Generalized ‘t Hooft anomalies

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arXiv:1805.12290, 1909.09027, 2002.02037, with Erich Poppitz
2008.05491 with Stephen Baker

Outline

- Introduction.
- New ‘t Hooft anomalies: construction and examples.

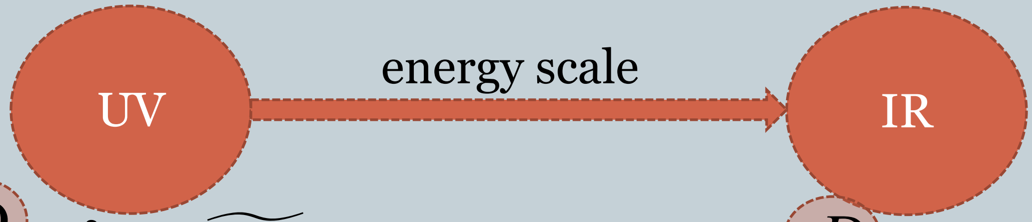
Why ‘t Hooft anomaly?

- ‘t Hooft anomalies: study nonperturbative phenomena in strongly coupled QFT. compare to Large-N AdS/CFT, lattice
- They put sever constraints on the IR spectrum of asymptotically free theories (composite quarks and leptons, dualities). Review by Rosner, 1998 Seiberg 1994
- Ordinary ‘t Hooft anomalies (0-form) were known since the 80s.
- New generalized ‘t Hooft anomalies → new constraints on the spectrum of a theory.

Gaiotto, Kapustin, Seiberg, Willett, 2014
Gaiotto, Kapustin, Komargodski, Seiberg, 2017

What is ‘t Hooft anomaly?

- Given a global symmetry G of a QFT, we may try to gauge G. (turn on a **background field** of G)
- **If obstructed** (anomalous), the theory has an **‘t Hooft anomaly**. ‘t Hooft, 1980 Frishman, Schwimmer, Banks, Yankielowicz 1981
- ‘t Hooft anomaly is RG invariant, useful for asymptotically free theories.



$$\frac{D}{32\pi^2}\int F_{\mu\nu}\widetilde{F^{\mu\nu}} \qquad \frac{D}{32\pi^2}\int F_{\mu\nu}\widetilde{F^{\mu\nu}}$$

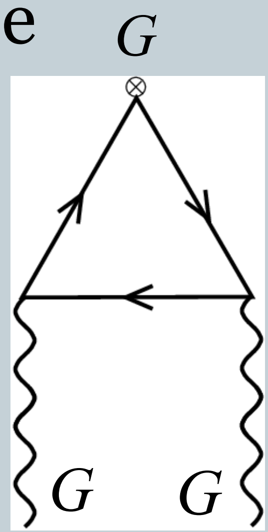
What is 't Hooft anomaly?

- **The anomaly is unremovable phase** in the partition function upon the transformation:

G is a 0 − form symmetry $\lambda \to U\lambda, \quad U = e^{i\varepsilon^a T^a}$

$$\left[D\lambda\right]\left[D\bar{\lambda}\right] \to e^{-2i\varepsilon D Q^T}\left[D\lambda\right]\left[D\bar{\lambda}\right] \qquad Q^T = \frac{1}{32\pi^2}\int F_{\mu\nu}\widetilde{F_{\mu\nu}}$$

fields of G



- Also, G can be $G = G_1 \times G_2 \times \ldots$: mixed anomalies.
- In all the **traditional** (0 − form) anomalies we take

$Q^T \in \mathbb{Z}$ a la BPST instantons

What is ‘t Hooft anomaly?

- The UV/IR matching of the anomaly: (1) CFT, (2) composite massless fermions, (3) Goldstone bosons (or domain walls). **Unique gapped vacuum is excluded**.

- E.g. QCD with 3 fundamentals: $G = U(1)_B \times SU(3)_L \times SU(3)_R$

- Anomalies: $\left[SU(3)_L\right]^3 = N_c$, $U(1)_B\left[SU(3)_L\right]^2 = N_c$.

- The matching is via Goldstone bosons:

$$SU(3)_L \times SU(3)_R \to SU(3)_V$$ Witten, 1983

What is ‘t Hooft anomaly?

- The option $Q^T \in \mathbb{Z}$ is **not** the most general one.
- In 2014 it was realized that $Q^T \in \mathbb{Q}$ leads to new generalized anomalies.

  Gaiotto, Kapustin, Seiberg, Willett, 2014
- Generalized ‘t Hooft anomalies can play many important roles.

New ‘t Hooft anomaly

New 't Hooft anomaly

- We consider a $SU(N_c)$ Yang-Mills theory with fermions in some irr. rep. R

- Define the theory: we sum over $SU(N_c)$ gauge connections

$$Z = \underbrace{\int \left[DA_\mu \right] \left[D\lambda \right] \left[D\bar{\lambda} \right]}_{\text{sum over } SU(N_c) \text{ connections}} e^{-S}$$

$$Q^T \in \mathbb{Z}$$

New 't Hooft anomaly

Main question

- Given YM theory with fermions in Irr. rep., what is the most **general background** we can turn on?

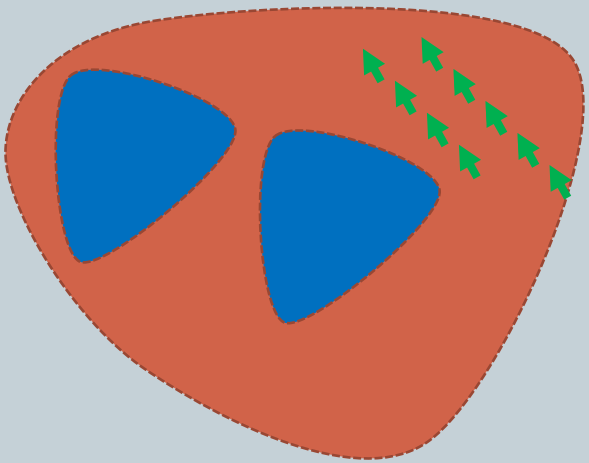
$$Z \to Z[\text{background} = \text{source}]$$

- The background has to be **compatible** with the theory. In general:

$$Q^T_{\text{background}} \in \mathbb{Q}$$

New ‘t Hooft anomaly

- The background: manifold (metric) + G-bundle (gauge connections)



🔶General Lore: Examine all possible anomalies due to gauging internal + spacetime (continuous and discrete) symmetries.

New 't Hooft anomaly

- As an example, we discuss turning on a background in the center of $G = SU(N_c)$.

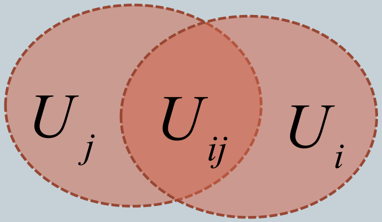
$$Z_{N_c} = e^{i\frac{2\pi k}{N_c}} I_{N_c \times N_c} \qquad k = 0,1,2,\ldots,N_c - 1$$

- Depending on the representation, the fermions may not see the full Z_{N_c}. E.g., adjoints.

1-form 't Hooft anomaly

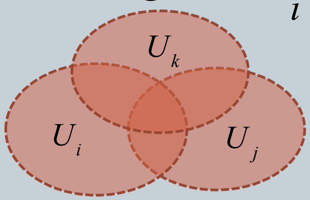
- We take a manifold $\mathtt{M}$ with fermion λ in Rep. R of a group G defined on a collection of covers $\{U_i\}$ of $\mathtt{M}$.

- We also take transition functions g_{ij}^R of G on the overlap $U_{ij} = U_i \cap U_j$:



$$\lambda_i = \left(g_{ij}^R\right)^{-1} \lambda_j$$

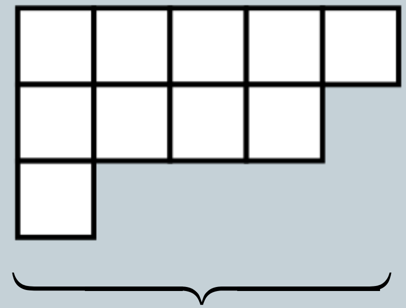
- The cocycle condition $g_{ij}^R g_{jk}^R g_{ki}^R = 1$ on $U_i \cap U_j \cap U_k$.



1-form 't Hooft anomaly

- If we take $G = SU(N_c)$ and g_{ij} in the defining Rep.:

$$g^R_{ij} \sim \underbrace{g_{ij} g_{ij} \cdots g_{ij}}_{\text{number of boxes}=n}$$



number of boxes=n

- Only the Nality= $n \bmod N_c$ matters.

- **Side note**: a similar construction on the lattice

$$\chi^*_x \left(U_{x,\mu}\right)^n \chi_{x+\mu}$$

1-form ‘t Hooft anomaly

- We consider the center elements:

$$Z_{N_c} : g_{ij} \sim e^{i\frac{2\pi k}{N_c}} I, \quad k = 0,1,\ldots,N_c - 1$$

- In Rep. R: $g_{ij}^R \sim e^{i\frac{2\pi k}{N_c}n} I$.

- If $\gcd(N_c, n) = p > 1$, then the faithful Rep. is $SU(N_c)/Z_p$ and the cocycle condition:

$$g_{ij}^R g_{jk}^R g_{ki}^R = 1 \quad \Rightarrow \quad g_{ij} g_{jk} g_{ki} = e^{i\frac{2\pi}{p}n_{ijk}}$$

New ‘t Hooft anomaly

- For adjoint fermions and $\mathbb{M}$ to be a 4-torus.

- An explicit background is constructed as follows: gauge connection in the 1-2 plane (similar expression in the 3-4 plane)

$$A_1 = \frac{2\pi m_{12}}{L^2}\vec{H}.\vec{v}x^2,\; A_2 = 0$$

$$m_{12} = 0,\dots,N_c - 1$$

Cartan generators weights of defining rep.

$$\vec{H} = \begin{bmatrix} \vec{v}_1 & & & & \\ & \vec{v}_1 - \vec{\alpha}_1 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \vec{v}_1 - \vec{\alpha}_1 - \dots - \vec{\alpha}_{N_c-1} \end{bmatrix}$$

New 't Hooft anomaly

- Then

$$F_{12} = -\frac{2\pi m_{12}}{L^2}\vec{H}.\vec{\nu}_1,\; F_{34} = -\frac{2\pi m_{34}}{L^2}\vec{H}.\vec{\nu}_1,$$

$$\vec{H}.\vec{\nu}_1 = -\frac{1}{N_c} + \text{integer}$$

$$\Bigg\} \quad Q^T = \frac{1}{32\pi^2}\int_{T^4} F_{\mu\nu}\widetilde{F^{\mu\nu}} = m_{12}m_{34}\left(1 - \frac{1}{N_c}\right)$$

- There is an anomaly between the discrete chiral symmetry $Z_{2N_cN_f}$ and the center background Z_{N_c} .

$$Z \to e^{-i\frac{2\pi}{N_c}} Z$$

New 't Hooft anomaly

- For $SU(N_c = 2)$ and $N_f = 2$ there are 3 scenarios:

M.A., Poppitz, 2018
Cordova, Dumetriscu, 2018
Bi, Senthil, 2018

(1) $SU(2)_f \times Z_8 \to SO(2) \times Z_2$

(2) $SU(2)_f \times Z_8 \to SU(2)_f \times Z_4 \times \mathrm{TQFT}$

(3) CFT

Currently under investigation by the lattice groups at Jena and MIT.

(4) (excluded) $SU(2)_f \times Z_8 \to SU(2)_f \times Z_8 \times \mathrm{TQFT}$

Proposed by: Bi, Senthil, 2018

Excluded by: Wan, Wang, 2018
Cordova, Ohmori, 2019

New ‘t Hooft anomaly

- In scenario (2) the $0-$ form anomalies are matched via composite massless fermions~$\lambda\lambda\lambda$ transforming in fund. of $SU(2)_f$. M.A., Poppitz, 2018

- The breaking $Z_8 \to Z_4$ is probed via $\langle \lambda\lambda\lambda\lambda \rangle \neq 0$, while $\langle \lambda\lambda \rangle = 0$ matching the new anomaly.

New ‘t Hooft anomaly

- When you **couple the axions** to a gauge theory, new phenomena happen in the IR (a new anomaly in the color-flavor-baryon-number backgrounds).
- Deconfinement on axion domain walls.
- Modifying models of natural inflation.

$$V(a) = \Lambda_{QCD}^4\left[1-\cos\left(\frac{a}{f}\right)\right] \Rightarrow V(a,\sigma) = \Lambda_{QCD}^4\left[2-\cos\left(\frac{a}{f}-\frac{\sigma}{\Lambda_{QCD}}\right)-\cos\left(\frac{a}{f}+\frac{\sigma}{\Lambda_{QCD}}\right)\right]$$

breaks down for large-field excursions
Hadronic physics is important!

M.A., Poppitz 2020
M.A., Baker 2020

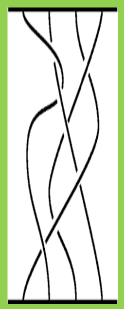
Deconfinement on axion domain walls

- Anomaly inflow:

M.A., Poppitz 2020

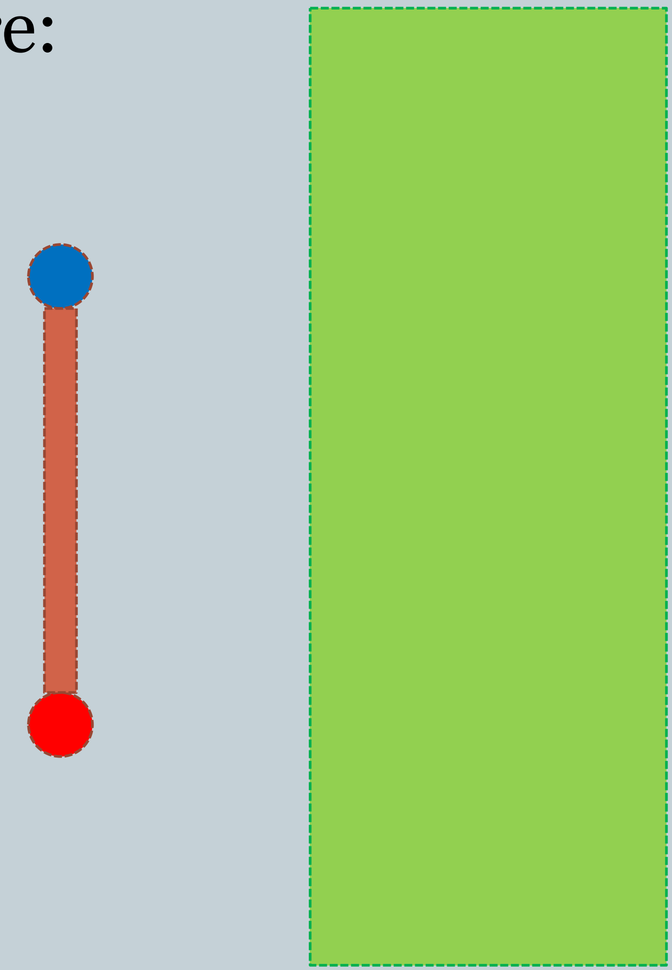
Chern-Simons theory!

anomaly



Deconfinement on axion domain walls

- The picture:



M.A., Poppitz 2020

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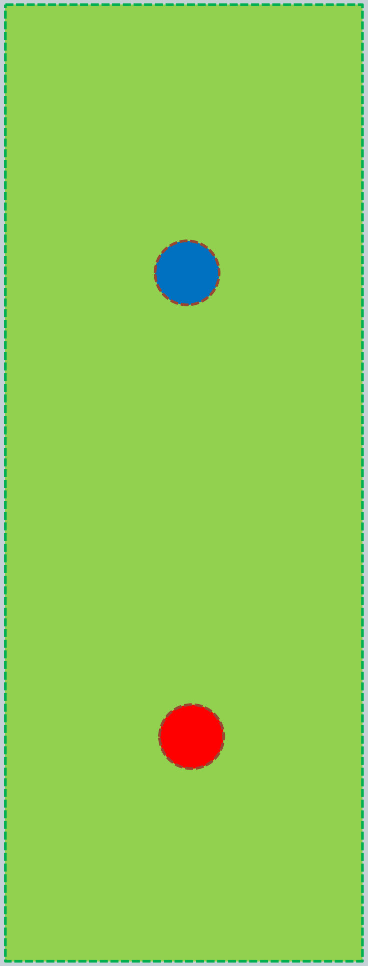

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Deconfinement on axion domain walls

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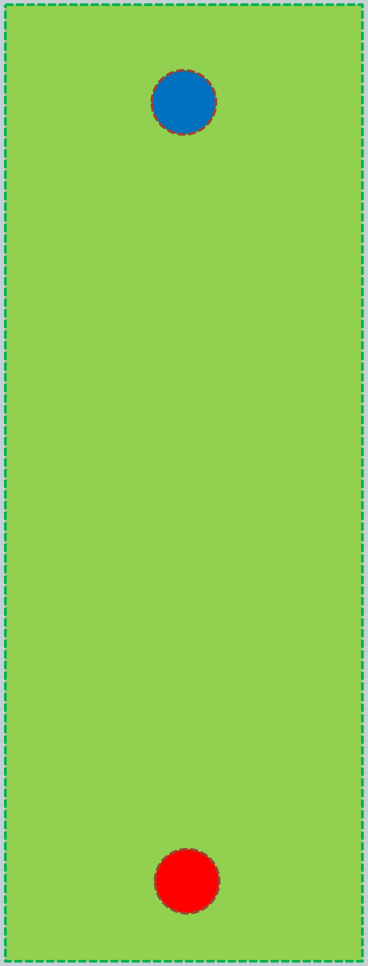

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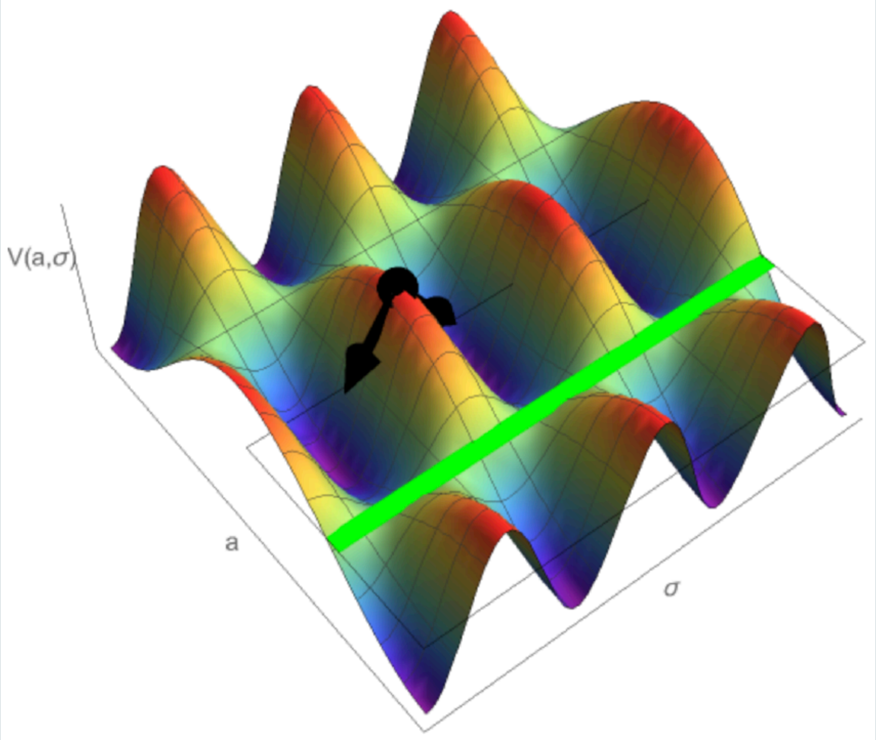
Modifying models of natural inflation

- Hadronic DOF play an important role in models of axion inflation



M.A., Baker 2020