Gravitational Effects of Radiation on Large Scale Structure

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Structure Formation

The theory of Structure Formation explains how density fluctuations in the early universe evolve into structures like galaxies or galaxy clusters observed today.

Important observables include correlation functions of the fluctuations in the matter density $\delta_m = \delta\rho_m/\overline{\rho}$:

power spectrum :

$$P(k)\delta^3(\vec{k}+\vec{k'}) = \langle \tilde{\delta}_m(\vec{k})\tilde{\delta}_m(\vec{k'})\rangle$$

or the *bispectrum* :

$$B(\vec{k},\vec{k'},\vec{k''})\delta^3(\vec{k}+\vec{k'}+\vec{k''}) = \langle \tilde{\delta}_m(\vec{k})\tilde{\delta}_m(\vec{k'})\tilde{\delta}_m(\vec{k''})\rangle$$

Alternatively one can study the clustering of galaxies or galaxy clusters using the halo model.

Effect of Radiation

Radiation involves massless particles which are free-streaming in the current epoch.

This introduced a new length scale relevant to structure formation - the free-streaming scale For massless particles this is just the particle horizon :

$$\lambda_{fs} = \left(\frac{3}{2}Ha\right)^{-1}$$

Evolution of matter density fluctuations is suppressed at scales below the free-streaming scale as compared to evolution of matter density fluctuations.

Matter fluctuations therefore evolve in a scale-dependent way $\delta_m = \delta_m(a, k)$

Effect of Radiation

The process of structure formation is highly non-linear - the formation of structures on small scales is affected by large scale density perturbations. Massless particles influence this coupling between small and large scales in a non-trivial way.

One can study this effect by computing the response of small scale observables to large scale matter density perturbations using the Separate Universe (SU) technique.

Separate universe technique

The long wavelength perturbation $\delta_m(a, k_L)$ is absorbed into the background cosmic expansion to yield a set of 'local' cosmological parameters describing a "*Separate Universe*" (SU).

$$\bar{\rho}_w(a) = \bar{\rho}(a)\,(1 + \delta_m(a, k_L)) \quad \text{(local matter density)}$$

$$a_w = a\left(1 - \frac{1}{3}\delta_m(a, k_L)\right) \quad \text{(local scale factor)}$$

$$H_w = H\left(1 - \frac{1}{3}\frac{\partial \delta_m(a, k_L)}{\partial ln(a)}\right) \quad \text{(local Hubble rate)}$$

Separate universe simulations

We perform N-body simulations in SUs at two scales $k_{L\uparrow} = 0.0005\ \mathrm{Mpc}^{-1}$ and $k_{L\downarrow} = 0.05\ \mathrm{Mpc}^{-1}$.

We compute the response of small scale observables to the large scale density perturbation $\delta_m(a, k_L)$.

The scale dependence of $\delta_m(a, k_L)$ causes the response of small scale perturbations to become scale dependent. This is the primary effect that we investigate here.

We use $N_\nu = 28$ massless neutrinos in addition to the CMB photons as the extra radiation component in the universe.

Power spectrum response

The matter power spectrum in the SU differs from that in the global universe. The response

$$\Delta \log(P) = \left(R_{growth}(k_L, k) + R_{SU}(k)\right) \delta_m = R_{tot} \delta_m \quad (1)$$

encodes this difference

In what follows, we refer to R_{growth} as the “growth response”

The power spectrum response can be used to compute the squeezed limit bispectrum

$$\lim_{k_L \to \infty} B(k, k', k_L) = B^{sq}(k, k_L) = R_{tot} P(k_L) P(k) \quad (2)$$

Power spectrum response

Computed using one-loop perturbation theory, the power spectrum response for different SUs should have the same dependence on small scales k with an overall scaling dependent on the large scale k_L

Figure: Growth response at z=1

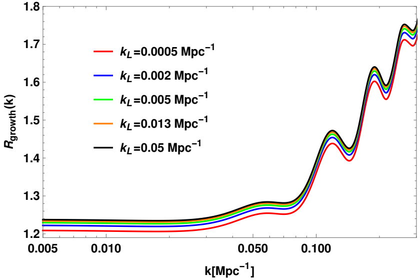
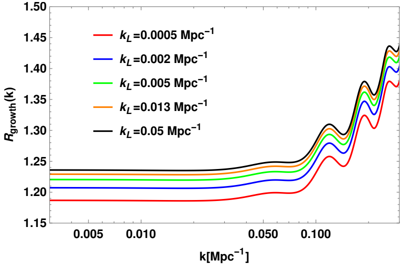


Figure: Growth response at z=3



Difference between massless neutrinos and photons?

The relative growth response for various k_L does not vary significantly for different massless neutrino density as long as the total radiation density remains constant. Clearly massless neutrinos are equivalent to photons in this respect.

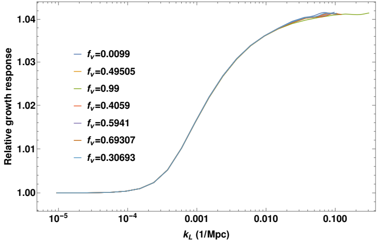


Figure: Growth response vs k_L for $\Omega_{rad} = 0.00101$ at $z = 0$

The figure on the left shows that the difference between the relative growth response for photons and neutrinos (with the same total energy density) is $\lesssim 0.05\%$

Power spectrum response from simulations

The one loop calculations agree with the results of our simulations in their regime of validity

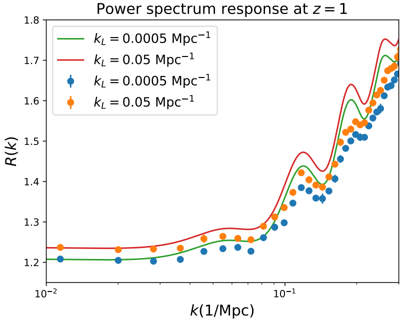


Figure: Power Spectrum Response for different k_L at $z = 1$

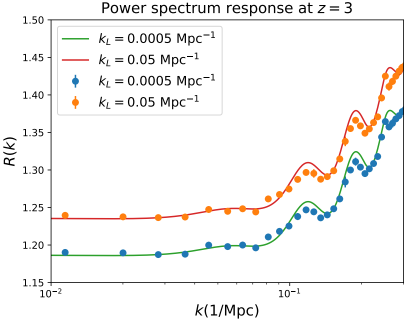


Figure: Power Spectrum Response for different k_L at $z = 3$

Scale dependent halo bias

The cumulative halo mass function $n(M)$ encodes the average mass-energy density of halos of mass $\geq M$

In the separate universe, we compute the (scale dependent) *lagrangian response bias* :

$$\overline{b}_L(M, k_L) = \frac{\Delta \log n(M)}{\Delta \delta_c}(k_L)$$

Computing the response bias is equivalent to computing the clustering bias of halos

$$b(k, M) = \frac{P_{h\delta}(k, M)}{P_{\delta\delta}(k)}$$

The scale dependence of the evolution of δ_c thus translates into the scale dependence of the clustering bias

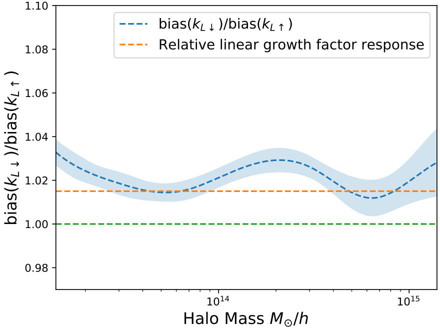


Figure: Relative Lagrangian bias between $k_{L\uparrow}$ and $k_{L\downarrow}$ at $z = 0$. The dashed orange line shows the ratio of the power spectrum growth responses

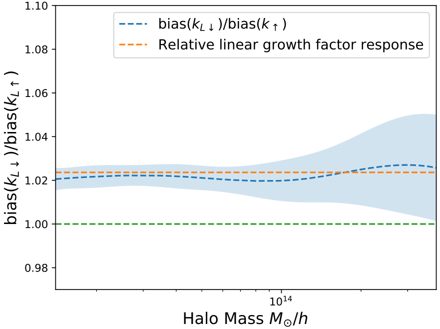


Figure: Relative Lagrangian bias between $k_{L\uparrow}$ and $k_{L\downarrow}$ at $z = 1$. The dashed orange line shows the ratio of the power spectrum growth responses

Bias models

As shown in the previous slide, the scale dependence of the cumulative lagrangian bias can be modelled by the scale dependence of the power spectrum growth response to reasonable accuracy

$$\frac{\overline{b}_L(M, k_{L\downarrow})}{\overline{b}_L(M, k_{L\uparrow})} = \frac{R_{growth}(k_{L\downarrow})}{R_{growth}(k_{L\uparrow})} \tag{3}$$

This follows from the Ansatz that the cumulative halo mass function is a universal function of the power spectrum so that

$$\overline{b}_L(M, k_L) = \frac{d \log n[M; P_W]}{d\delta_c} \propto R_{growth}(k_L) \tag{4}$$

Effects on observables

For the more realistic case of $N = 3$ massless neutrinos, the scale dependence of the halo bias is most relevant for objects with a high bias and thus, with a high mass

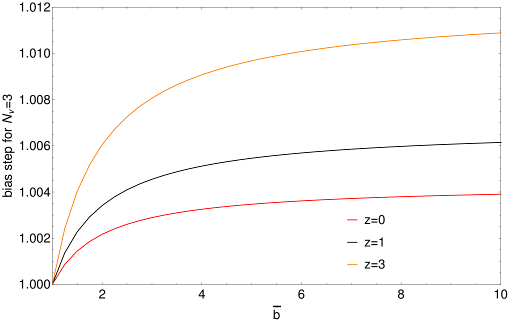


Figure: Eulerian bias step between $k_{L\downarrow}$ and $k_{L\uparrow}$ at $z = 0, 1, 3$ computed for $N_\nu = 3$ massless neutrinos

Summary

- Free-streaming components in the universe cause the evolution of matter density perturbations to become scale-dependent. In addition, the response of small scale observables to large scale density perturbations becomes scale-dependent as well.
- In particular, the radiation-induced response of the Lagrangian halo bias and the power spectrum acquire a scale dependence that can be adequately explained by the one-loop power spectrum growth response computed analytically.
- Massless neutrinos and photons affect the coupling between structure formation at large and small scales in the same way - i.e. the gravitational effects of radiation are independent of its composition.
- The scale dependence of the Eulerian bias thus obtained becomes more important in cosmological observations of objects with high bias at higher redshifts.