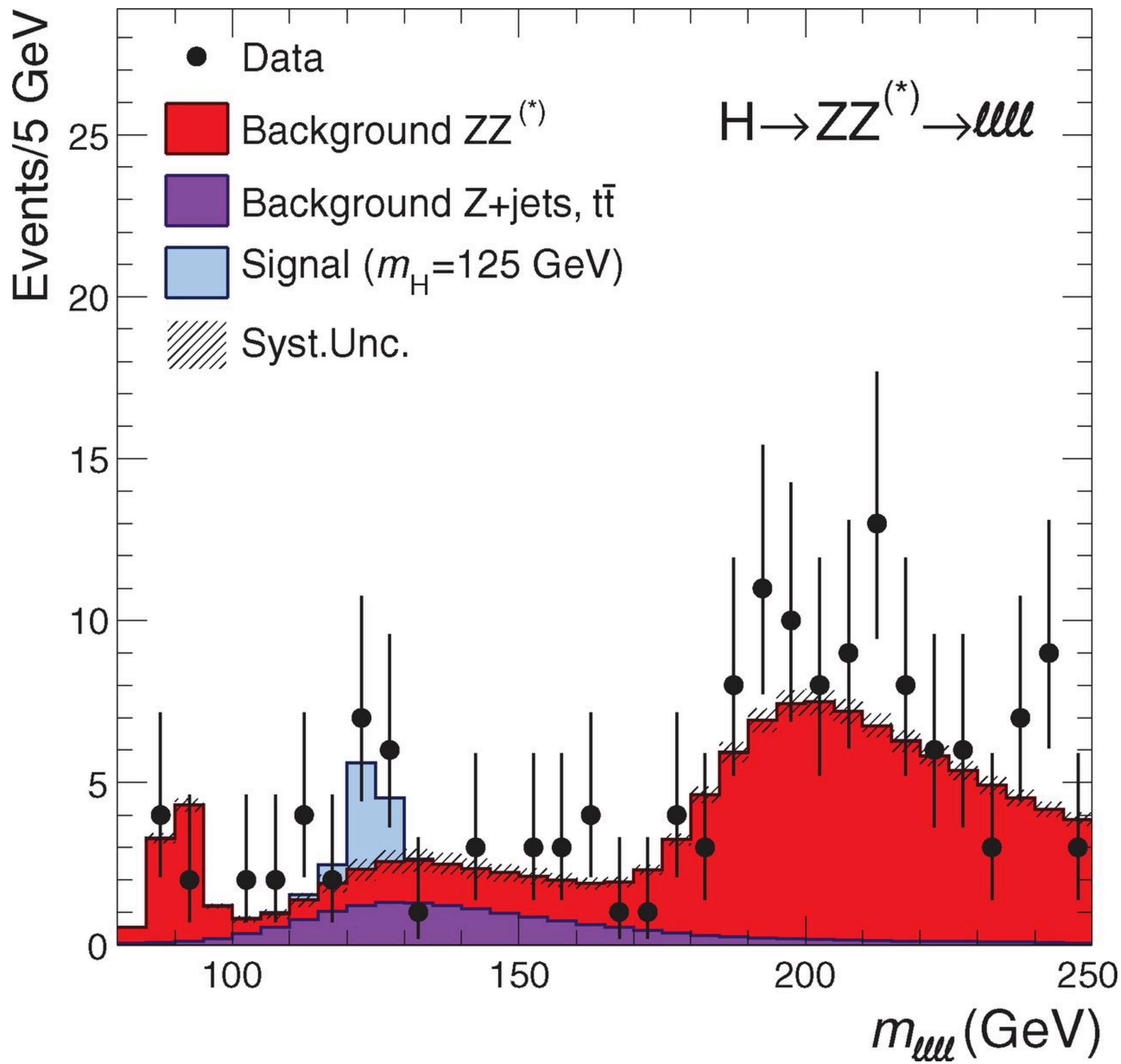
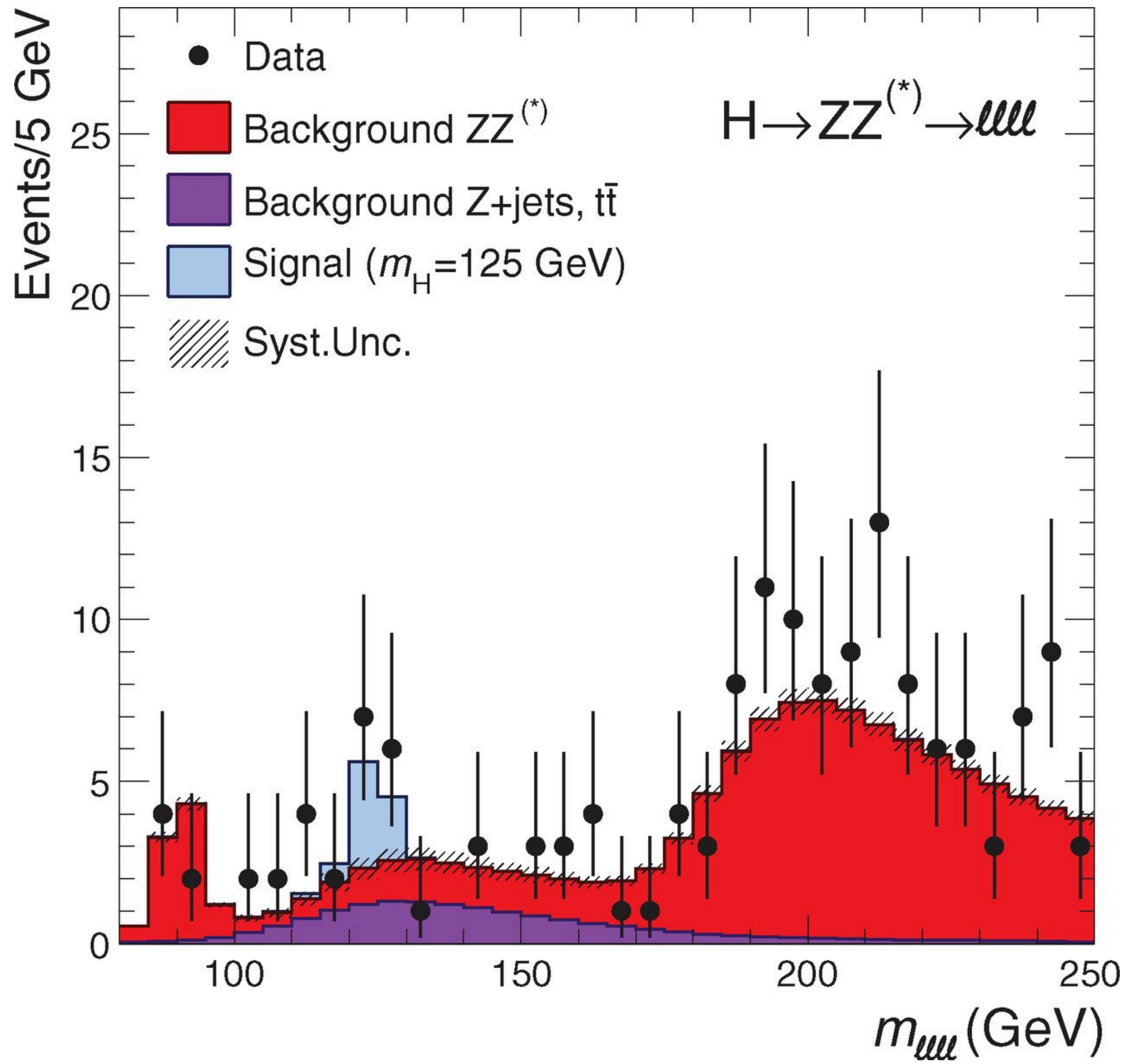
PHYS 391
Day 15

Week	Tu	Th	F
8	HW4 due Fourier Series	Fourier Transform	Lab 4 time
9	Sampling Theory Lab 4 due Wednesday	No class	No labs
10	Fast Fourier Transform	Windowing HW5 due	Lab 5 time
Finals			Lab 5 due

ATLAS Higgs Discovery



ATLAS Higgs Discovery



Observe: 7+6 = 13 events
Expect: ~4 (background-only)

$P_{4.0}(\geq 13)$ ~ 0.03%
$\Longrightarrow$ 3.5 sigma excess

This is a local significance, in null hypothesis could have occurred at any mass - how to define look-elsewhere range?

Fourier Transforms

Fourier Transform Notes:
https://pages.uoregon.edu/torrence/391/fftnotes.pdf

Have you ever discussed Fourier Series or Fourier Transform in another class?

PHYS 391 – Fourier Transform Primer

Eric Torrence with modifications by Dean Livelybrooks

1 Introduction

This document is intended to give you just enough information to follow the discussion of the experimental aspects of the discrete Fourier transform and related operations like the power spectrum. This will not be a mathematically rigorous treatment of the Fourier series or integral Fourier transform. A better description can be found in any decent textbook on mathematical methods for physics.

The key concepts for this class are the experimental aspects related to the Nyquist criterion (sampling rates, etc.), aliasing, the need for windowing your data, and the relationship between the amplitude or power spectrum and the raw Fourier transform. Without some mathematical foundation, however, these concepts will be meaningless.

Parts of this write-up were derived from "Mathematical Methods for Physicists" by Arfken, parts of the experimental description were motivated by a similar treatment in "Advanced LabVIEW Labs" by Essick, and parts came directly from Wikipedia.

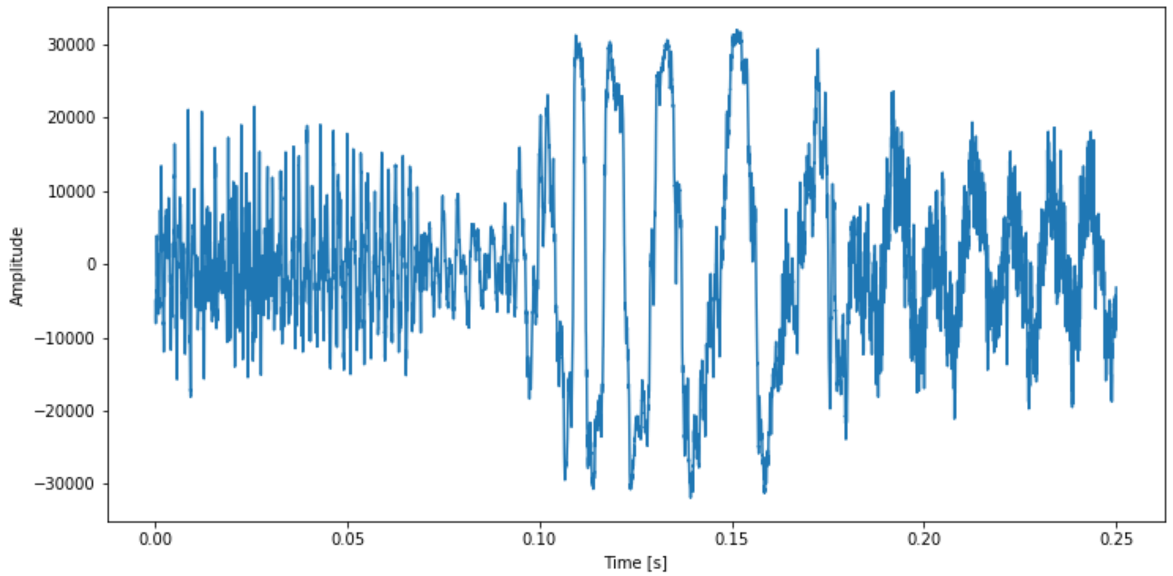
2 Fourier Series

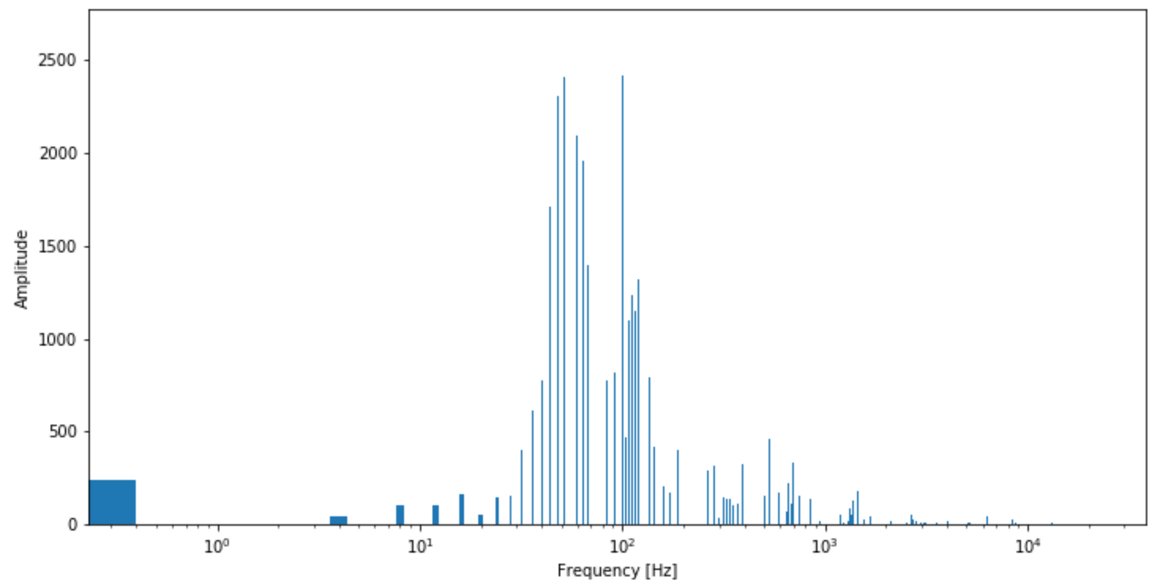
In 1807, Baron Jean Baptiste Joseph Fourier asserted that any arbitrary function defined over an interval $(-\pi, +\pi)$ could be represented by the series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \tag{1}$$

Big Idea

How do we turn a time series into an amplitude spectrum?
… and what does it mean?





Time spectrum **Frequency spectrum**

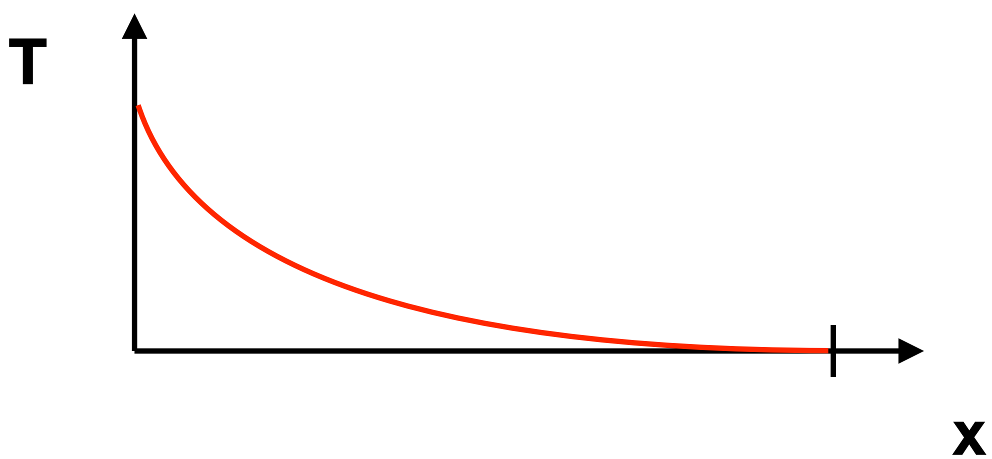
From: https://www.ritchievink.com/blog/2017/04/23/understanding-the-fourier-transform-by-example/

(includes python examples!)

Joseph Fourier

Studying heat flow in 1820s

e.g. temperature along a rod



Any function f(x) over an interval, can be written as a series sum of sine and cosine functions

Works for continuous or discontinuous functions…

Fourier Series

For function f(x) over [-π, +π]:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

where:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(nx) dx$$

Fourier Series

For function f(x) over [-π, +π]:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

Const. **Even** **Odd**

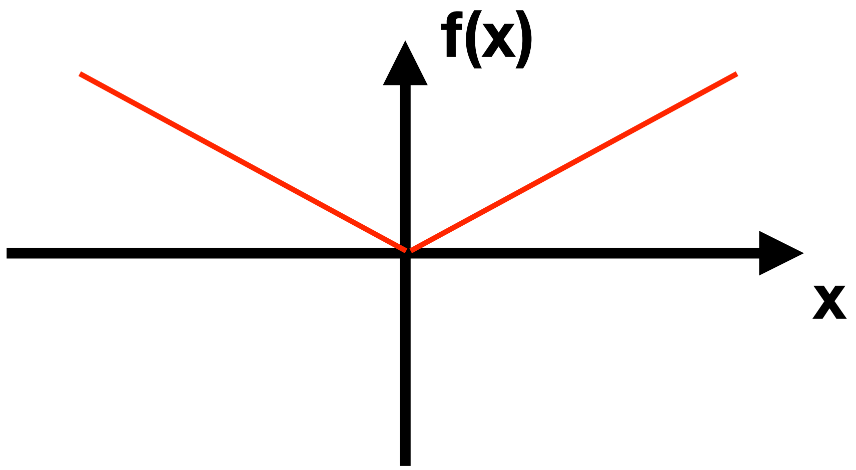
where:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos(nx) dx$$

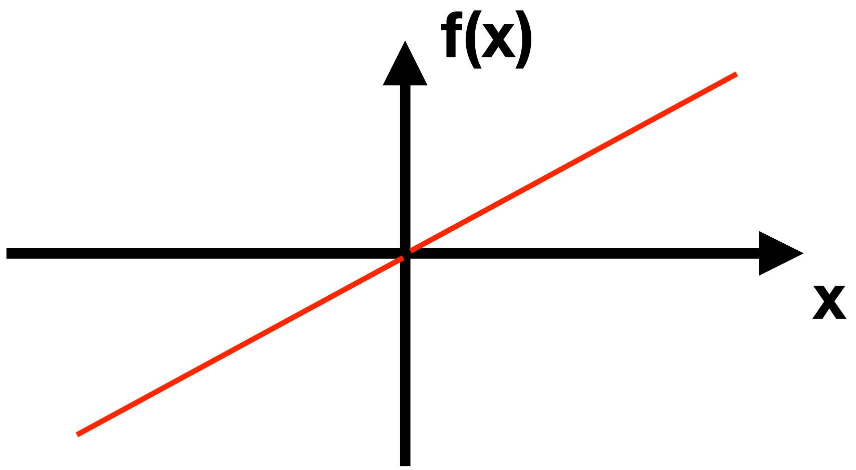
$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(nx) dx$$

Even and Odd

- Even function: f(x) = f(-x)

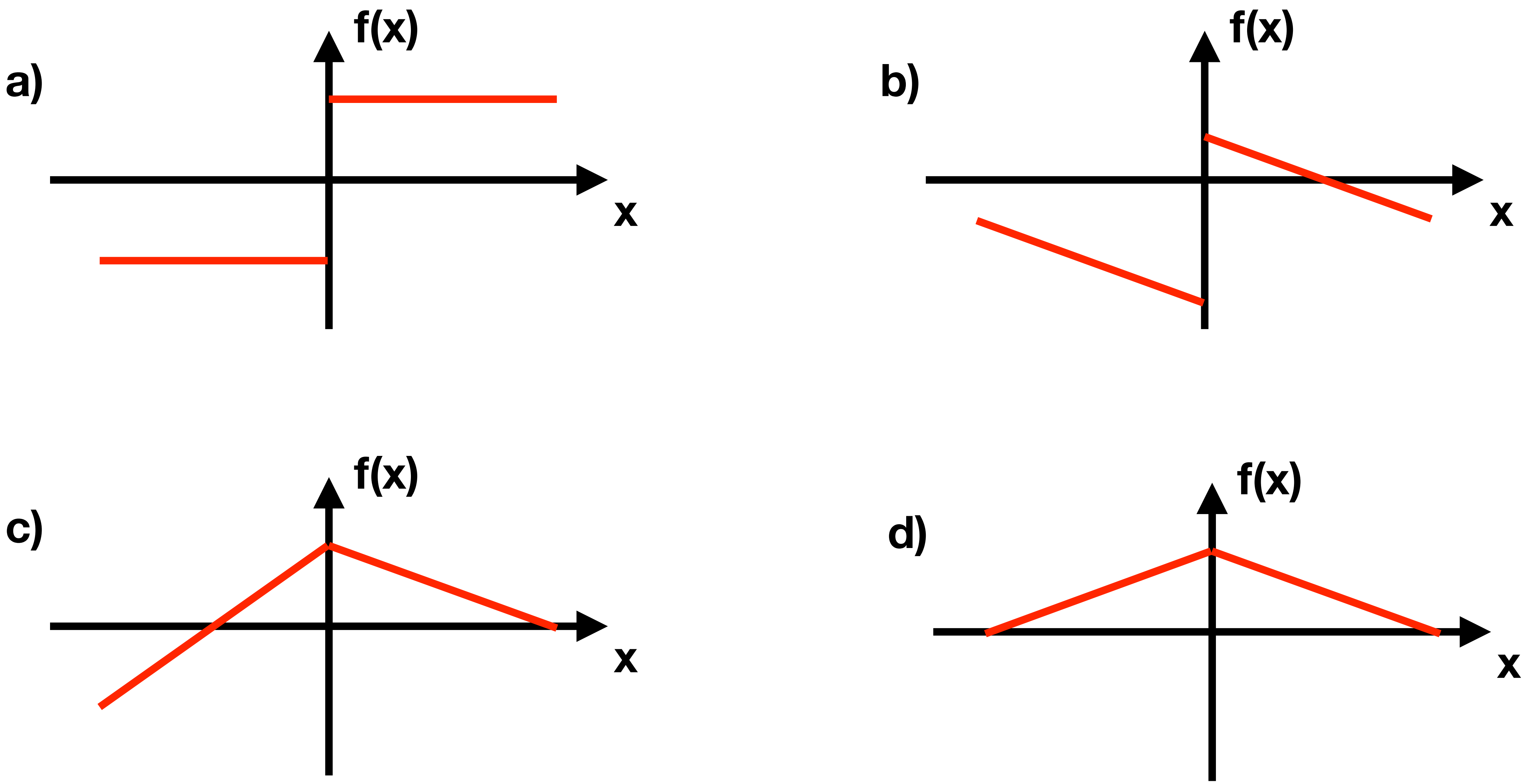


- Odd function: f(x) = -f(-x)



Most functions are neither purely even nor purely odd

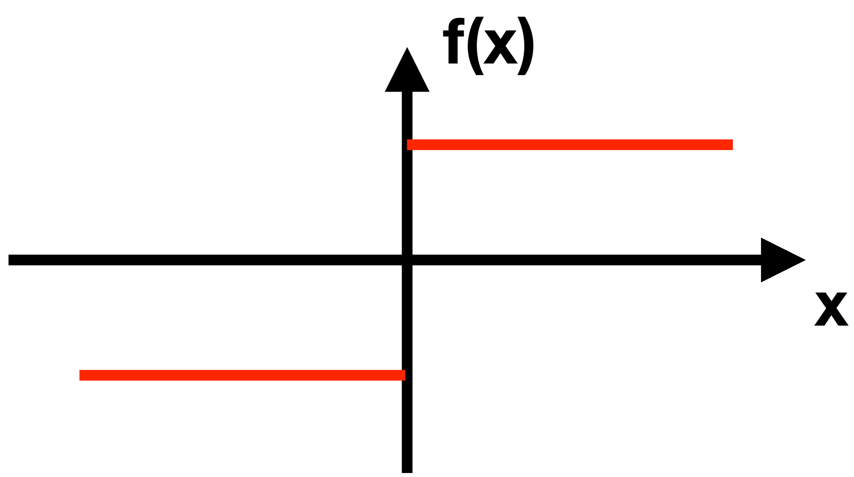
Even/Odd quiz



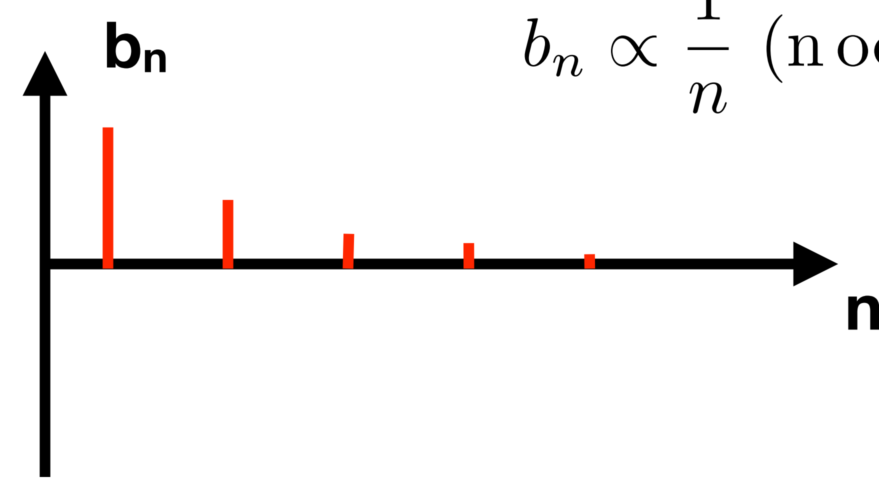
Which are even or odd functions?
Which have even or odd Fourier components?

Big Idea

- “Sharp features” need high-frequency components



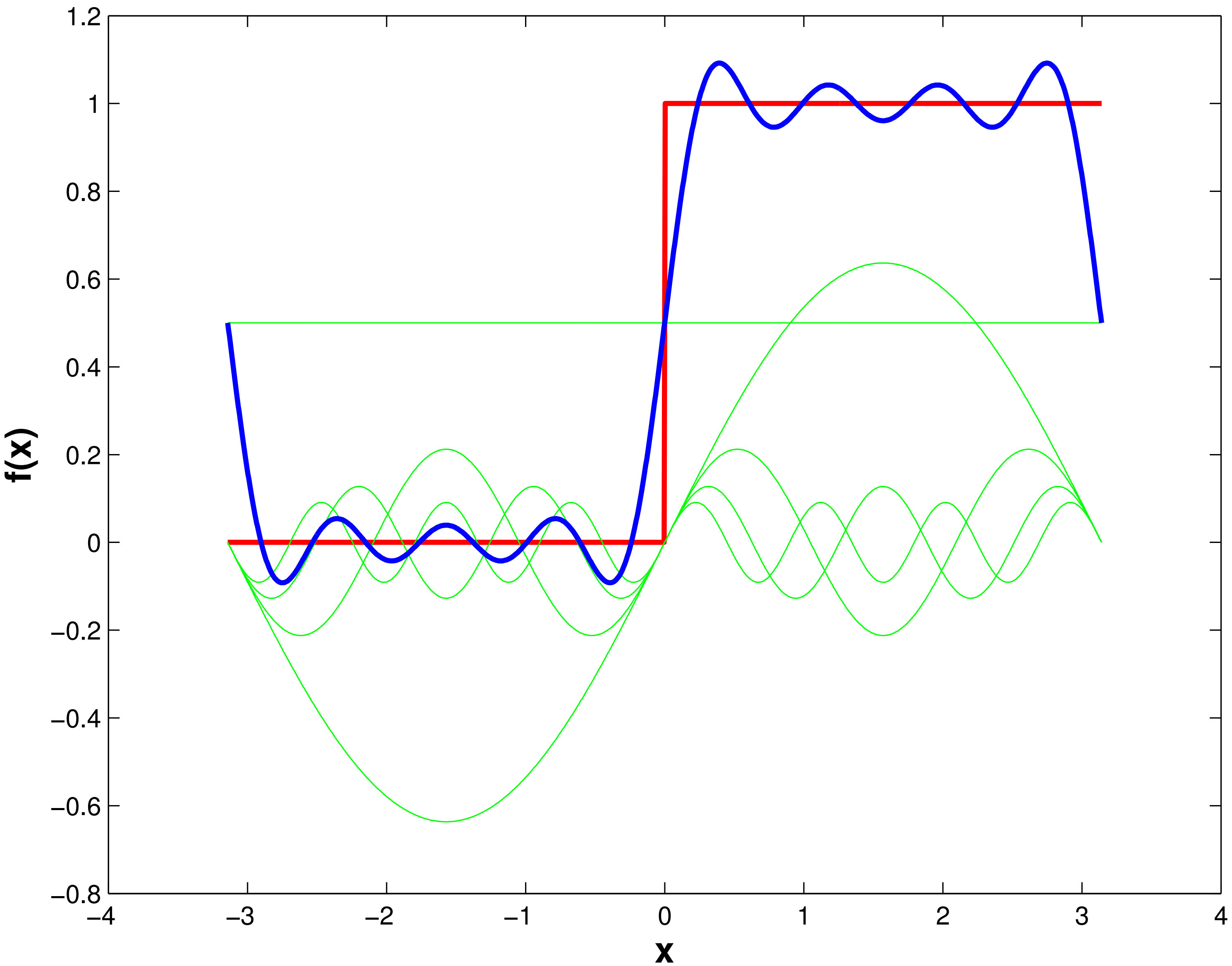
$\Rightarrow$



$$b_n \propto \frac{1}{n} \; (\mathrm{n\,odd})$$

- Removing those high-frequency components will “round off” the sharp features

Example



Step function (red) and Fourier Series (blue)
with first 4 non-zero terms (green)

Cool Resources

Interactive demo of Fourier Series:

http://www.jezzamon.com/fourier/

3 Blue 1 Brown Videos:

Fourier Series: **https://www.youtube.com/watch?v=r6sGWTCMz2k&vl=en**

Fourier Transform: **https://www.youtube.com/watch?v=spUNpyF58BY**

Complex Representation

- Using the Euler relationship:

$$e^{ix} = \cos x + i \sin x$$

- Can re-write Fourier Series as

$$f(x) = \sum_{n=-\infty}^{+\infty} c_n \, e^{inx}$$

where

$$c_n = \begin{cases} \frac{1}{2}(a_n - ib_n), & n > 0, \\ \frac{1}{2}(a_n + ib_n), & n < 0, \end{cases}$$