Below is the periodic table of elements, with the size of one atom of each element indicated. The figure also indicates two trends: (i) moving to the right in a given row tends to decrease the atomic size, and (ii) going down along a given column tends to increase the size.

(a) Explain these two trends. For example, why is O smaller than C, and why is Rb larger than Na?

**Answer:** The outer electrons of O are in the same shell as those of C, but O’s nucleus has more protons and so pulls harder on the outer electrons. The O electrons are closer to the nucleus than C’s electrons, and so O is smaller than C. Rb’s outer electron is in a higher shell ($n = 5$) than Na ($n = 3$) and thus farther away from the nucleus.

(b) What is the electronic configuration of the ground state of Mg (atom number Z=12)? Why is Mg in the third row and the second column?

**Answer:** $(1s)^2(2s)^2(2p)^6(3s)^2$. There are two electrons in the outer shell, which has
n = 3 as principal quantum number.

(c) Sketch the radial part of the wave function (as a function of $r \geq 0$) of a 4p state of an electron, taking into account the following three features of the wave function: the value at $r = 0$, the value at $r \to \infty$, the number of nodes.

**Answer:** The number of nodes is 2 ($p \leftrightarrow l = 1$ and $4 = 2 + 1 + 1$), and both at $r = 0$ and $r \to \infty$ we have $\psi(r) \to 0$. See figure 2.

(d)*[extra credit] If an electron were a fermion with spin $s = 3/2$ (instead of $s = 1/2$), which elements would be noble gases? Name the first two.

**Answer:** 4 (instead of 2) electrons could fill each different orbital, so the periodic system is "stretched" out by a factor of 2. The regular noble gas atoms are He ($Z=2$), Ne ($Z=10$), Ar ($Z=18$) etc., so in the phantasy world that would be $Z=4$ (Be), $Z=20$ (Ca), $Z=36$ (Kr) etc.
Consider a particle with mass $m$ moving in the 1-D potential $V(x)$ sketched in Figure 2 (now 3). The particle has a fixed energy $E$. We consider three cases:

1. $-V_0 < E < 0$.
2. $0 < E < V_0$, where the particle is moving in from the left.
3. $E > V_0$, where the particle is moving in from the right.

(a) In which of these cases do we have bound states with discrete energies? In which cases do we have tunneling?

**Answer:** In case 1 there are discrete bound states, when the particle is most likely to be found in the well. In cases 1 and 2 there is tunneling: in case 1 the particle can be found in the region with $V = 0$, a classically forbidden region, and even to the right of the barrier! In case 2 the particle can be found inside the barrier as well as to the left of the barrier.

(b) Is momentum in the $x$ direction conserved? Explain your answer.

**Answer:** Momentum is *not* conserved, because $V$ depends on $x$. In other words, $\hat{p}_x$
does not commute with the potential energy part of the Hamiltonian:

$$[\hat{p}_x, V(x)] = \hbar i dV/dx \neq 0.$$  

This statement is independent of which of the three cases we’re in.

(c) The wave function in the well (where the potential is $-V_0$) is of the form

$$\psi(x) = A \exp(-iEt/\hbar) \sin(kx + \delta),$$

with $A, \delta$ constants. Verify that $\psi$ is a solution of the Schrödinger equation, and express $k$ in $E, V_0$ and $m$.

**Answer:** We have

$$\hbar i \partial \psi / \partial t = E \psi,$$

and

$$-(\hbar^2/2m) \partial^2 \psi / \partial x^2 = (\hbar^2 k^2 / 2m) \psi,$$

so $\psi$ satisfies the Schrödinger equation if and only if

$$E = \frac{\hbar^2 k^2}{2m} - V_0 \leftrightarrow k = \sqrt{\frac{2m(E + V_0)}{\hbar^2}}.$$
In the photoelectric effect, electrons are ejected out of a metal by illuminating the surface with (laser) light (see Figure). Suppose we measure both the kinetic energy $E$ of the ejected electrons and the number of electrons emitted per second, $N$. Explain how $E$ and $N$ change in the following cases:

1. We increase the intensity of the laser light, but keep the wavelength the same.
   **Answer:** $E$ is determined by the energy of 1 photon, $h\omega = hc/\lambda$ and the “work function” of the metal, $W$, which is just the minimum energy needed to eject one electron. That is, we have
   $$E = \frac{hc}{\lambda} - W.$$ 
   So, keeping the wavelength the same means we keep $E$ the same. The increased intensity just means we send in more photons per second, hence eject more electrons per second, and so $N$ increases.

2. We increase the wavelength of the laser light, but keep the same light intensity.
   **Answer:** Increasing the wavelength decreases $E$ (see part 1). In order to keep the intensity of the incoming light the same, although decreasing the energy of each photon, we have to increase the number of photons hitting the surface per second. So $N$ will increase.

3. *[extra credit] We change the metal: instead of Cs we use Pb (refer back to Figure 1, if necessary).
   **Answer:** Cs has a smaller ionization potential than Pb. It is much easier to eject an electron from a Cs metal than from a Pb metal. That is $W_{Cs} < W_{Pb}$. So $E$ will decrease. $N$ stays the same.
Here is a list of statements concerning quantum mechanics. Identify which are true (T) and which are false (F).

(a) (T) One needs quantum mechanics to explain the spectrum of blackbody radiation, as classical physics gives the wrong answer. This was the effect that prompted Planck to introduce his constant.

(b) (T) One needs quantum mechanics to explain the structure of atoms, as classical physics gives the wrong answer. Recall the Bohr model, and how we quantized $H$!

(c) (F) Quantum entanglement can be used to communicate superluminally. No way! If this were true quantum mechanics would blatantly contradict special relativity.

(d) (F) When Young’s double-slit experiment is performed with neutrons it does not produce an interference pattern, because neutrons are particles, not waves. Everything is a wave! People have performed this experiment and saw nice fringes.

(e) (T) Pauli’s exclusion principle states that two identical fermions cannot be in the same state. That’s right.

(f) (F) Pauli’s exclusion principle states that two identical fermions cannot be in the same spatial location. Fermions with different spins can be in the same location.

(g) (T) The De Broglie wavelength of a tennis ball is extremely small, and that’s why a tennis ball normally behaves like a particle, not like a wave. Recall $\lambda = h/p$ and realize that $h$ is a tiny constant.

(h) (T) The faster the wave function of a given particle oscillates in space, the more kinetic energy the particle has. The kinetic energy is determined by the second spatial derivative of the wave function. Cf. problem 2c.

(i) (T) The $x$, $y$ and $z$ components of the spin of an electron do not commute with each other, and that’s why we cannot measure all three components accurately at the same time.

(j) (F) The energy of a photon is $E = hc$, with $h$ Planck’s constant and $c$ the speed of light. Correct is $E = h\omega = h\nu = hc/\lambda$.

(k) (F) Heisenberg’s uncertainty principle for position and momentum does not apply to photons, because a photon is always moving at the speed of light. The speed of a photon may be fixed, but its momentum can be anything: recall $p = E/c = h\omega/c$.

(l) (F) A laser emits photons, but an ordinary light bulb does not. Light consists of photons, no matter where the light came from.