Ordinary Least Squares (OLS) – assesses the sums of squares of the residuals, the extent of differences between the sample observed v-c matrix, S, and the model implied v-c matrix, $\Sigma(\hat{\theta})$

$$\text{OLS} = \quad trace[S - \Sigma(\hat{\theta})]^2$$

Note functional similarity of the matrix formulation for v-c matrices to the more familiar OLS expression for individual scores on a single variable:

$$\Sigma(X - \overline{X})^2$$

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Estimation Methods

- Generalized Least Squares (GLS) – like the OLS method except residuals are multiplied by S^{-1}, in essence scales the expression in terms of the observed moments

$$(1/2)trace[(S-(\Sigma(\hat{\theta}))S^{-1}]^2$$

Estimation Methods

- Maximum Likelihood – Based on the idea that if we know the true population v-c matrix, Σ, we can estimate the probability (log-likelihood) of obtaining any sample v-c matrix, S.

$$(\ln\left|\Sigma(\hat{\theta})\right| - \ln|S|) + [trace(S \cdot \Sigma(\hat{\theta})^{-1}) - k]$$

where k = the order of the v-c matrices or number of measured variables

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Estimation Methods: Maximum Likelihood

- In SEM, both S and $\Sigma(\hat{\theta})$ are sample estimates of Σ, but the former is unrestricted and the latter is constrained by the specified SEM model
- ML searches for the set of parameter estimates that maximizes the probability that S was drawn from $\Sigma(\theta)$, assuming that $\Sigma(\hat{\theta})$ is the best estimate of $\Sigma(\theta)$

Estimation Methods: Maximum Likelihood

$$(\ln\left|\Sigma(\hat{\theta})\right| - \ln\left|S\right|) + [trace(S \cdot \Sigma(\hat{\theta})^{-1}) - k]$$

- Note in equation that if S and $\Sigma(\hat{\theta})$ are identical, first term will reduce to zero
- If S and $\Sigma(\hat{\theta})$ are identical, $S \cdot \Sigma(\hat{\theta})^{-1}$ will be an identity matrix, the sum of its diagonal elements will therefore be equal to k and the second term will also be equal to zero

Estimation Methods: Maximum Likelihood

- ML is scale invariant and scale free (value of fit function the same for correlation or v-c matrices or any other change of scale; parameter estimates not usually affected by transformations of variables)
- The ML fit function is distributed as χ^2
- ML depends on sufficient *N*, multivariate normality, and a correctly specified model

Other Estimation Methods

- Asymptotically Distribution Free (ADF) Estimation
  - Adjusts for kurtosis; makes minimal distributional assumptions
  - Requires raw data and larger *N*
  - Computationally expensive
  - Outperforms GLS and ML when model is correct, *N* is large, and data are not multivariate normal (not clear how much ADF helps when non-normality small or moderate)

Other Estimation Methods

- Unweighted Least Squares – Very similar to OLS, uses ½ the trace of the model residuals
  - ULS solutions may not be available for some parameters in complex models
  - ULS is not asymptotically efficient and is not scale invariant or scale free
- Full Information Maximum Likelihood (FIML) – equivalent to ML for observed variable models
  - FIML is an asymptotically efficient estimator for simultaneous models with normally distributed errors
  - Only known efficient estimator for models that are nonlinear in their parameters
  - Allows greater flexibility in specifying models than ML (multilevel for example)

Estimation Methods

- Computational costs (least to most):
  OLS/ULS, GLS, ML, ADF
- When model is overidentified, value of fit function at convergence is approximated by a chi-square distribution: $(N\text{-}1)(F) = \chi^2$
- Necessary sample size 100-200 minimum, more for more complex models
- If multivariate normal, use ML
- If robustness an issue, use GLS, ADF, or bootstrapping
- ML most likely to give misleading values when model fit is poor (incorrect model or data problems)

Intermission

"Frankly, Harold, you're beginning to bore everyone with your statistics."

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Model Testing and Evaluation

- After estimating an SEM model, overidentified models can be evaluated in terms of the degree of model fit to the data
- Goodness of Fit (GOF) is an important feature of SEM modeling because it provides a mechanism for evaluating adequacy of models and for comparing the relative efficacy of competing models

Evaluating Model Fit

- After estimation has resulted in convergence, it is possible to represent the degree of correspondence between the observed and model implied v-c matrices by a single number index
- This index is usually referred to as *F*, the fitting function (really lack of fit function)
- The closer to zero, the better the fit
- The exact definition of the fit function varies depending on the estimation method used (GLS, ML, etc.)

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Goodness of Fit

- If $\Sigma = \Sigma(\hat{\theta})$, then the estimated fit function (F) should approximate zero, and the quantity $(N - 1)(F)$ approxi-mates the central chi-square distribution with

  df = (# sample moments – # estimated parameters)
- GOF can be evaluated by determining whether the fit function differs statistically from zero
- When $\Sigma \neq \Sigma(\hat{\theta})$, then the noncentral chi-square distribution applies
- Noncentral chi-square depends on a noncentrality parameter (λ) and df (if $\lambda = 0$ exactly, central χ^2 applies)
- Lambda is a measure of population "badness of fit" or errors of approximation

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Goodness of Fit

- The χ^2 test is interpreted as showing no significant departure of the model from the data when $p \geq .05$
- When $p < .05$, the interpretation is that there is a statistically significant departure of one or more elements from the observed data
- As in other NHST applications, the use of the χ^2 test in SEM provides only partial information and is subject to misinterpretations:
  - Given (N-1) in the formula, for any nonzero value of F, there is some sample size that will result in a significant χ^2 test
  - The χ^2 test does not provide clear information about effect size or variance explained
  - It may be unrealistic to expect perfectly fitting models, especially when N is large
  - Type I and Type II errors have very different interpretations in SEM

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Goodness of Fit

- Recognition of problems in using the χ^2 test, led to the use of the χ^2/df ratio (values of about 2-3 or less were considered good).
- Further research on properties of fit indices led to development of many fit indices of different types
- Many of the fit indices make comparisons between a model of interest or target model and other comparison models
- Some incremental or comparative indices provide information about variance explained by a model in comparison to another model

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Goodness of Fit

- Two widely used comparison models are the saturated model and the null or independence model
- The saturated model estimates all variances and covariances of the variables as model parameters; there are always as many parameters as data points so this unrestricted model has 0 df
- The independence model specifies no relations from one measured variable to another, independent variances of the measured variables are the only model parameters
- The restricted, target model of the researcher lies somewhere in between these two extremes

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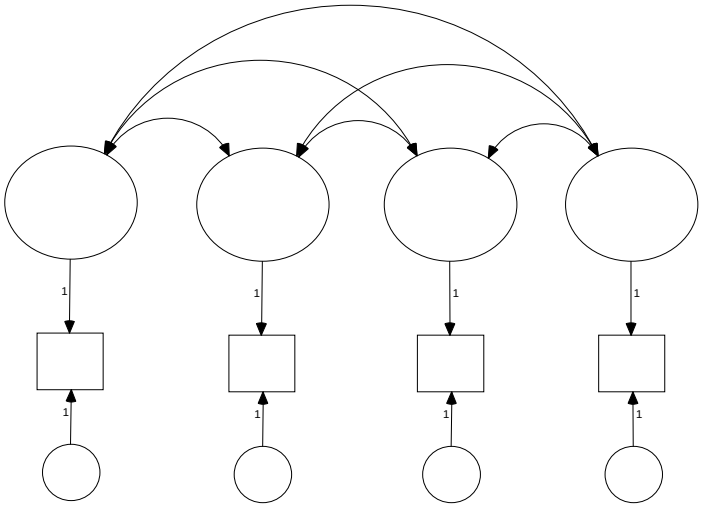
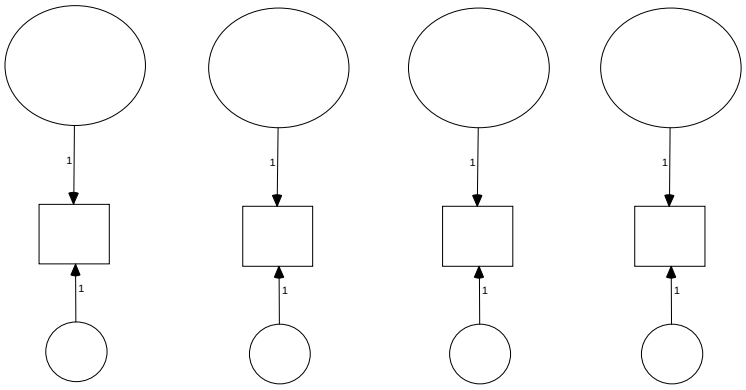
Saturated Model



79

Independence Model



80

Absolute Fit Indices

- GFI = $1 - \frac{F_T}{F_S}$ Where F_T is the fit of the target model and F_S is the fit of the saturated model

- AGFI = the GFI adjusted for df; AGFI penalizes for parameterization:

$$1 - \frac{k(k+1)}{(2 \cdot df)} \cdot (1 - GFI)$$

- GFI and AGFI intended to range from 0 to 1 but can take on negative values

Absolute Fit Indices

- Akaike's Information Criterion (AIC) – Intended to adjust for the number of parameters estimated

  AIC = $\chi^2 + 2t$, where t = # parameters estimated

- Corrected AIC – Intended to take N into account

  CAIC = $\chi^2 + (1+\log N)t$

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Absolute Fit Indices

- ECVI – rationale differs from AIC and CAIC; measure of discrepancy between fit of target model and fit expected in another sample of same size

$$\text{ECVI} = [\chi^2/(n-1)] + 2(t/(N-1))$$

Absolute Fit Indices

- For AIC, CAIC, and ECVI, smaller values denote better fit, but magnitude of the indices is not directly interpretable
- For these indices, compute estimates for alternative SEM models, rank models in terms of AIC, CAIC, or ECVI and choose model with smallest value
- These indices are useful for comparing non-nested models

Absolute Fit Indices

- RMR or RMSR – the Root Mean Square Residual is a fundamental measure of model misfit and is directly analogous to quantities used in the general linear model (except here the residual is between each element of the two v-c matrices):

$$\text{RMR} = \sqrt{2\Sigma\Sigma \frac{[S - \Sigma(\hat{\theta})]^2}{k(k+1)}}$$

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Absolute Fit Indices

- SRMR – Standardized RMR
  - The RMR is expressed in the units of the raw residuals of the variance-covariance matrices
  - The SRMR is expressed in standardized units (i.e., correlation coefficients)
  - The SRMR therefore expresses the average difference between the observed and model implied correlations
  - SRMR is available in AMOS only through tools-macros

Absolute Fit Indices

- RMSEA – Root Mean Square Error of Approximation
  - Designed to account for decrease in fit function due only to addition of parameters
  - Measures discrepancy or lack of fit "per df"

$$\text{RMSEA} = \sqrt{\hat{F} / df}$$

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Incremental Fit Indices

- Normed Fit Index (NFI; Bentler & Bonett, 1980) – represents total variance-covariance among observed variables explained by target model in comparison to the independence model as a baseline

$$\text{NFI} = \quad (\chi^2_B - \chi^2_T) / \chi^2_B$$

Where B = baseline, independence model and T = target model of interest

- Normed in that $\chi^2_B \geq \chi^2_T$, hence 0 to 1 range

Incremental Fit Indices

- Tucker-Lewis Index (TLI), also known as the Non-Normed Fit Index (NNFI) since it can range beyond 0-1
- Assumes multivariate normality and ML estimation

$$\text{TLI} = \frac{[(\chi^2_B / df_B) - (\chi^2_T / df_T)]}{[(\chi^2_B / df_B) - 1]}$$

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Incremental Fit Indices

- A third type of incremental fit indices depends on use of the noncentral χ^2 distribution
  - If conceive of noncentrality parameters associated with a sequence of nested models, then: $\lambda_B \geq \lambda_T \geq \lambda_S$
  - So the noncentrality parameter and model misfit are greatest for the baseline model, less for a target model of interest, and least for the saturated model
  - Then: $\delta = (\lambda_B - \lambda_T)/\lambda_B$
  - δ assesses the reduction in misfit due to model T
  - A statistically consistent estimator of delta is given by the BFI or the RNI (can range outside 0-1)
  - CFI constrains BFI/RNI to 0-1

Incremental Fit Indices

$$\text{BFI/RNI} = \frac{[(\chi^2_B - df_B) - (\chi^2_T - df_T)]}{(\chi^2_B - df_B)}$$

$$\text{CFI} = \frac{1 - \max[(\chi^2_T - df_T), 0]}{\max[(\chi^2_T - df_T), (\chi^2_B - df_B), 0]}$$

Hoelter's Critical N

- Hoelter's "critical N" (Hoelter, 1983) reports the largest sample size for which one would fail to reject the null hypothesis there there is no model discrepancy
- Hoelter does not specify a significance level to be used in determining the critical N, software often provides values for significance levels of .05 and .01

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Which Indices to Use?

- Use of multiple indices enhances evaluation of fit
- Recognize that different indices focus on different aspects or conceptualizations of fit
  - Don't use multiple indices of the same type (e.g., RNI, CFI)
  - Do make sure to use indices that represent alternative facets or conceptualizations of fit (e.g., SRMR, CFI)
  - Should always report χ^2, df, and at least two other indices

Cutoff Criteria

- Hu & Bentler
  - SRMR .06
  - RMSEA .05 - .08
  - RNI, TLI, CFI .95
- Exact vs. close fit debate
- Marsh, et al.

Comparing Alternative Models

- Some indices include inherent model comparisons
  - Comparisons are sometimes trivial
  - Stronger to compare target model to a plausible competing model that has theoretical or practical interest
- Nested models can be tested using subtracted difference between χ^2 for each model (df is difference in df between the two models)
- Can rank models using indices like AIC, BIC when not nested
- Can compare variance explained by comparing TLI, CFI or other incremental fit results (.02 or more, Cheung & Rensvold)

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Evaluating Variance Explained

- Inspect R^2 for each endogenous variable; consider the size of the uniqueness as well, especially if specific and error variance can be partitioned
- Incremental fit indices provide an indication of how well variance is explained by the model
- Can be strengthened by using plausible comparison models

Kinds of SEM Models

- Regression models
- Measurement Models
- Structural Models
- Hybrid or full models

SEM Models With Observed Variables

- Directly observed explanatory or predictor variables related to some number of directly observed dependent or outcome variables
- No latent variables
- These SEM models subsume GLM techniques like regression

Model Specification

The general SEM Model with observed variables only is:

$$\mathbf{y} = \beta\mathbf{y} + \Gamma\mathbf{x} + \zeta$$

where:

- $\mathbf{y}$ = a p X 1 column vector of endogenous observed variables (Y's);
- $\mathbf{x}$ = a q X 1 column vector of exogenous observed variables (X's);
- β = an p X p coefficient matrix defining the relations among endogenous variables;
- Γ = an p X q coefficient matrix defining the relations from exogenous to endogenous variables;
- ζ = an p X 1 vector of residuals in the equations.

Model Specification

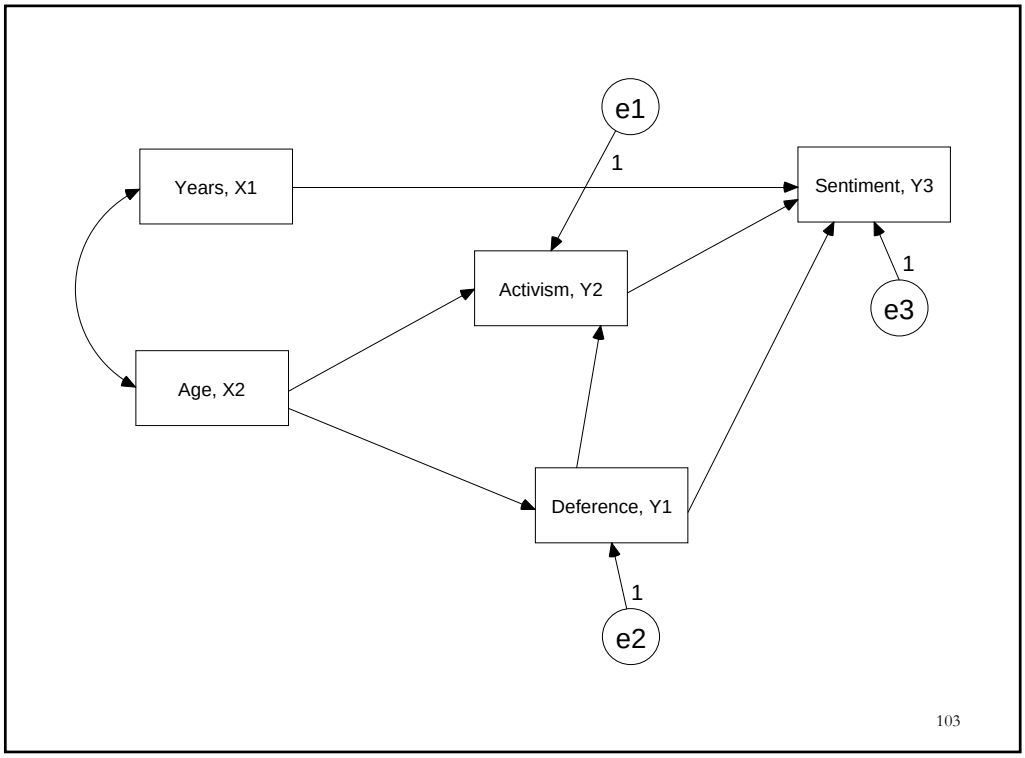
- The endogenous variables (Y's) can be represented by a set of structural equations relating the y's, x's, and residuals through beta, gamma, and zeta matrices
- These matrices specify the relations among the observed variables
- The residuals are also assumed to be uncorrelated with the X's

Model Specification

- The two major types of structural equation models with observed variables are recursive and nonrecursive models
  - Recursive models have no reciprocal relations or feedback loops
  - When this is true, the Beta matrix is a lower triangular matrix
  - Nonrecursive models allow feedback loops

Observed Variable Example

- An example of these models is presented by Bollen (1989). The union sentiment model is presented in the path diagram below
- In this model, worker's sentiment towards unions, Y_3, is represented as a function of support for labor activism, Y_2, deference towards managers, Y_1, years in the textile mill, X_1, and workers age, X_2


e1
1
Years, X1
Sentiment, Y3
Activism, Y2
1
e3
Age, X2
Deference, Y1
1
e2
103

Specification of the Structural Equations

Given the model above, the following set of structural equations define the model:

$$Y_1 = \gamma_{12} X_2 + \zeta_1$$

$$Y_2 = \beta_{21} Y_1 + \gamma_{22} X_2 + \zeta_2$$

$$Y_3 = \beta_{31} Y_1 + \beta_{32} Y_2 + \gamma_{31} X_1 + \zeta_3$$

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Equations in Matrix Form

Y matrix - a p X 1 column vector of endogenous observed variables

β matrix - a p X p coefficient matrix defining the relations among endogenous variables

Γ matrix - a p X q coefficient matrix defining the relations from exogenous to endogenous variables

X matrix - a q X 1 column vector of exogenous observed variables

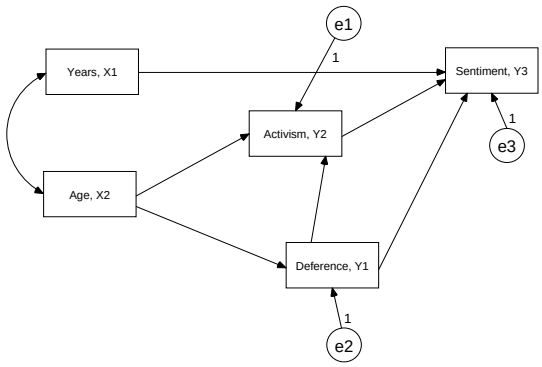
ζ matrix - a p X 1 vector of residuals in the equations

phi (Φ) matrix - a q X q variance-covariance matrix among the exogenous variables

psi (ψ) matrix - a p X p variance-covariance matrix among the residuals (zetas)

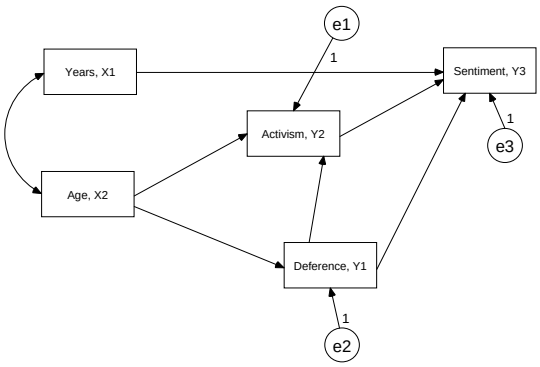
Matrix Practice

Construct the weight matrices necessary to specify the union sentiment model (beta, gamma, and zeta)



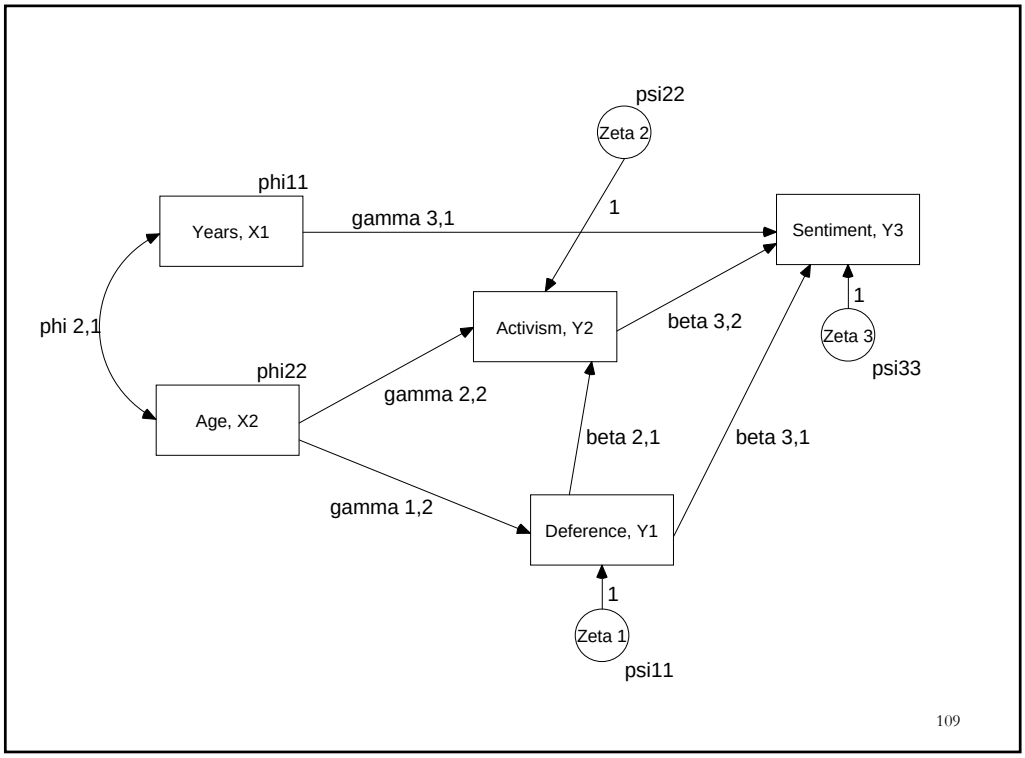
Matrix Practice

Construct the variance-covariance matrices (phi and psi) necessary to specify the union sentiment model



Equation in Matrix Form

$$\begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \mathbf{Y}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{B}_{21} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{B}_{31} & \boldsymbol{B}_{32} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \gamma_{12} \\ \mathbf{0} & \gamma_{22} \\ \gamma_{31} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix}$$


psi22
Zeta 2
phi11
Years, X1
gamma 3,1
1
Sentiment, Y3
1
Zeta 3
psi33
phi 2,1
Activism, Y2
beta 3,2
phi22
Age, X2
gamma 2,2
beta 2,1
beta 3,1
gamma 1,2
Deference, Y1
1
Zeta 1
psi11
109

Bootstrapping

- A statistical resampling/simulation technique that provides a means to estimate statistical parameters (most often a standard error)
  - Can estimate standard errors even when formulas are unknown (R^2)
  - Obtain independent estimates when assumptions are violated (e.g., nonnormal data)
  - Can be also be used for comparison of:
    - Competing models (Bollen & Stine, 1992)
    - Different estimation techniques (i.e., ML, ADF, GLS)

Bootstrapping

- Two types of bootstrapping:
  - Nonparametric – sample treated as a pseudo-population; cases randomly selected with replacement
  - Parametric – Samples randomly drawn from a population distribution created based on parameters specified by researcher or by estimation from sample
- Size of original sample and number of simulated samples both important
- Representativeness of original sample linked to degree of bias in bootstrapped results

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AMOS Example

Confirmatory Factor Analysis (CFA) Models

- Exploratory Factor Analysis (EFA) models depend on an emergent, empirically determined specification of the relation between variables and factors
- CFA models provide an a priori means of specifying and testing the measurement relations between a set of observed variables and the latent variables they are intended to measure
- CFA models allow for the explicit representation of measurement models
- CFA models are particularly useful in research on instrument validity

Model Specification

The general CFA model in SEM notation is:

$$\mathbf{X_i = \Lambda_X \xi_k + \theta_\delta}$$

where:

- $\mathbf{X_i}$ = **a column vector of observed variables;**
- ξ_k = **ksi, a column vector of latent variables;**
- Λ_X = **lambda, an i X k matrix of coefficients defining the relations between the manifest (X) and latent (ξ) variables;**
- θ_δ = **theta-delta, an i X i variance/covariance matrix of relations among the residual terms of X**

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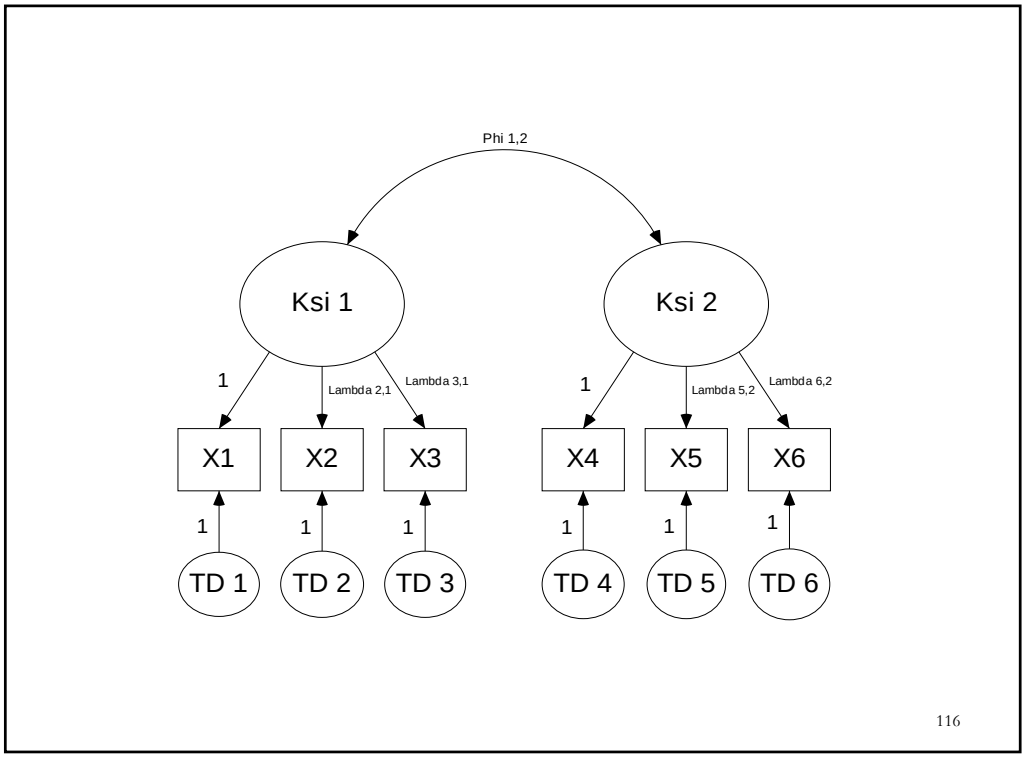
Confirmatory Factor Analysis (CFA) Models

- This general equation indicates that each manifest variable can be represented by a structural equation relating lambda, ksi, and theta-delta
- Conceptually this means that a set of manifest variables can be represented in terms of the constructs they are intended to measure (ksi's), and variance specific to the variable that is unrelated to the construct (the residual)


Phi 1,2
Ksi 1
Ksi 2
1
Lambda 2,1
Lambda 3,1
1
Lambda 5,2
Lambda 6,2
X1
X2
X3
X4
X5
X6
1
1
1
1
1
1
TD 1
TD 2
TD 3
TD 4
TD 5
TD 6
116

Specification of the Structural Equations

Given the model above, the following set of structural equations define the model:

$$\mathbf{X_1} = \Lambda_{11}\,\xi_1 + \theta_{\delta 11}$$
$$\mathbf{X_2} = \Lambda_{21}\,\xi_1 + \theta_{\delta 22}$$
$$\mathbf{X_3} = \Lambda_{31}\,\xi_1 + \theta_{\delta 33}$$
$$\mathbf{X_4} = \Lambda_{42}\,\xi_2 + \theta_{\delta 44}$$
$$\mathbf{X_5} = \Lambda_{52}\,\xi_2 + \theta_{\delta 55}$$
$$\mathbf{X_6} = \Lambda_{62}\,\xi_2 + \theta_{\delta 66}$$

Equations in Matrix Form

- Of course these equations can also be represented in matrix form which we will do in class
- An additional matrix is also necessary for model estimation, the phi (Φ) matrix, which is a k X k variance-covariance matrix specifying the relationships among the latent (ξ) variables
- The model implied v-c matrix for CFA is:

$$\Sigma(\hat{\theta}) = \mathbf{\Lambda_X \Phi \Lambda_X} + \theta_\delta$$

Model Identification

- Since latent variables are inherently unobserved, they have no scale or units of measurement. In order to represent the latent variable a scale must be defined. This is usually done arbitrarily by one of two methods:
  - Method 1: the coefficient (Λ_X, "loading") for one of the manifest variable is not estimated, but is constrained to an arbitrary value (typically 1.0). This defines the units of measure for all remaining lambdas and for the variance of the relevant ksi
  - Method 2: the variances of the latent variables (ξ_{kk}) are set to 1.0

Model Identification

- Identification tested by examining whether the number of observed variances/covariances vs. the number of parameters to be estimated
- Using the t-rule (see Bollen, p. 243), for the example above with 6 manifest variables there are (q)(q + 1)/2 elements in the variance/covariance matrix = (6 X 7)/2 = 21 elements
- The number of freely estimated parameters in the model is t = 4 lambda estimates (remember the constraints placed by Method 1 above), 3 phi estimates, and 6 theta-delta estimates = 13 parameters. So the t-rule is satisfied (and by the way the model therefore has 21 - 13 = 8 degrees of freedom)

Model Estimation

- As long as the SEM model is overidentified, iterative estimation procedures (most often Maximum Likelihood) are used to minimize the discrepancies between S (the sample variance-covariance matrix) and $\Sigma(\hat{\theta})$ (the model implied estimate of the population variance-covariance matrix)
- Discrepancies are measured as F, the fitting function. The CFA model can be used to attempt to reproduce S through the following equation (note that now the phi matrix becomes part of the model):

$$\Sigma(\hat{\theta}) = \boldsymbol{\Lambda}_X \boldsymbol{\Phi} \boldsymbol{\Lambda}_X + \boldsymbol{\theta}_\delta$$

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Model Estimation

- This equation indicates that a variance/covariance matrix is formed through manipulation of the structural matrices implied by the specified CFA model. This, then provides a basis for evaluating goodness-of-fit (GOF) as in:

$$\chi^2 = (N - 1)\, F$$

AMOS Example

SEM Models With Observed and Latent Variables

- Klein refers to these models as "hybrid" models
- Also known as the full LISREL model
- Full Models include:
  - A combination of directly observed and unmeasured latent variables
  - Some number of explanatory or predictor variables with relations to some number of dependent or outcome variables

SEM Models With Observed and Latent Variables

- Full Models include:
  - Measurement models
    - Specify relations from Latent Ksi's to Measured X's (exogenous)
    - Specify relations from Latent Eta's to Measured Y's (endogenous)
  - Structural, path, or "causal" models
    - Specify relations from Ksi's to Eta's
    - Specify relations from one Eta to another Eta
  - Structural residuals for latent Etas (equation residuals)
  - Measurement residuals for X's and Y's

Fundamental Equations for the Full Model

- The structural equation m[illegible]el:

$$\eta = \beta\eta + \Gamma\xi + \zeta$$

Ksi, a latent, exogenous variable

Zeta, a regression coefficient relating equation residuals to etas

Eta, a latent, endogenous variable

Beta, a regression coefficient relating one eta to another

Gamma, a regression coefficient relating a Ksi to an Eta

- The measurer[illegible] Y:

$$y = \Lambda_y\eta + \varepsilon$$

Epsilon, a regression coefficient relating a measurement residual to a manifest Y variable

A manifest, exogenous variable

Lambda X, a matrix of regression coefficients relating Ksi's to X variables

A manifest, endogenous variable

Lambda Y, a matrix of regression coefficients relating Eta's to manifest Y variables

measuren[illegible]

$$x = \Lambda_x\xi + \delta$$

Delta, a regression coefficient relating a measurement residual to a manifest X variable

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Model Specification

The full SEM Model includes the following matrices:

y = a p X 1 vector of observed outcome measures

x = a q X 1 vector of observed predictor variables

η = an m X 1 vector of endogenous latent variables

ξ = an n X 1 vector of exogenous latent variables

ε = a p X 1 vector of measurement errors in y

δ = a q X 1 vector of measurement errors in x

Model Specification

Full SEM Model matrices continued:

- $\mathbf{\Lambda_Y}$ – a p X m weight matrix representing paths from endogenous latent variables (η) to observed Y variables
- $\mathbf{\Lambda_X}$ – a q X n weight matrix representing paths from exogenous latent variables (ξ) to observed X variables
- $\mathbf{\Gamma}$ – an m X n coefficient matrix defining the relations from exogenous to endogenous variables
- β – an m X m coefficient matrix defining the relations among endogenous variables
- ζ – an m X 1 vector of equation residuals in the structural relationships between η and ξ

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Model Specification

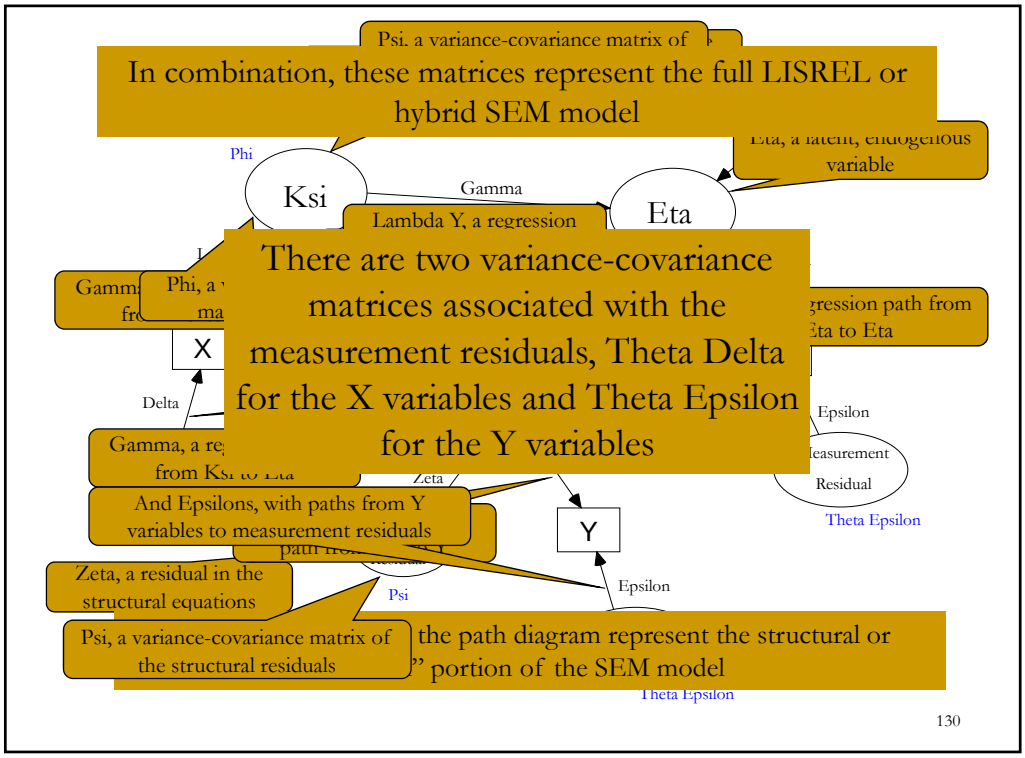
The full SEM Model also includes the following variance-covariance matrices:

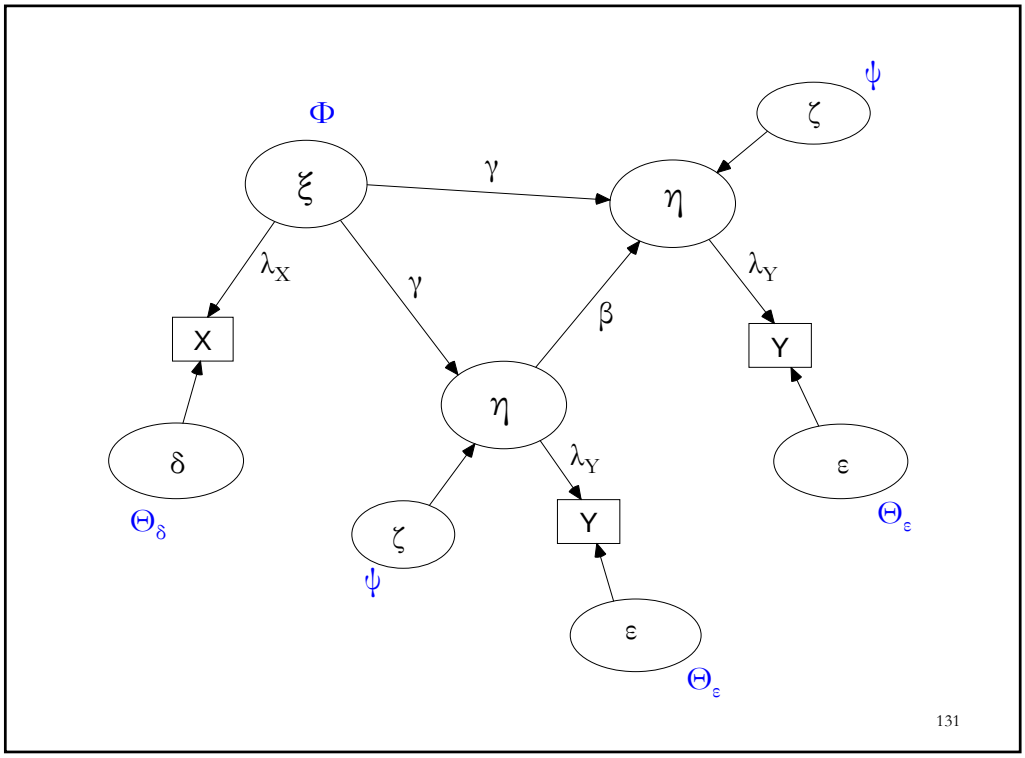
Φ – an n X n matrix of ξ

Ψ – an m X m matrix of ζ

Θ_δ – a q X q matrix of measurement residuals (δ)

Θ_ε – a p X p matrix of measurement residuals (ε)


Psi, a variance-covariance matrix of
In combination, these matrices represent the full LISREL or hybrid SEM model
Phi
Ksi
Gamma
Eta
variable
Lambda Y, a regression
There are two variance-covariance matrices associated with the measurement residuals, Theta Delta for the X variables and Theta Epsilon for the Y variables
X
Delta
Epsilon
Residual
Theta Epsilon
Zeta
And Epsilons, with paths from Y variables to measurement residuals
Y
Zeta, a residual in the structural equations
Psi
Epsilon
Psi, a variance-covariance matrix of the structural residuals
the path diagram represent the structural or
" portion of the SEM model
Theta Epsilon
130


Φ
ψ
ζ
ξ
γ
η
λX
γ
β
λY
X
Y
η
δ
λY
ε
ΘδΘε
ζ
Y
ψ
ε
Θε
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Full Models

- Some debate on how to properly evaluate fit for the full model
  - Conduct multiple steps, measurement models first, full models later
  - Analyze model all in one step
  - See special issue of journal SEM, volume 7(1), 2000

Invariance Testing with SEM Models

- A powerful approach to testing group differences
  - Groups can be levels of typical categorical variables (e.g., gender, ethnicity, treatment vs. control, etc.)
  - Groups can be different samples as in cross-validation
- Unlike typical applications in GLM, invariance testing allows more varied hypotheses about the way in which groups differ. One can test the statistical equivalence of:
  - Model structure
  - Regression coefficients
  - Correlations
  - Variances
  - Residuals
  - Means, Intercepts

Invariance Testing with SEM Models

- Invariance testing involves the specification of multiple hypotheses about how groups differ
- Hypotheses are usually arranged from more global or general hypotheses to more specific
  - For example, it usually makes sense to first test a configural or structural hypothesis that the same SEM model fits well from group to group
  - This general hypothesis can then be followed by more specific tests of equality constraints in particular parameters

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Invariance Testing with SEM Models

- Some analysts prefer to test a first hypothesis that examines the equality of variance-covariance matrices
  - If there are no significant differences, there is no reason to proceed
  - If significant differences are found then additional invariance tests are performed
- Hypotheses should be ordered a priori
  - This creates a hierarchy of invariance hypotheses
  - The hierarchy is nested in that each step adds constraints to the prior step(s)
- Chi square difference tests and differences in GOF indices are used to determine if a "lack of invariance" has occurred from one step to another

Comparing Nested Models

- Models are considered nested when:
  - Some function of the free parameters of one model equals another free parameter in a second model, or
  - Free parameter(s) in one model are expressed as fixed constants in a second model
- Simplest example is when free parameters in one model are a subset of the free parameters in a second model
- Examples:
  - Two models with and without correlated residuals
  - Two models one which has some λ's to be equal
  - Two models one of which fixes a parameter to a constant value (e.g., $\Phi_{11} = 2.0$)

Invariance Testing with SEM Models

- One common testing hierarchy:
  - Model form
  - Lambda matrices equal
  - Phi matrices equal
  - Theta Delta matrices equal
- When invariance fails, post hoc testing can be done using "partial invariance" tests examining each individual element in the matrix

Invariance Testing with SEM Models: Example of Results

Comparison	χ^2	df	χ^2_Δ	df_Δ	CFI	RMSEA	TLI	SRMR
I. Calibration: Southwestern Sample	387	8	–	–	.983	.065	.974	.024
II. Cross-validation: Standardization Sample								
A. Freely Estimated	632	16	–	–	.985	.059	.978	.024
B. Λ Fixed			739	20	107	4 .983	.057	.977 .029
C. Λ, Φ Fixed	929	23	190	3	.978	.060	.974	.033
D. All Parameters Fixed	1444	29	516	6	.966	.067	.965	.037

Note. Step II.A. represents testing of both samples together, hence the df are doubled and the chi-square is equivalent to the sum of the chi-square for each separate group.

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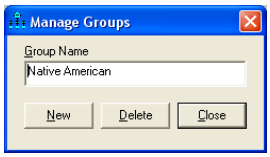
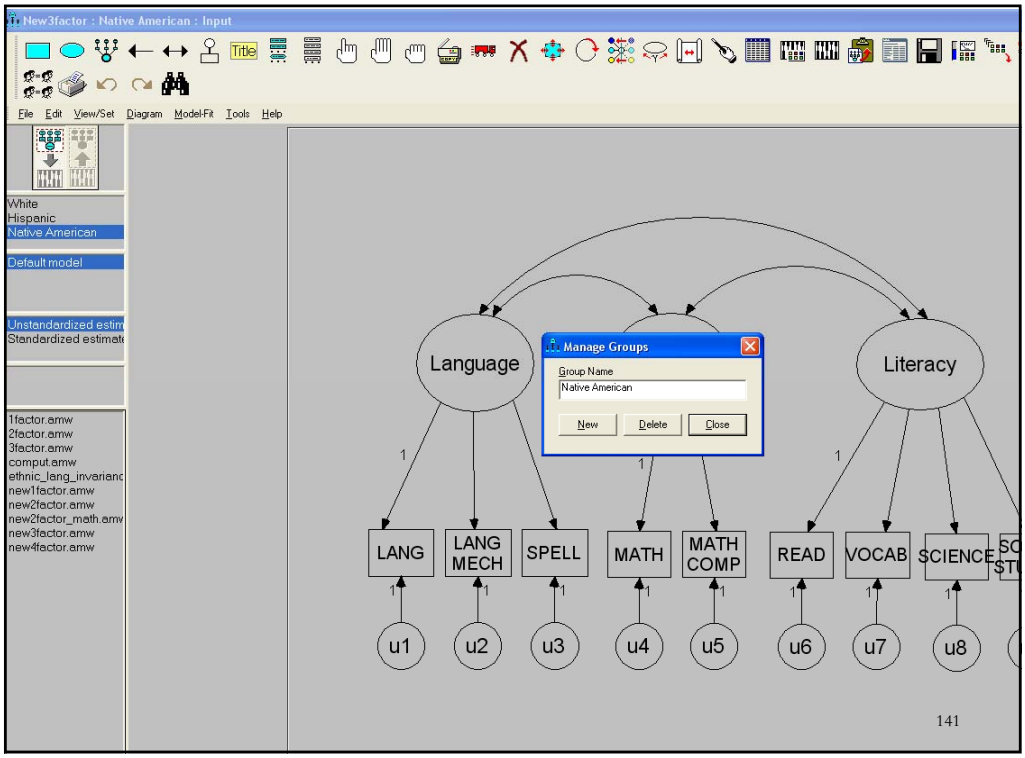
Invariance Testing with SEM Models

- Multiple groups can be compared at once
- In the same way as GLM techniques like ANOVA, significant omnibus tests are followed by more specific, focused tests to isolate differences
  - This is necessary when more than two groups are being compared
  - Also necessary given multiple parameters in a particular matrix
  - Post hoc testing to determine which individual elements of a matrix are significant is referred to as "partial" invariance tests

Steps in Invariance Testing with AMOS

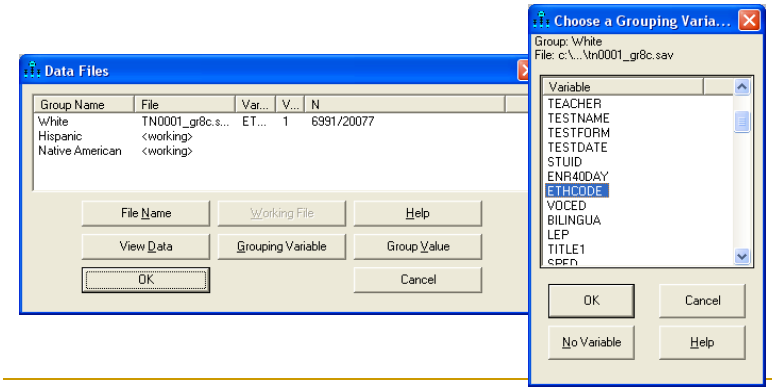
- Define the groups or samples
  - Double click on the groups window to open the "manage Groups" dialog box
  - Create multiple groups and group names:




New3factor : Native American : Input
File Edit View/Set Diagram Model-Fit Tools Help
White
Hispanic
Native American
Default model
Unstandardized estim
Standardized estimate
1factor.amw
2factor.amw
3factor.amw
comput.amw
ethnic_lang_invarianc
new1factor.amw
new2factor.amw
new2factor_math.amv
new3factor.amw
new4factor.amw
Manage Groups
Group Name
Native American
New
Delete
Close
Language
Literacy
LANG
LANG MECH
SPELL
MATH
MATH COMP
READ
VOCAB
SCIENCE
u1
u2
u3
u4
u5
u6
u7
u8
1
141

Steps in Invariance Testing with AMOS

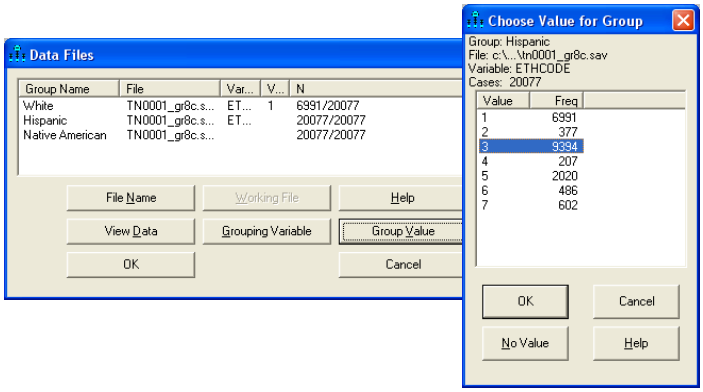
- Associate the data with the group names
  - Method 1: use coded variable values in data set



142

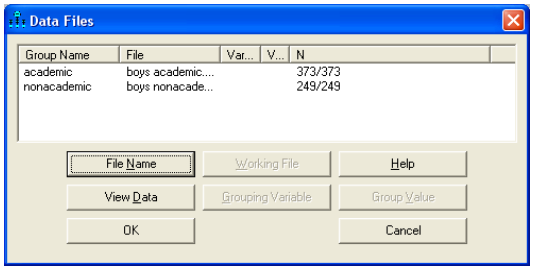
Steps in Invariance Testing with AMOS

- Define code values for the grouping variable:



Steps in Invariance Testing with AMOS

- Associate the data with the group names
  - Method 2: use separate data files for each group or sample



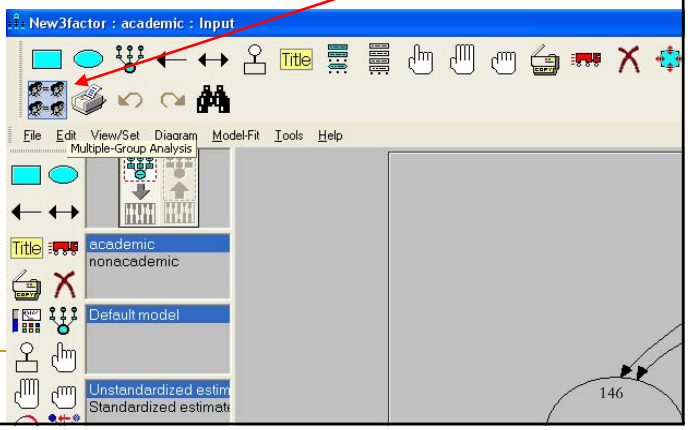
Steps in Invariance Testing with AMOS

- Define one of the groups as the referent group
- For this group, label all parameters to be tested
  - This can be done manually if you want to control parameter labels
  - Less laborious is using the AMOS macro:

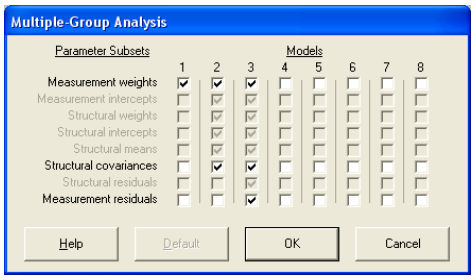
    tools ⟶ macros ⟶ name parameters
- Use view ⟶ matrices to create constraints from one group to another by copying the labels from one group into the matrix for the other group (we will look at this in detail in class)

Automatic Method of Invariance Testing with AMOS

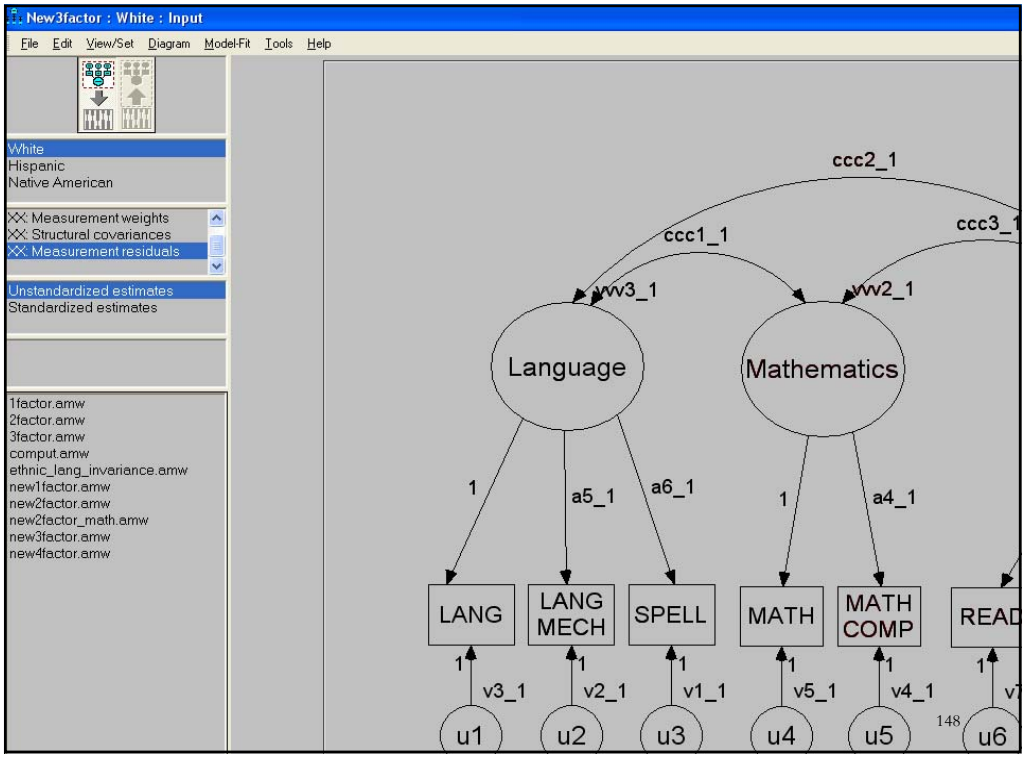
- After setting up data files and groups and specifying SEM model, click on the multiple groups icon:



Automatic Method of Invariance Testing with AMOS



147

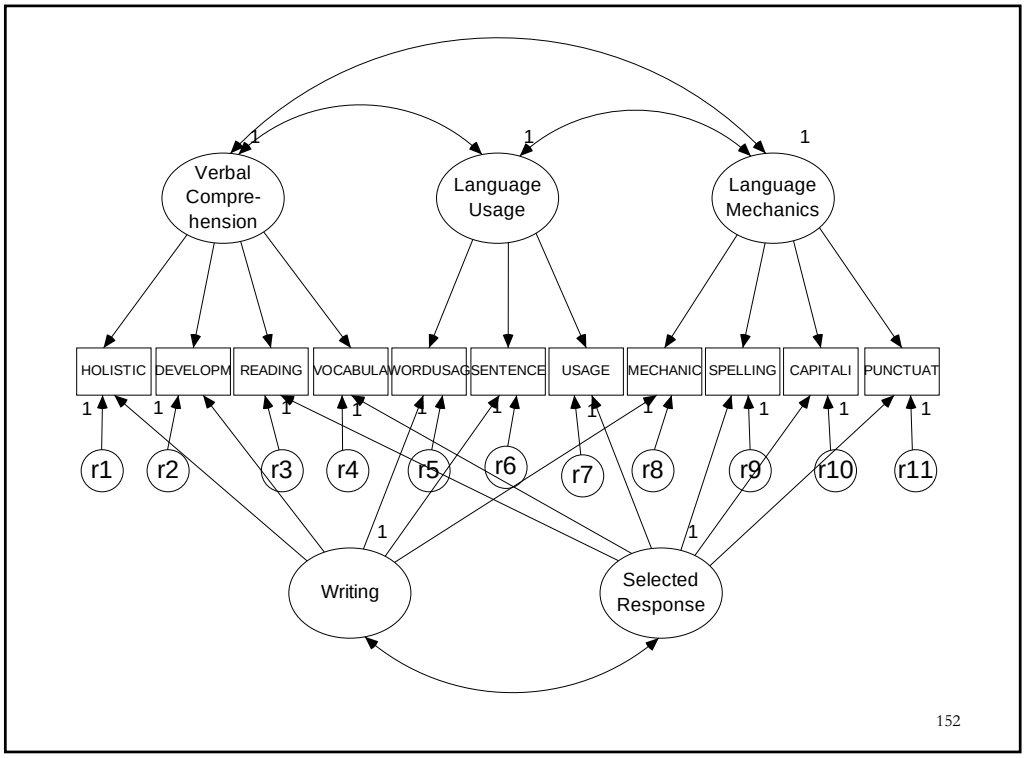

New3factor : White : Input
File Edit View/Set Diagram Model-Fit Tools Help
White
Hispanic
Native American
XX: Measurement weights
XX: Structural covariances
XX: Measurement residuals
Unstandardized estimates
Standardized estimates
1factor.amw
2factor.amw
3factor.amw
comput.amw
ethnic_lang_invariance.amw
new1factor.amw
new2factor.amw
new2factor_math.amw
new3factor.amw
new4factor.amw
ccc2_1
ccc1_1
ccc3_1
vv3_1
vv2_1
Language
Mathematics
1
a5_1
a6_1
1
a4_1
LANG
LANG MECH
SPELL
MATH
MATH COMP
READ
1
1
1
1
1
1
v3_1
v2_1
v1_1
v5_1
v4_1
u1
u2
u3
u4
u5
u6
148

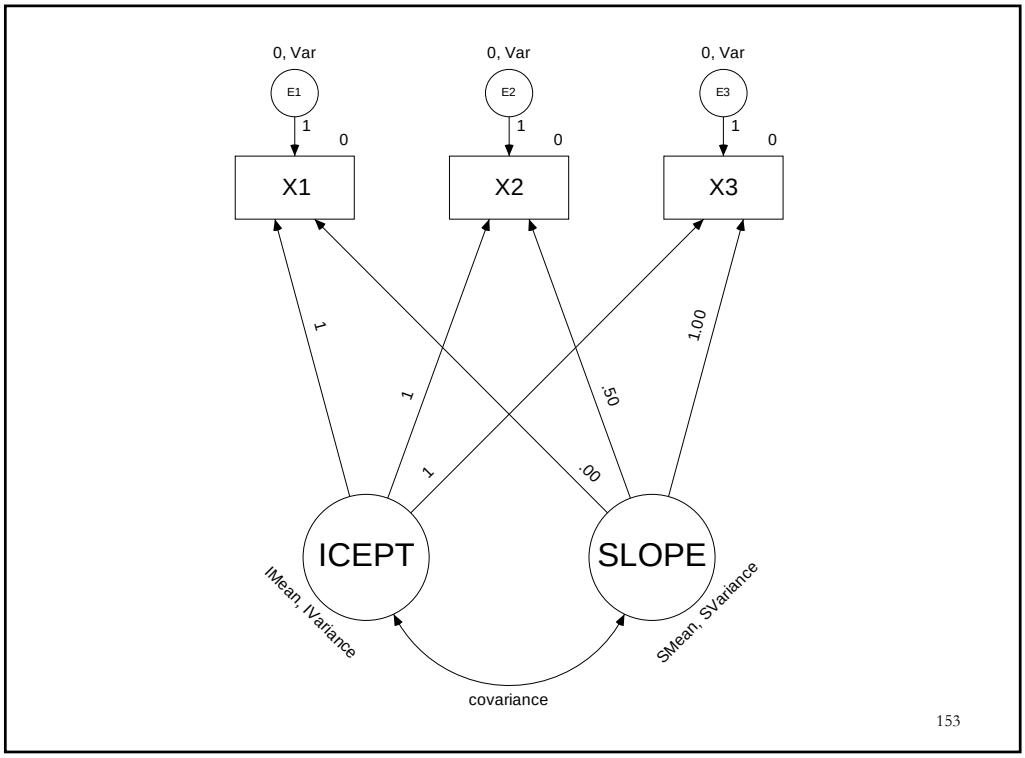
AMOS Example


STATISTICS DEPT.
YOU ARE HERE
(SAMPLING ERROR ±4%)
© Original Artist

150

Other Models


Verbal Compre-hension
Language Usage
Language Mechanics
HOLISTIC
DEVELOPM
READING
VOCABULA
WORDUSAG
SENTENCE
USAGE
MECHANIC
SPELLING
CAPITALI
PUNCTUAT
r1
r2
r3
r4
r5
r6
r7
r8
r9
r10
r11
Writing
Selected Response
1
152


0, Var
0, Var
0, Var
E1
E2
E3
1
1
1
0
0
0
X1
X2
X3
1
1
1
.50
1.00
.00
ICEPT
SLOPE
IMean, IVariance
SMean, SVariance
covariance
153

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