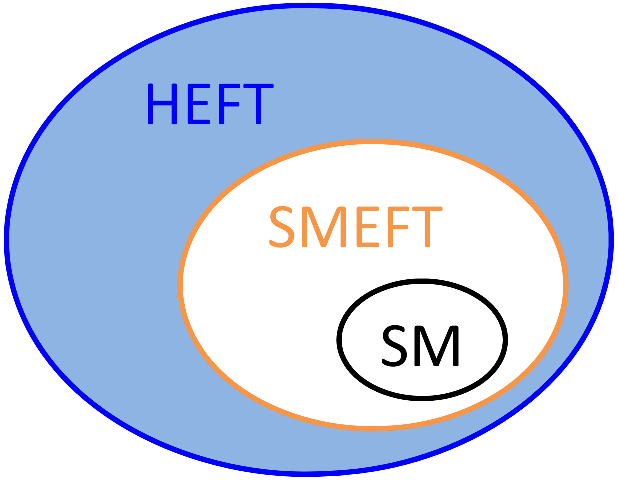
# SMEFT vs HEFT

Pacific NW Theory Workshop, Sep 21-25, 2020

Xiaochuan Lu

University of Oregon

arXiv: 2008.08597,
with Timothy Cohen, Nathaniel Craig, and Dave Sutherland

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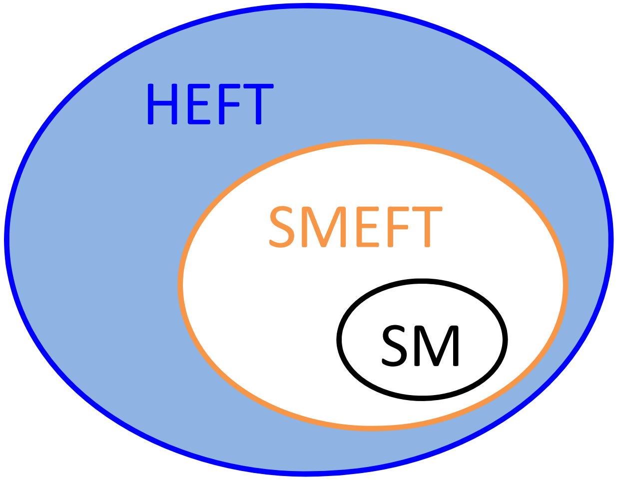


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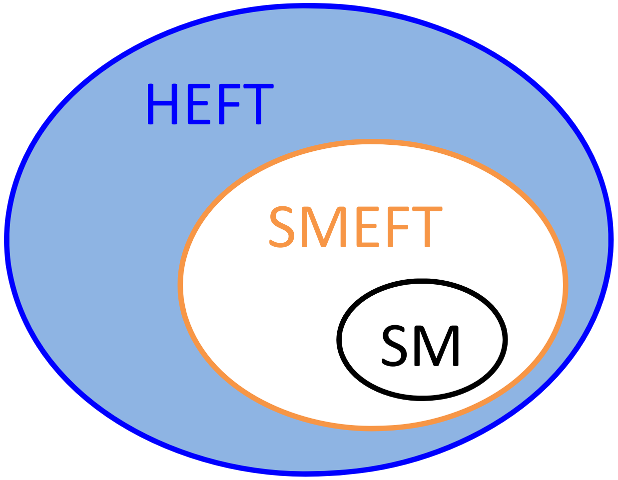


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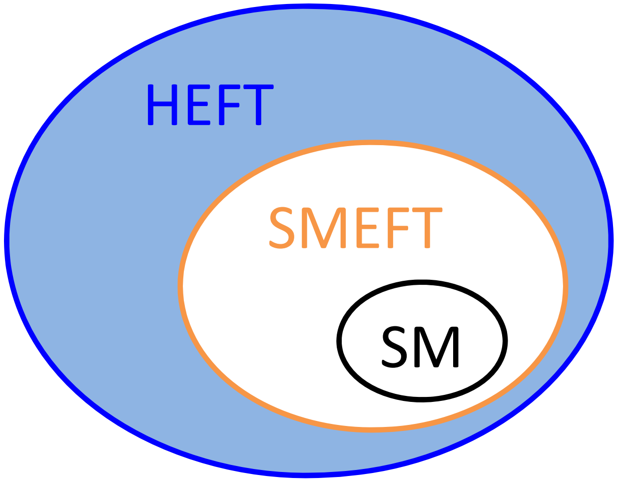


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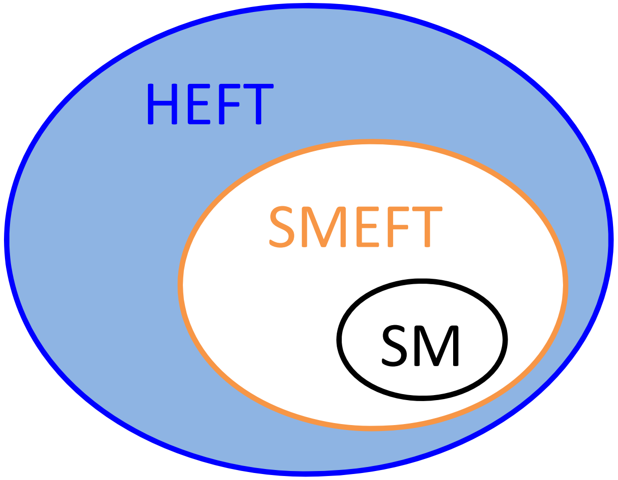


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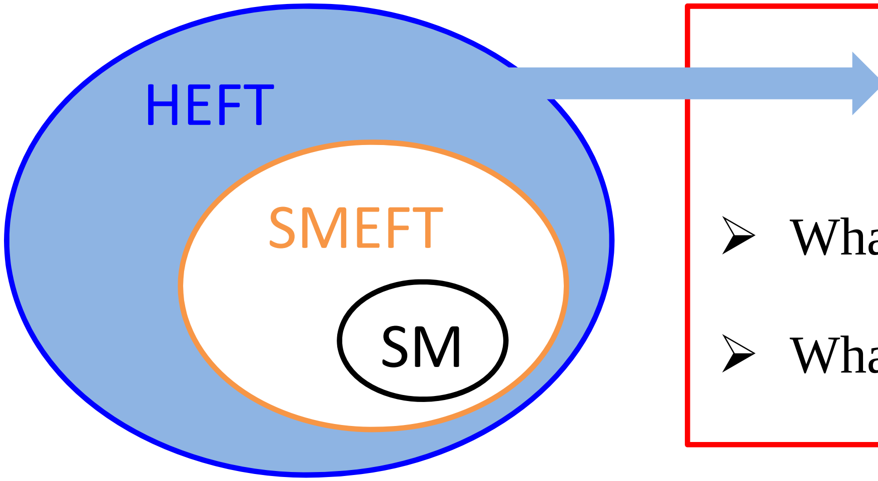


# Outline

- What is HEFT?
- What UV physics generate HEFT?

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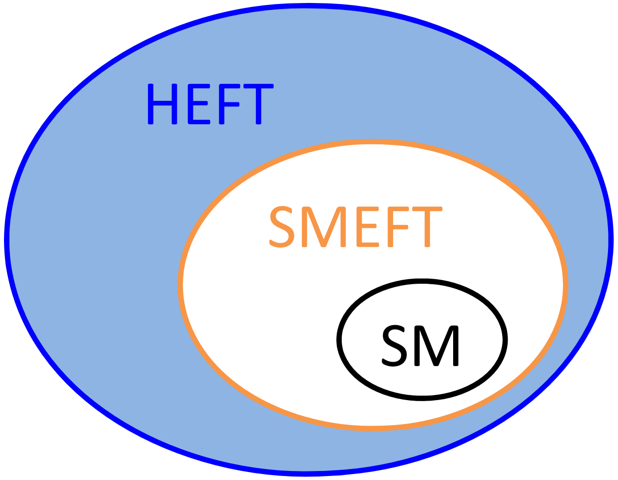



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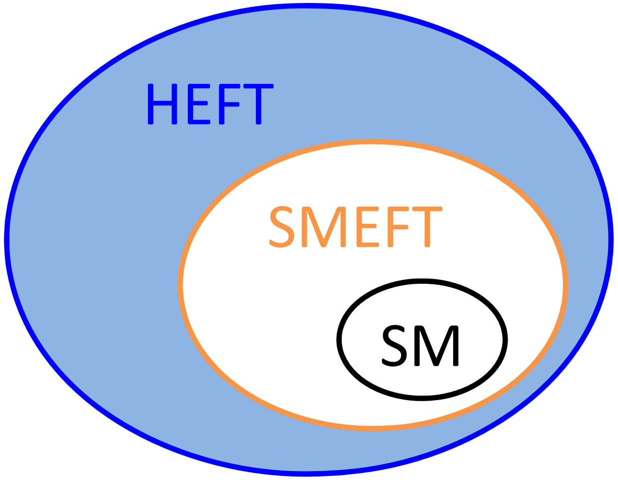


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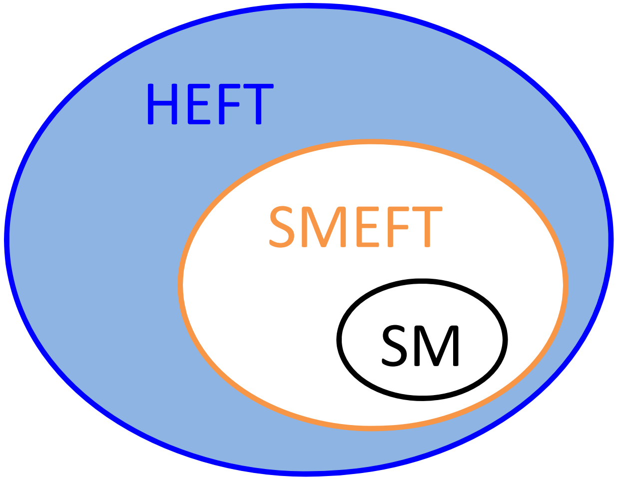



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$$\begin{cases} \text{SMEFT:} & (x, y) \\ \text{HEFT:} & (r, \theta) \end{cases} \Rightarrow \quad r = \sqrt{x^2 + y^2}$$

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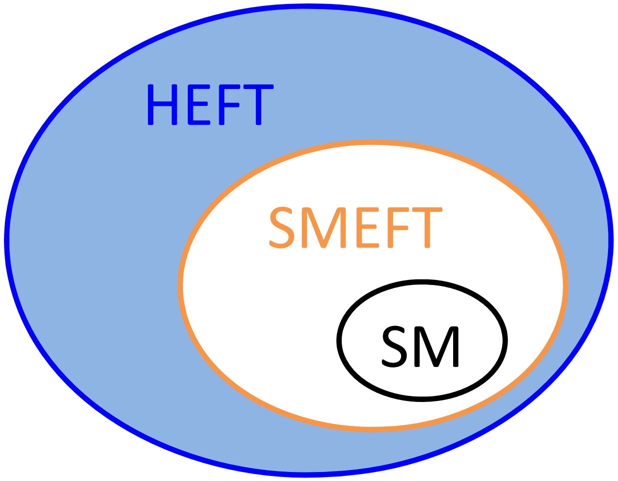


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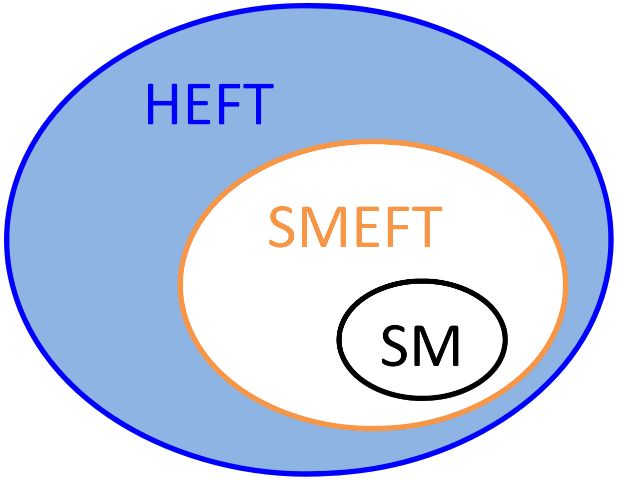


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$$\sqrt{x} = \sqrt{x_0 + (x - x_0)}$$
$$= \sqrt{x_0} + \frac{1}{2\sqrt{x_0}}(x - x_0) + \cdots$$

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## Adam Falkowski and Riccardo Rattazzi: (arXiv: 1902.05936)

UV cut-off. Our distinction between analytic and non-analytic lagrangians coincides with the distinction, in use in the Higgs EFT community, between linear (so-called SMEFT) and non-linear (so-called HEFT) effective theory, or equivalently between $h$ being or not being part of a $SU(2)_W$ doublet. We however believe our classification is more adequate and enlightening from a physical point of view.

$$V(H) \supset \sqrt{2H^\dagger H} = v + h$$

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Field redefinition:
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$$= \left(v_1 + h_1\right)^2 = 2H_1^\dagger H_1$$

## Rodrigo Alonso, Elizabeth E. Jenkins, Aneesh V. Manohar: (arXiv: 1605.03602)

One can convert the SMEFT Lagrangian to HEFT form using Eq. (2.11) to switch from Cartesian and polar coordinates. One can attempt to convert from HEFT to SMEFT form using

$$\frac{\phi}{(\phi\cdot\phi)^{1/2}} = \boldsymbol{n} \tag{2.30}$$

with $(\phi\cdot\phi)^{1/2}$ some function of $h$. This substitution gives a Lagrangian $L(\phi)$ that need not be analytic in $\phi$. However, if there is an $O(4)$ fixed point, then there is a suitable change of variables such that the resulting Lagrangian is analytic in $\phi$.

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi^1 + i\phi^2 \\ \phi^3 + i\phi^4\end{pmatrix}$$

$$\mathcal{L}_{\text{Kinetic}} = \frac{1}{2} g_{ij}(\phi)\left(\partial_\mu \phi^i\right)\left(\partial^\mu \phi^j\right)$$

$$\Downarrow$$

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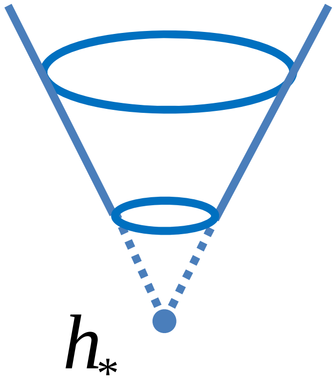
$\exists\, h_*$ such that $F(h_*) = 0$

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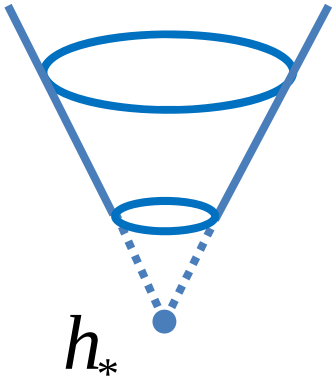


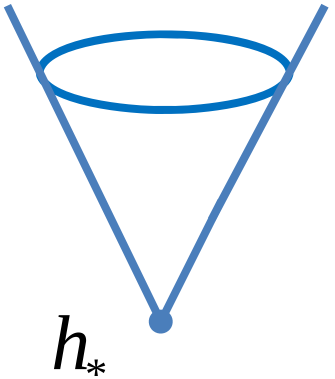


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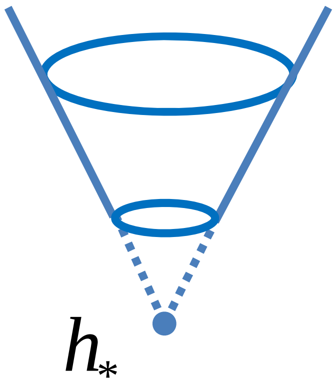


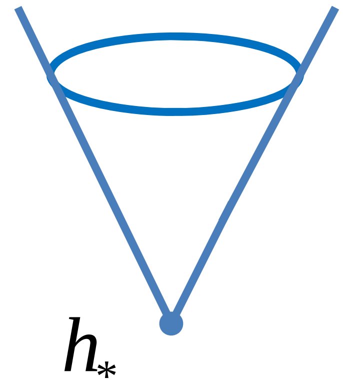


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us (arXiv: 2008.08597)

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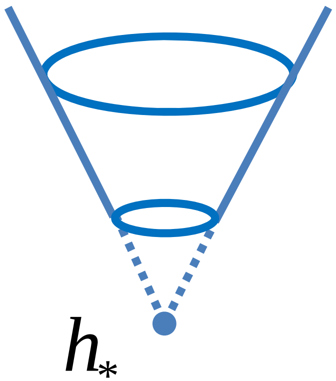
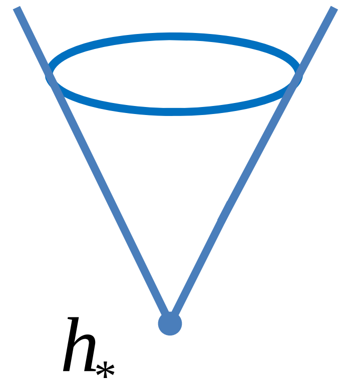
# UV theories that will generate HEFT?

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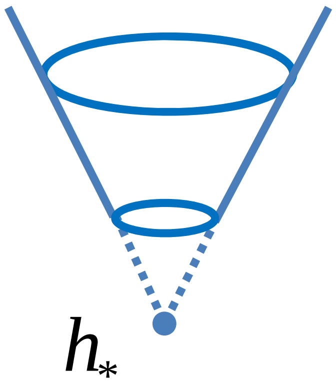


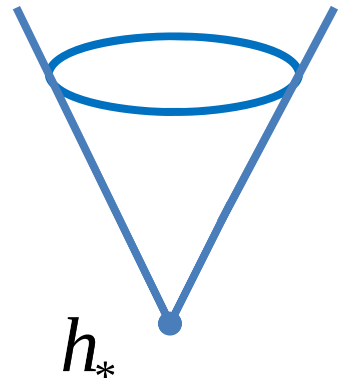


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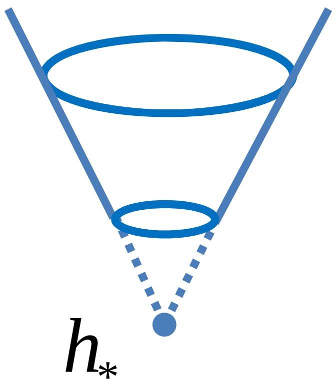


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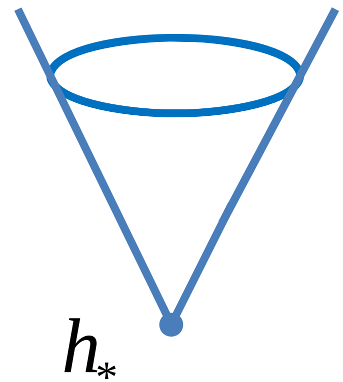


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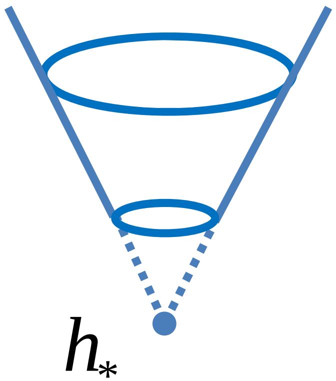


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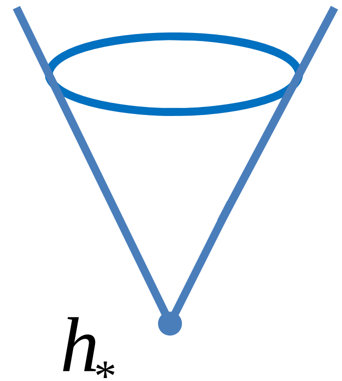


Leading Order Criterion

AJM (arXiv: 1605.03602)

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# Matching to all-order in fields

Coleman-Weinberg: $\mathcal{L}_{\text{UV}} \supset -\frac{1}{2}\Phi\left[\partial^2 + M^2 + U\left(\phi\right)\right]\Phi$

$$i \ln\det\left(\partial^2 + M^2 + U\right) = i\mathrm{Tr}\ln\left(\partial^2 + M^2 + U\right)$$

$$= i\int d^4x \int \frac{d^4p}{\left(2\pi\right)^4} \mathrm{tr}\left[\ln\left(-p^2 + M^2 + U\right)\right]$$

$$= \int d^4x \frac{1}{16\pi^2} \mathrm{tr}\left[\frac{1}{2}\left(M^2 + U\right)^2 \left(\ln\frac{\mu^2}{M^2 + U} + \frac{3}{2}\right)\right]$$

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If $\left[U, \partial_\mu U\right] \neq 0$: see App. D in 2008.08597

# Example: A heavy Singlet

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} - \frac{1}{2} S\left(\partial^2 + M^2 + \kappa |H|^2\right) S \qquad , \qquad m_S^2 = M^2 + \frac{1}{2}\kappa v^2$$

$$M^2 > 0 \quad , \quad \text{SMEFT} \qquad\qquad M^2 = 0 \quad , \quad \text{HEFT}$$

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$$R = \frac{1}{16\pi^2}\left[\frac{1}{2K^4}\frac{\kappa^2 M^2}{\left(M^2 + \kappa|H|^2\right)^2} + \frac{1}{2K^2}\frac{\kappa^2}{M^2 + \kappa|H|^2}\right] \quad , \quad K^2 \equiv 1 + \frac{1}{96\pi^2}\frac{\kappa^2|H|^2}{M^2 + \kappa|H|^2}$$

# Summary

- A geometric criterion to tell a HEFT Lagrangian from SMEFT
- An understanding of what UV theories would generate HEFT
- A functional method to match to all order in *fields* (not *derivatives* yet)
- A few UV examples show that the leading order criterion works