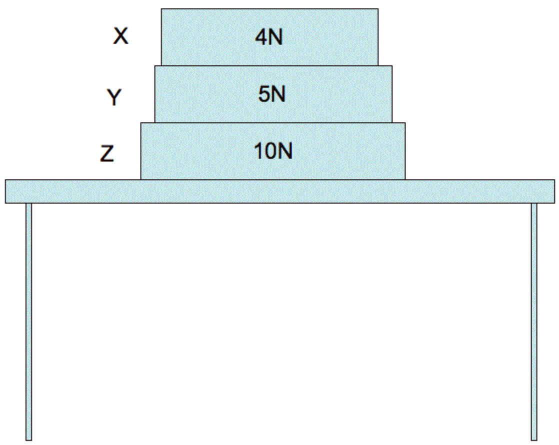
Solutions Final exam Phys 251H

10.15 AM–12.15 PM, Wednesday, December 5, 2007

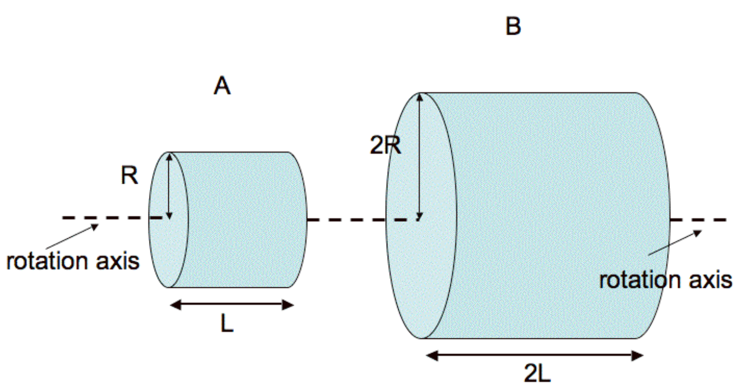
1 Three books (X, Y, and Z) rest on a table. The weight of each book is indicated in the Figure. What is the magnitude of the force exerted by book Z on book Y?



(i) 0, (ii) 5N, **(iii) 9N**, (iv) 10N, (v) 14N, (vi) 15N.

Book Z must compensate for the weights of both X and Y.

2 A and B are two solid cylinders made of aluminum. Their dimensions are shown in the Figure. What is the ratio of the moment of inertia (around the axis through the center-of-mass shown in the Figure) of B to that of A?



(i) 2, (ii) 4, (iii) 8, (iv) 16, **(v) 32**, (vi) 64

The mass of B is 8 times larger than that of A, and the average distance squared to the rotational axis is 4 times as large (the fact that the length of the cylinder is twice as large for B doesn't affect the average distance to the axis!): hence the ratio I_B/I_A is $8 \times 4 = 32$. This would in fact be true for *any* shape: if B is twice as large in all three dimensions, then its mass is 8 times as large and the average distance squared to any axis through its center of mass is 4 times as large.

3 You are on an accelerating train (on a straight and flat train track), whose speed as a function of time is given by

$$v(t) = kt^2 + v_0,$$

with k and v_0 positive constants. There is a suitcase with mass m on the floor, and the static friction coefficient between suitcase and floor is μ_s.

(a) At what time will the suitcase start sliding over the floor? Express this time in μ_s, k, v_0, g, m.

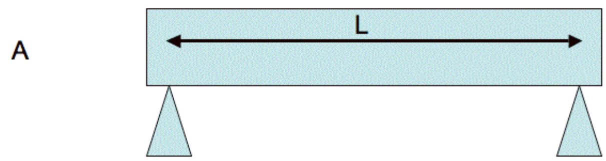
Answer: The acceleration of the train is $a(t) = 2kt$. There is a pseudo force (the train is a noninertial frame!) acting on the suitcase, equal to $F = -ma(t)$. When it is larger in magnitude than the friction force $F_f = \mu_s mg$ the suitcase will start sliding. This happens when $2mkt > \mu_s mg$, that is, when

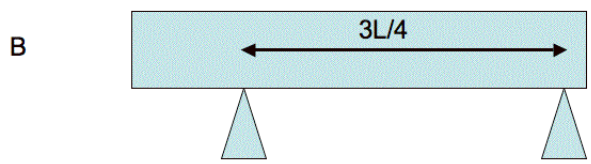
$$t > \frac{\mu_s g}{2k}.$$

(b) Give a dimensional analysis of your answer to part (a) (hint: in what units should k be expressed?).

Answer: from the definition of k it follows k has units m/s^3; μ_s is dimensionless. So, g/k has units $m/s^2/(m/s^3) = s$, as it should.

4 A uniform plank is resting on 2 supports, a distance L apart. The normal forces exerted by the supports in situation A are both equal to 30 N (and they point straight up). In situation B one of the support points has been moved over a distance of *exactly* $L/4$. What are the normal forces in situation B?





Answer: From left to right, **40N and 20N**. This is so that the sum of these two forces is still 60N, compensating for the weight of the plank, and so that the torque due to the normal forces around the support point on the right is still $\tau = -30N \times L = -40N \times 3L/4$, compensating for the torque due to gravity (which doesn't change by moving the left support point!), $\tau_W = W \times L/2 = 60N \times L/2$. Note that the fact that the normal forces are equal in situation A implies the center of mass of the plank is indeed located at a distance L/2 from the support points.

5 You throw a dart straight (aiming horizontally) toward the bull's eye on a dart board, with a speed $v = 15$ m/s. The dart ends up a distance L vertically below the bull's eye because of gravity. Neglect air friction, and use $g = 9.8$ m/s^2.
(a) If you stand 3.0 m from the dart board, what is L?
Answer: It takes a time $T = 3.0m/15(m/s) = 0.20s$ to reach the dart board. In that time the dart drops over a vertical distance $L = gT^2/2 = 0.20m$.
(b) At what angle (with the horizontal) does the dart enter the dart board (assuming the dart always points in the direction it is traveling)?
Answer The vertical speed of the dart at time T is $v_y = gT = 2.0m/s$ so that the angle with the horizontal is

$$\tan\theta = v_y/v_x = 2.0/15 \approx 0.13.$$

That is,

$$\theta \approx 0.13\text{rad} \approx 7.4^\circ.$$

6 Two stones are released from rest at a certain height, one a small time T after the other. Ignore friction.

(a) While falling, the difference in their speeds

(i) will increase

(ii) will decrease

(iii) will stay the same

(iv) depends on the ratio of their masses

Answer: Since the stones fall with the same acceleration—independent of their masses— the relative speed **stays the same**, namely $v_1(t) - v_2(t) = gT$, since

$$v_1(t) = gt;\ v_2(t) = g(t - T) = gt - gT \text{ for } t > T,$$

if we call $t = 0$ the time the first stone is dropped.

(b) While falling, their separation distance

(i) will increase

(ii) will decrease

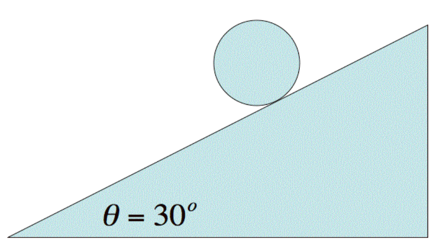
(iii) will stay the same

(iv) depends on the ratio of their masses

Answer: Since the relative speed stays the same, the difference in heights, then, must **increase** linearly with time. In fact, we have

$$h_2(t) - h_1(t) = gTt - \frac{1}{2}gT^2 \text{ for } t > T.$$

7 A solid sphere with mass M, radius R, and rotational inertia $I = \frac{2}{5}MR^2$ is rolling (without slipping) down an inclined plane making an angle of *exactly* $\theta = 30°$ with the horizontal (see Figure). Ignore air friction.



(a) Draw and name three forces acting on the sphere.

Answer: There is a normal force N normal to the slope, a friction force (between slope and sphere) F_f up the slope, and gravity $W = mg$ straight down.

(b) Write down the equations for the acceleration a along the slope, and for the angular acceleration α of the sphere.

Answer: Along the slope we have

$$Ma = Mg\sin\theta - F_f = Mg/2 - F_f.$$

There is a (positive) torque causing the angular acceleration:

$$\frac{2}{5}MR^2\alpha = F_f R.$$

(c) Solve the equations from part (b) using the condition for rolling without slipping. That is, express a in terms of M, R, g.

Answer: Rolling without slipping implies that $a = \alpha R$. This implies that $F_f = \frac{2}{5}Ma$ and substituting that in the equation $Ma = Mg/2 - F_f$ gives

$$a = \frac{5g}{14}.$$

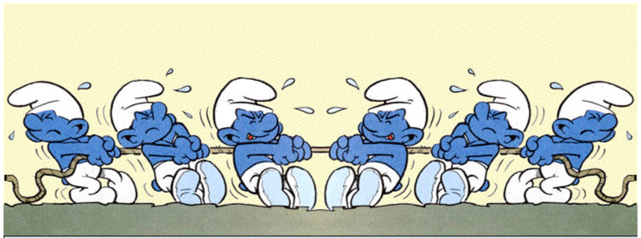
(d)***[extra credit] Suppose we have put a small black dot on the sphere, and suppose that point touches the inclined plane at time $t = 0$ when both translational and rotational speed of the sphere are zero. What is the acceleration of the black dot when it touches the inclined plane for a second time (after one rotation of the sphere)?

Answer: The black dot undergoes a combination of translation motion and pure circular motion. There is a tangential acceleration equal but opposite to the translational acceleration at that time. The net acceleration thus arises from the radial acceleration only. It equals v_T^2/R, with v_T the tangential velocity at that time (call it T) *in the frame of reference of the sphere* (because that's the frame of reference in which the motion of the black dot is circular!). The time T is determined by $2\pi = \alpha T^2/2$, so that $T^2 = 4\pi/\alpha = 4\pi R/a$. The tangential speed of the black dot relative to the sphere is then $v_T = \alpha RT = aT$. So the radial acceleration is $v_T^2/R = a^2T^2/R = 4\pi a$. That is,

$$a_R = \frac{10\pi}{7}g,$$

and its direction is normal to the slope. (See also the picture in Chapter 9 of a "cycloid.")

8 Two teams are competing in a tug-of-war contest (see Figure).


Tug-of-war

(a) Draw and compare the horizontal forces *on* the two teams and *on* the (massless) rope in the case that the team on the left is winning.

Answer: The tension T in the rope is uniform, so both teams pull equally hard on the rope. Thus, the rope pulls *equally* on both teams. The difference lies in the friction forces (with the ground), which is larger on the winning team: it is larger than T for the winning team, and smaller than T for the losing team.

(b) Which force exerted *by* the winning team allows it to win?

Answer: It is the force exerted on the ground by the winning team that is larger than the corresponding force exerted by the losing team.

9 On a package of Trader Joe's savory thin mini rice crackers you find the statement "NET WT 8 OZ (227g)." Name two things that are wrong with this statement from a nitpicking physicist's point of view (that OZ is not an SI unit is *not* the issue here).

Answer: (i) The gram is a unit for mass, not a unit for weight. (ii) The number of significant digits is inconsistent between 8oz and 227g.

10 You are standing on totally frictionless ice on a cold windless day. Ignoring air friction, which of the following actions would get you moving horizontally (relative to the ice), and which would *not*?

(i) Take off a shoe and throw it away: Yes!

(ii) Jump up and stick one arm out: No!

(iii) Swing one arm violently: No!

(iv) Pick up some snow from the ice and throw it away: Yes!