

## Final exam “Intro to Optics”

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### Problems & solutions

1 The standard 110V, 50Hz AC electric current from an outlet generates electromagnetic radiation with the same frequency of 50Hz.

(a) What is the *wavelength* of that radiation?

**answer:**  $\lambda = c/\nu = 3.0 \times 10^8 \text{m/s}/50\text{Hz} = 6.0 \times 10^6 \text{m}$ .

(b) What is the *energy* of one photon (in eV) with that wavelength? Can such a photon excite or ionize the hydrogen atom?

**answer:** The energy is  $h\nu = 2.1 \times 10^{-13}$  eV. This is *much* less than the energy required to excite the hydrogen atom, for which you need at least 10 eV, let alone to ionize it, for which you need at least 13.6 eV. You should know that the excitation energies of atoms are in the eV range.

(c) The wavelength of visible light ranges from  $\lambda_1 = 770\text{nm}$  down to  $\lambda_2 = 380\text{nm}$ . What colors do the wavelengths  $\lambda_1$  and  $\lambda_2$  correspond to, respectively? Why is that particular range of wavelengths visible?

**answer:**  $\lambda_1$  is deep red and  $\lambda_2$  is violet. These wavelengths correspond to single-photon energies in the eV range that can excite typical atoms or molecules, and that’s why they are visible: they can be detected by particular molecules in our eyes. For an alternative, less physical but more darwinistic answer, we can say our eyes adapted to the sun light, which is mostly in that same wavelength region.

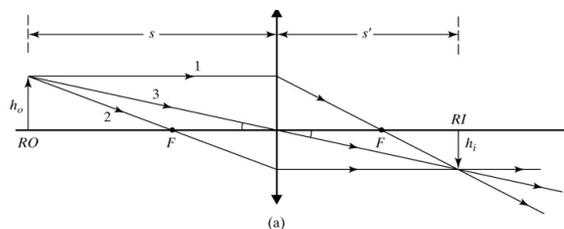
2 A candle stands 1m in front of a converging lens, with focal length  $f = 25\text{cm}$ .

(a) Where exactly should we put a screen in order to make the image of the candle visible? Is that image upside down?

**answer:** Use  $1/s + 1/s' = 1/f$  to get  $s' = +33.3\text{cm}$  (behind the lens). The image is upside down, because the magnification factor  $m = -s'/s = -1/3$  is negative.

(b) Draw three rays to illustrate how the image is formed.

**answer:** Not quite to scale for the above parameters, but this is Fig. 22(a) from Section 2



of the textbook.

(c) What happens when we move the screen closer to the lens? Is the image (i) larger and equally sharp, (ii) smaller and equally sharp, (iii) larger but blurred, (iv) smaller but blurred? Explain your choice!

**answer:** The image will become blurred because only at a distance  $s'$  is the image sharp. That does make it harder to say whether the image is smaller or larger, as the edge of the image won't be well defined anymore. Either by drawing rays or by making creative use of the relation  $m = -s'/s$  you'll see the bright part of the image gets smaller. On the other hand, the less bright part will be larger. Both (iii) and (iv) are not wrong!

(d)\* [extra credit] Suppose we block the upper half of the lens with an opaque piece of cardboard. Will we get (i) half the image, sharp, with the same brightness, (ii) the full image, sharp, but with half the brightness, (iii) a blurred full image, (iv) a blurred half image? Explain your choice!

**answer:** The cardboard blocks half of all rays, but the rays going through the lower half of the lens are sufficient to construct the full image (verify that yourself!). Moreover that image is still sharp as it is located at the correct image distance  $s'$  from the lens. So, (ii) is the right answer.

3 You are using a Fabry-Perot interferometer as a frequency filter. The length of the Fabry-Perot cavity is carefully stabilized to be “exactly”  $d = 10\text{cm}$ , and the mirrors you use are of high quality, with a reflection coefficient of  $r = 0.995$ .

(a) What is the *finesse*  $\mathcal{F}$  of the Fabry-Perot?

**answer:**  $\mathcal{F} = \pi r / (1 - r^2) = 313$ .

(b) What are the *free spectral range* and the *full-width half maximum* (both in Hz) of this Fabry-Perot?

**answer:**  $\text{FSR} = \nu_{fsr} = c/(2d) = 1.50 \times 10^9 \text{Hz}$  (=1.50 GHz). From that we get  $\text{FWHM} = \text{FSR}/\mathcal{F} = 4.79 \times 10^6 \text{Hz}$  (=4.79 MHz).

(c) For the following you may want to use  $\Delta\nu/\nu = \Delta\lambda/\lambda$ . You are using light with a wavelength of  $\lambda = 500.0000\text{nm}$  and a bandwidth  $\Delta\lambda = 6.5 \times 10^{-4}\text{nm}$ . How many lines can the

Fabry-Perot interferometer transmit within this bandwidth? What will the bandwidth of the transmitted (frequency-filtered) light be?

**answer:** First, for the unfiltered light we have  $\Delta\nu = \nu\Delta\lambda/\lambda = 780$  MHz. This is smaller than the FSR (see question (b)). That means *at most one* line will be transmitted. Since  $d$  is an integer multiple of  $\lambda$ , there will indeed be 1 transmitted line at that wavelength.

The frequency spread of the transmitted light will be equal to the FWHM from part (b). This corresponds to a bandwidth  $\Delta\lambda = \lambda\text{FWHM}/\nu = \lambda^2\text{FWHM}/c = 39.9 \times 10^{-16}\text{m} = 3.99 \times 10^{-6}\text{nm}$ , more than hundred times smaller than it was before filtering.

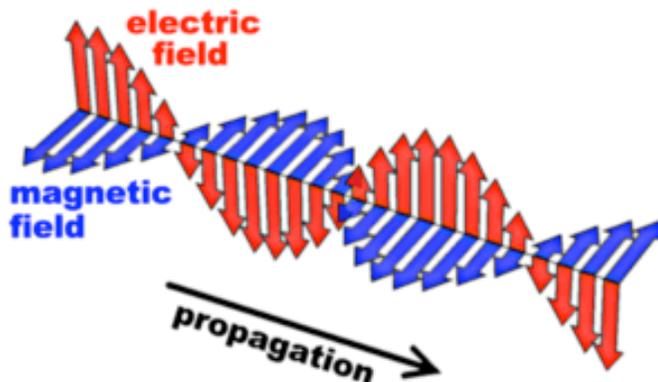
(d) Compare the (longitudinal) *coherence lengths* of the unfiltered and filtered light. Which is better for demonstrating interference of light in, for example, Young's double-slit experiment?

**answer:** For the unfiltered light we have  $l_t = \lambda^2/\Delta\lambda = 38.5\text{cm}$ , for the filtered light we have  $l_t = c/\text{FWHM} = 62.7\text{m}$ , more than a hundred times longer than for the unfiltered light. The latter is in principle better for demonstrating interference, namely when the interference involves optical path length differences longer than about 38 cm.

4 Suppose we have an electromagnetic wave of this form:

$$\begin{aligned}\vec{E} &= E_0\hat{x}\exp(ikz - i\omega t) \\ \vec{B} &= \frac{E_0}{c}\hat{y}\exp(ikz - i\omega t),\end{aligned}$$

where  $\hat{x}$  and  $\hat{y}$  are unit vectors in the  $x$  and  $y$  directions, resp., and  $E_0$  is the amplitude of the electric field. (See figure for an illustration.)



(a) What type of wave is this: plane, spherical, or cylindrical?

**answer:** plane wave: the phase of the wave is constant in *planes*  $z = \text{constant}$ .

(b) What polarization does this wave have, linear, circular, or elliptical? Write down the

corresponding Jones vector.

**answer:** The electric field component is always along the  $x$  direction, so the polarization is *linear* along the  $x$  direction. It is described by a Jones vector

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(c)\* [extra credit] Suppose the wave is a radio wave. In which direction should you point an antenna (which is just a straight thin metal wire) in order to receive the maximum signal? Why?

**answer:** You should point it along the  $x$  direction: the electrons in the wire experience the strongest force from the electric-field component of the (radio) wave, which is in the  $x$  direction. So you want to enable a current in the wire in the  $x$  direction.

(d) Suppose we send the wave through a polarizer that transmits linear polarization described by the following Jones vector:

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Write down the Jones matrix that describes this polarizer.

**answer:**

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

(e) The Jones vector from part (d) describes, as always, the direction of the *electric* field of the transmitted wave. What is the direction of the *magnetic* field of the transmitted wave?

**answer:** For electromagnetic waves the magnetic field is always perpendicular to the electric field, and perpendicular to the propagation direction. Thus, in this case, the magnetic field would have to be in the direction

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

that is, in a direction making an angle of 135 degrees with the  $x$ -axis.

- 5 You are, once more, performing Young's double-slit experiment. The distance between the two slits is  $a = 0.1\text{mm}$  and the size of the slits is  $b = 0.01\text{mm}$ . You put a screen 1m away from the slits, and you hope to see bright and dark fringes on the screen.

(a) Someone suggests to you to use a 100W light bulb for the experiment. Why would that be silly? Does it help if you put a linear polarizer between the light bulb and the two slits?

**answer:** The large bandwidth  $\Delta\lambda$  of the light from a light bulb or, equivalently, the very short coherence length [about half a visible wavelength only!] makes it *much* harder to see fringes in Young's double-slit experiment [you would be able to see fringes only when the path length difference is less than half a wavelength]. The intensity of the light bulb has nothing to do with this, nor does the unpolarized character of the light.

(b) Instead of a light bulb you use a He-Ne laser emitting light with a wavelength of  $\lambda = 633\text{nm}$ . What will the distance between the dark fringes on the screen be?

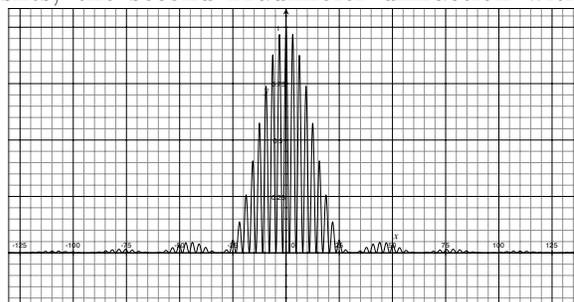
**answer:**  $\Delta y = \lambda L/a = 10^4 \lambda = 6.33\text{mm}$ .

(c) Explain qualitatively what *diffraction* (due to the finite (nonzero) size  $b$  of the slits) does to the fringe pattern.

**answer:** As long as  $b$  is much smaller than a wavelength, the interference pattern on the screen extends far on the screen. That is, the bright fringes far from the center still have (almost) the same intensity as the central fringes. When  $b$  becomes larger, the different rays of light from each *one* slit will interfere with themselves. That gives rise to the Fraunhofer diffraction pattern, and it makes far-away fringes less and less bright because there is more destructive interference from within one slit. See the next question/answer for the quantitative answer.

(d)\* [extra credit] Sketch the intensity on screen as a function of the vertical coordinate  $y$ . How many fringes will be visible in the central maximum?

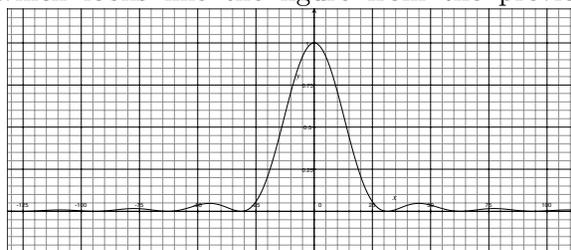
**answer:** Since the slit separation is 10 times more than the slit size, the 10th maximum of the Young interference pattern will coincide with the zero of the Fraunhofer diffraction. So in the central maximum you will have 19 (9 on each side plus the central maximum) bright fringes. Here is a plot of the function  $f(x) = \cos^2(x)\text{sinc}^2(x/10)$  (the first factor describes Young's interference pattern of 2 slits, the second Fraunhofer diffraction with



a slit 10 times smaller than the slit separation)

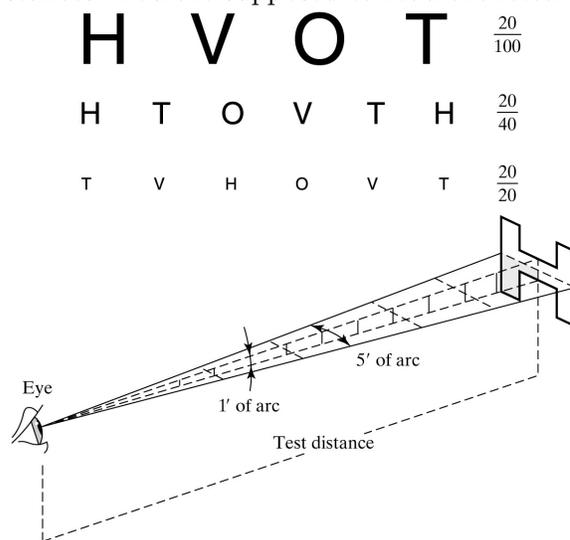
(e) What happens to the fringe pattern when you cover one of the two slits?

**answer** You won't have interference between the two slits anymore: all that is left is Fraunhofer diffraction of the single slit, which looks like the figure from the previous



question but without the rapid oscillations:

6 The Dutch ophthalmologist Snellen (not to be confused with the Dutch astronomer/mathematician Snellius for whom Snell's law is named) invented the standard vision test, the eye chart, in 1862. Letters of different size are used that subtend, at various different distances (200ft, 100ft, 80ft, . . . , 20ft, 10ft), an angle of 5' of arc (1' of arc  $\equiv$  1/60th of a degree). The *details* that you need to distinguish different letters have a size of 1' of arc at those distances. You are supposed to read the letters at a fixed distance of 20ft from



the eye chart.

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For example, if, from a distance of 20ft, you can only read the letters that subtend 5' of arc at a distance of 100ft, then your vision is "20/100". Snellen found that "normal" vision corresponds to being able to read letters that subtend 5' of arc at a distance of 20ft. Hence "20/20" corresponds to normal vision.

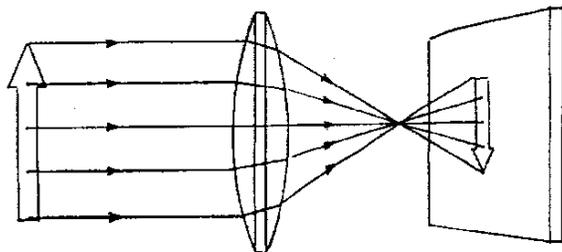
(a) Given that a *resolution* of 1' of arc corresponds to an angle of  $29 \times 10^{-5}$ rad [you may verify this, for extra credit] what is the pupil size consistent with that resolution, if you assume a (green) wavelength of  $\lambda = 550$ nm?

**answer:**  $1'$  of arc =  $1/60$  times  $2\pi/(360) = 29 \times 10^{-5}$  rad. Using the Rayleigh criterion (derived from Fraunhofer diffraction)  $\Delta\theta_{\min} = 1.22\lambda/D$  for the minimum angular resolution, with  $D$  the diameter of the pupil, gives  $D = 2.3\text{mm}$ .

(b) Is your resolution better or worse with *red* light? Is your resolution better or worse in the dark?

**answer** Using the same criterion,  $\Delta\theta_{\min} = 1.22\lambda/D$ , you see that a larger (redder) wavelength corresponds to a larger angle, hence to a *worse* resolution. In the dark your pupil size  $D$  will increase, hence the angle you can resolve becomes smaller, hence your resolution becomes *better*. It's a pity there is also less light to profit from this...(Birds like the owl have better resolution than people because their pupils are larger. They also have many more "rods" on their retina so they are more sensitive to light and can see better in the dark. They don't have so many "cones" so they do miss out on color vision.)

7\* [extra credit] The following is a drawing from an elementary school textbook, "explaining" how an image is formed by a converging lens. Identify aspects of this drawing that are misleading. That is, find *wrong* conclusions implied by this drawing.



**answer:** All rays drawn from the object are parallel to the optical axis. That seems to imply the image will be sharp no matter at what distance you put the screen. Moreover, it seems to imply you don't even need a lens to get a sharp image (in fact, the image without lens would seem to be right side up and equal size as the object). It also seems to imply that if you block half the lens you get only half the image. But *all* of these implications are wrong!